Math 105, Sample questions from old tests
Chapter 2
Fall 2009

1. Consider the piecewise defined function below:

$$f(x)=\begin{cases} x^2, & if \ \ x>2 \\ x-1, & if \ \ x<2 \\ 3, & if \ \ x=2 \end{cases}$$

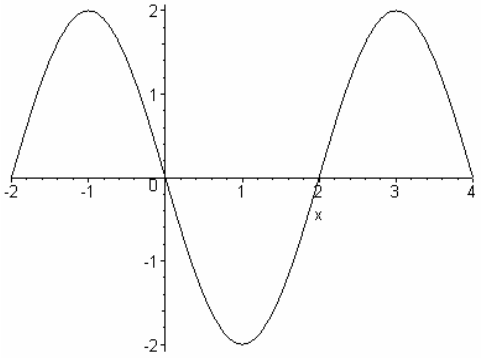
   a) Find $f(3),\ f(2),\ f(-1)$. Be sure to show all work.

   b) Sketch the graph of $f(x)$.

2. Let $h(x)=x^2+4$. What is the domain of $h(x)$?
   a) Find the average rate of change of $h(x)$ between the values $x=1$ and $x=3$.
   b) Find the average rate of change of $h(x)$ between the values $x=a$ and $x=a+h$.
   c) Find $f(x)=h(\sqrt{x})$ and $g(x)=\sqrt{h(x)}$.
   d) What is the domain of $f(x)$? What is the domain of $g(x)$?

3. Determine the average rate of change of the function $g(x)=\dfrac{1}{x+1}$ between the values $x=0$ and $x=h$. Simplify your answer as much as legally possible.

4. Consider the graph below:



   a) Does it represent the graph of a function?_____ Why or why not?

   b) In interval notation, state the domain and range of this graph:
   Domain:_____ Range:_____

   c) Over what interval(s) is the graph increasing?

5. Page 167, Problem 25, information from a graph. In addition to questions in the book answer:
   a) What is the domain of *f*?

   b) What is the range of *f*?

   c) What is the domain of *g*?

   d) What is the range of *g*?

6. Page 191, Problems 20 and 21, function transformations from a graph.

7. Consider the function $f(x) = \frac{1}{x}$.

   a) Sketch the graph of $f(x)$ on the interval $-4 \le x \le 4$, by plotting points. Be sure to label your axes correctly with your scale.

   b) Then sketch the following functions, not by plotting points but by starting with the graph of $f(x) = \frac{1}{x}$ above. Be sure to label your graphs clearly.

      i) $y = \dfrac{2}{x}$

      ii) $y = \dfrac{1}{x-2}$

      iii) $y = 2 - \dfrac{1}{x+1}$

8. Let $f(x) = -2x^2 + 8x - 6$.
   a) Express the quadratic function $f(x)$ in standard form.

   b) Find its vertex.

   c) Find its *x*- and *y*-intercepts.

   d) Find the function's maximum or minimum value, and identify it as a maximum or minimum. How do you know whether it is a maximum or a minimum?

   e) Graph the function, clearly showing all information you found above.

9. Let $f(x)=\frac{1}{x}$ and let $g(x)=\sqrt{4+x}$. Find the following functions and the domain of each:

   a) $\frac{f(x)}{g(x)}=$ Domain: _____

   b) $f(x)-g(x)=$ Domain: _____

   c) $f(g(x))=$ Domain: _____

   d) $f(f(x))=$ Domain: _____

   e) $g(f(x))=$ Domain: _____

10. Find the functions f and g, such that the function $F(x)=(x+7)^4$ can be expressed in the form $f \circ g$.

11. Let $f(x)=\frac{2}{x+4}$ and let $g(x)=1+\sqrt{x+3}$. Answer the following questions for both functions.

    a) What is the domain of f? What is the domain of g?

    b) Are the functions one-to-one? Briefly justify your answers (a graph using what you know of function transformations is fine.)

    c) Find the inverse function of $f(x)$ and $g(x)$.

    d) Use the Inverse Function Property to show that $f(x)$ and the function you found are inverses of each other. Do the same for $g(x)$.

    e) What is the domain of f^{-1}? What is the range of f^{-1}? What is the domain of g^{-1}? What is the range of g?