

22 points

Math 105, Precalculus

Name ANSWER KEY

Homework 2, Sections 2.3 & 2.4

Due by 1:00 pm, October 8, 2009

Show all work neatly. No work means no credit.

1. Let $f(x) = (x-1)^2 + 1$.

2 pts.

a) Find $f(a)$. $f(a) = (a-1)^2 + 1 = a^2 - 2a + 1 + 1 = a^2 - 2a + 2$

2 pts.

b) Find $f(a+h)$. $f(a+h) = (a+h-1)^2 + 1 = (a+h-1)(a+h-1) + 1 = a^2 + 2ah + h^2 - 2a - 2h + 2$

3 pts.

c) Find and reduce the difference quotient $\frac{f(a+h) - f(a)}{h}$, where $h \neq 0$.

(using answers to parts a) & b) above: $\frac{f(a+h) - f(a)}{h} = \frac{a^2 + 2ah + h^2 - 2a - 2h + 2 - (a^2 - 2a + 2)}{h} = \frac{2ah + h^2 - 2h}{h} = \frac{h(2a + h - 2)}{h} = 2a + h - 2$

d) Find the average rate of change of $f(x)$ between $x=0$ and $x=h$.

$\frac{f(0) - f(h)}{0 - h} = \frac{(-1)^2 + 1 - [(h-1)^2 + 1]}{-h} = \frac{1 + 1 - (h^2 - 2h + 1 + 1)}{-h} = \frac{2 - h^2 + 2h - 2}{-h} = \frac{-h^2 + 2h}{-h} = h - 2$

3 points. get 1 for set-up, -1 for subtracting terms, -1 for cancelling terms

$\frac{h(-h+2)}{-h} = (-1)(-h+2) = h-2$

NOTE: using result in part c), we can let $a=0$ and $a+h = 0+h = h$. then $\frac{f(a+h) - f(a)}{h} = \frac{f(0+h) - f(0)}{h} = \frac{2 \cdot 0 + h - 2}{h} = \frac{h-2}{h} \cdot h = h-2$

2. Let the function f be $f(x) = \sqrt[3]{x}$. If the following transformations are applied to its graph (in the given order), write the equation for the final transformed graph: shift 3 units to the right, stretch vertically by a factor of 5, reflect in the x-axis.

I. becomes $f(x-3)$

II. becomes $5 \cdot f(x-3)$

III. becomes $-5 \cdot f(x-3)$

$g(x) = -5\sqrt[3]{x-3}$

3 pts. (1 for each transformation)

horiz. shift

vert. stretch

$$= 4x^2 - 3$$

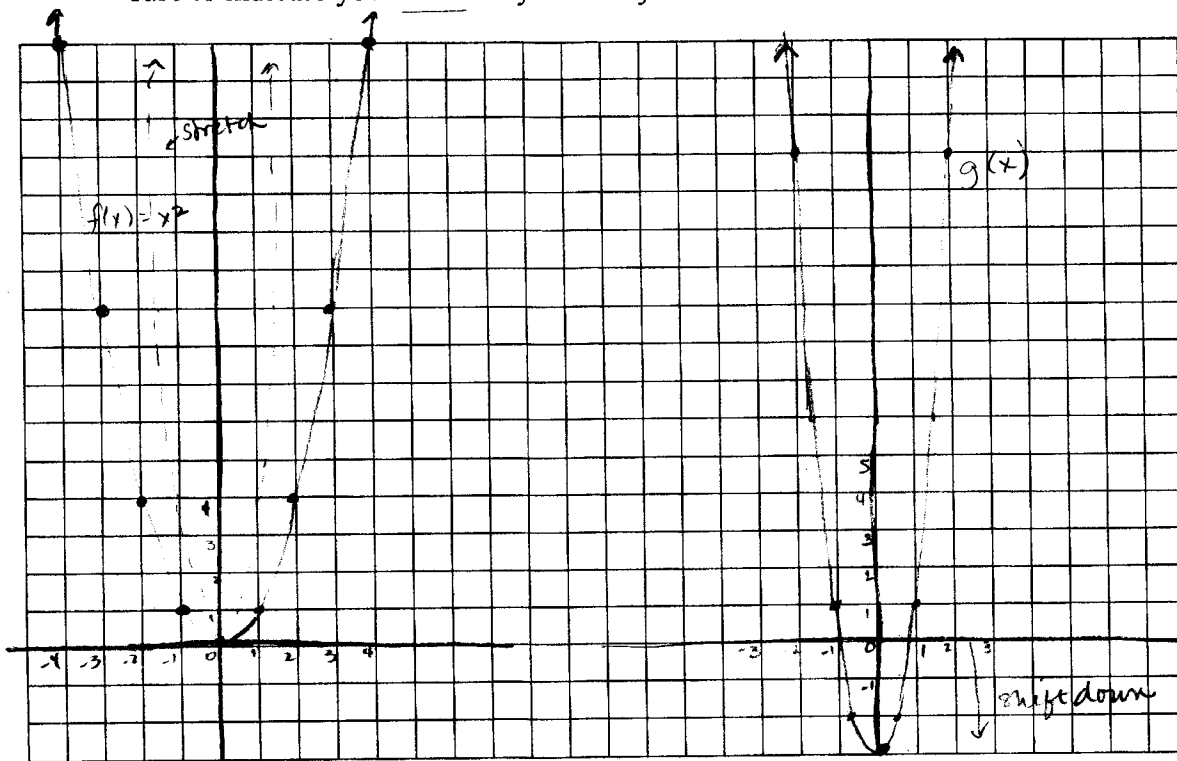
3. Let $g(x) = (2x)^2 - 3$.

a) Explain how the graph of this function is obtained from the graph of $f(x) = x^2$ by listing the function transformations in order. Basic graph: $f(x) = x^2$

2 pts.
(1 each -
-1 if quantity or
vert. stretch is wrong)

- ① horizontal stretch by factor of $1/2$ (or vertical stretch by factor of 4)
- ② Shift down by 3.

b) Graph the function $g(x) = (2x)^2 - 3$, not by plotting points but by first graphing the function $f(x) = x^2$ and then by applying the function transformations. Be sure to indicate your scale very carefully.



3 pts.

4. Determine whether the function $f(x) = \frac{x^2 + 1}{x^3 - 2x}$ is even, odd or neither. Show all work in making your determination.

1 pt.

① find $f(-x) = \frac{(-x)^2 + 1}{(-x)^3 - 2(-x)} = \frac{x^2 + 1}{-x^3 + 2x}$. Compare: does $f(x) = f(-x)$? No \therefore not even.

$$\frac{x^2 + 1}{x^3 - 2x} \neq \frac{x^2 + 1}{-x^3 + 2x} \text{ (not even)}$$

does $f(-x) = -f(x)$?

$$-f(x) = \frac{(-1)(x^2 + 1)}{x^3 - 2x} = \frac{x^2 + 1}{-1(x^3 - 2x)} = \frac{x^2 + 1}{-x^3 + 2x} = f(-x) \checkmark$$

$\therefore f(x)$ is an odd function.

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