Math 105 Fall 2009

Final exam sample questions, from old tests, etc.

A note on this exam: it will be comprehensive and will require you to solve problems using information from the entire course. The questions may require you to use what you learned in several different contexts to solve a problem. Among the skills I expect you to be able to draw on in any context are:

1. Write the equation of a line.
2. Use laws of exponents correctly in any context.
3. Factor by factoring out greatest common factors, even if the common factor is a polynomial, exponential, log, or trig function.
4. Solve any quadratic, determining whether it has real roots and if so, whether the roots are rational or irrational.
5. Use the technique of completing the square.
6. Find function inverses, including function inverses of log and exponential functions

So here are some typical questions:

1. Find all real solutions to the following, using any mathematically correct technique.
   a) $y^2(y-7)-9(y-7)=0$
   b) $x^2+3x=-1$
2. Find all solutions to the following inequalities. You may express your solutions in interval or in set notation.
   a) $|3x+4|<7$
   b) $2x^2 \geq 5x$
3. Find an equation of the line that satisfies the following conditions. Express your answers in slope-intercept form.
   a) Through the two points (-2, 3) and (1, -9)
   b) Through the point (5, 2) and parallel to the line $2x+5y=3$.
4. Find the distance between the points (-2, 3) and (1, -9) and the equation of the circle through the point (-2,3) with center at (1, -9).
5. Let $f(x)=\sqrt[3]{x+1}$ and $g(x)=e^{2x}$.
   a) What is the domain of $f(x)$? _____
   b) What is the domain of $g(x)$? _____
   c) Find the following functions and their domains:
      i) $f \circ g$
      ii) $g \circ f$

6. Let $f(x)=\sqrt{\frac{x}{4}+2}$, $x \geq -8$.

   a) Why is there the restriction on the domain of f?

   b) Find the function f^{-1} and its domain.

   c) How does the graph of $f(x)=\sqrt{\frac{x}{4}+2}$ compare to the graph of $g(x)=\sqrt{x}$ in terms of function transformations?

7. Let $g(x)=3+e^{2x}$.

   a) What is the domain of *g*?

   b) What is the range of *g*?

   c) How does the graph of $g(x)=3+e^{2x}$ compare to the graph of $f(x)=e^{x}$ in terms of function transformations?

   d) Find the function inverse f^{-1}. Show that f and the function you found are indeed inverses by using function composition.

8. Let $f(x)=\frac{4}{x}$.

   a) Find the average rate of change between $x=1$ and $x=2$.

   b) Find the average rate of change between $x=a$ and $x=a+h$.

9. Let $f(x)=\begin{cases} -x^2+2, \text{ if } x \leq 0 \\ x-1, \text{ if } x>0 \end{cases}$

   a) Find the following, showing all work:
      i) $f(2)$

      ii) $f(0)$

      iii) $f(-1)$

   b) Graph the piecewise defined function $f(x)$ on a domain of at least [-3…3]

10. Let $f(x)=2x^2-x$. Is *f* an even function, an odd function, or neither? Justify your answer.

11. Find functions f and g such that the function $F(x)=\sqrt[5]{(x-4)}$ can be expressed in the form $f \circ g$.

12. Let $f(x) = x^2 - 8x + 10$.

    a) Express $f(x)$ in standard form.

    b) Find the vertex (both coordinates).

    c) What is the *y*-intercept?

    d) What is/are the *x*-intercepts, if any?

13. Consider the polynomial: $P(x) = x^3 + 2x^2 - 5x + 2.$

    a) Use any method to show that $x = 1$ is a zero (root) of $P(x)$.

    b) Find all zeros of the polynomial $P(x)$.

14. Let $P(x) = x^4 - x^3 - 6x^2$

    a) Find all real zeros of $P(x)$. _____

    b) Sketch the graph of $P(x)$, accurately showing all intercepts and long term behavior.

15. Find the quotient and remainder: $\dfrac{9x^3 - 3x + 1}{x^2 - x}$

16. Let $g(x) = \dfrac{12 + 3x^2}{x^2 - 2x - 3}$. Find the following:

    a) *x*-intercepts, if any _____

    b) *y*-intercept, if any _____

    c) Vertical asymptote(s), if any _____

    d) Horizontal asymptote, if any _____

    e) Graph the function $g(x) = \dfrac{12 + 3x^2}{x^2 - 2x - 3}$

17. Evaluate the following expressions, eliminating the logarithms. (Your answer in each case should be a number.)

    a) $\log 20 + \log 5 =$

    b) $4^{2\log_4 5} =$

    c) $6 \ln \sqrt[3]{e} =$

18. Rewrite the expression as a single logarithm: $\ln(x + 4) + \ln(x - 4) - 4\ln x =$

19. Solve for *x*.

    a) $5^{3x-7} - 7 = 18$

b) $1 = \log 4 - \log(x+1)$

c) $\ln x^2 = \ln 2 + \ln x$

20. True/False. Determine whether each of the following statements is true or false. Show work to justify your answer.

a) _____ $e^{\ln x - \ln y} = x - y$

b) _____ $\log(2000x) = 3 + \log x + \log 2$

c) _____ If $\log y = 3 + 2x$ then $y = (1000)\cdot(100^x)$

d) _____ If $y = e^x \cdot e^2$, then $\ln y = 2x$.

21. Consider the angle $-\frac{5\pi}{4}$.

a) Express $-\frac{5\pi}{4}$ radians as degrees:_____

b) What is the reference angle for this angle (in radians)? _____

c) In what quadrant does the terminal side of this angle lie? _____

d) What is the exact value of $\sin -\frac{5\pi}{4}$? _____

What is the exact value of $\tan -\frac{5\pi}{4}$? _____

22. Consider the angle $250°$.

a) Express $250°$ as radians:_____

b) What is the reference angle for this angle (in radians or degrees)? _____

c) In what quadrant does the terminal side of this angle lie? _____

23. Find the exact values of the following functions of the angle θ, when $\cos\theta = \frac{7}{8}$, and $\sin\theta < 0$. First figure out in which quadrant the terminal side of θ must lie:
Quadrant _____. a) $\sin\theta =$ _____; b) $\tan\theta =$ _____;

c) $\sec\theta =$ _____; d) $\csc\theta =$ _____; e) $\cot\theta =$ _____

24. Find the exact value for the following:

a) $\cos -\frac{14\pi}{3} =$ _____

b) $\tan(135°) =$ _____

25. Evaluate the expressions: $\sin^2(-150°) + \cos^2(210°) =$ and $4\tan^2\frac{\pi}{5} - 4\sec^2\frac{\pi}{5}$.

26. Find the radius of the circle if an arc of length 15 m on the circle subtends a central angle of $30°$.(Hint: $S = r\theta$) _____