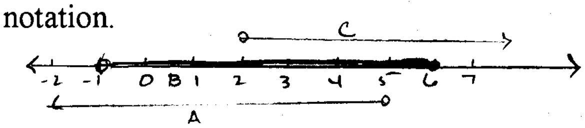
ANSWER KEY (corrected)

Math 105, Test 1
Problems from old tests
Fall 2009

1. If $A = \{x \mid x < 5\}$, $B = \{x \mid -1 < x \le 6\}$ and $C = \{x \mid x > 2\}$, find the indicated sets. You may express your answer in either set or interval notation.



   a) $A \cup B = \{x \mid x \le 6\}$ or $(-\infty, 6]$

   b) $A \cap B$ $\{x \mid -1 < x < 5\}$ or $(-1, 5)$

   c) $A \cup C = \mathbb{R}$ or $(-\infty, \infty)$

   d) $B \cap C = \{x \mid 2 < x \le 6\}$ or $(2, 6]$.

2. Factor the following completely:

   a) $8x^3 - 32xy^2 = 8x(\underbrace{x^2 - 4y^2}_{\text{Difference of 2 squares}}) = \boxed{8x(x-2y)(x+2y)}$

   b) $x^3 - 2x^2 - 8x = x(x^2 - 2x - 8) = \boxed{x(x-4)(x+2)}$

3. Find all real solutions to the following. Use the technique of completing the square at least once.

   (complete sq.)

   a) $x^2 + 2x - 4 = 0$; $x^2 + 2x = 4$; $x^2 + 2x + 1 = 5$; $(x+1)^2 = 5$; $x + 1 = \pm 5$; $\boxed{x = -1 \pm \sqrt{5}}$

   OR $x = \dfrac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2} = \dfrac{-2 \pm \sqrt{4+16}}{2} = \dfrac{-2 \pm \sqrt{20}}{2} = \dfrac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$

   b) $2x^2 + x = 3$

   $2x^2 + x - 3 = 0$ $(2x+3)(x-1) = 0$ Let: $2x + 3 = 0$; $x - 1 = 0$; $2x = -3$; $\boxed{x = -3/2 \text{ or } x = 1}$

   c) $x^2 - 2x + 5 = 0$ $x^2 - 2x = -5$; $x^2 - 2x + 1 = -4$ DONE: NO Real roots, OR

   $b^2 - 4ac = (-4)^2 - 4(1)(5) = 16 - 20 = -4$ (discriminant is negative: no real roots.)

   d) $\dfrac{1}{x+1} - \dfrac{2}{x^2} = 0$ $\dfrac{x^2}{x^2(x+1)} - \dfrac{2(x+1)}{x^2(x+1)} = 0$; $x^2 - 2x - 2 = 0$ $x = \dfrac{+2 \pm \sqrt{4 - 4(1)(-2)}}{2} = \dfrac{2 \pm \sqrt{12}}{2} = \dfrac{2 \pm 2\sqrt{3}}{2} = \boxed{1 \pm \sqrt{3}}$

   e) $|x - 3| = 5$ $x - 3 = 5$ $\boxed{x = 8}$; $x - 3 = -5$ $\boxed{x = -2}$

4. Solve the inequalities. Express your answer in set or interval form.

   a) $\dfrac{4}{x} \le -1$; $\dfrac{4}{x} + 1 \le 0$ $\dfrac{4}{x} + \dfrac{x}{x} = \dfrac{4+x}{x} \le 0$ Let $4 + x = 0$ $x = -4$ $x = 0$

   (Note: x cannot $= 0$), so $\boxed{\{x \mid -4 \le x < 0\} \text{ or } [-4, 0)}$

   b) $|x + 4| > 2$ split: $x + 4 > 2$; $x > -2$; $x + 4 < -2$ $x < -6$. Solution: $\{x \mid x < -6 \text{ or } x > 2\}$ or $(-\infty, -6) \cup (2, +\infty)$

   c) $2x^2 + x \ge 1$ $2x^2 + x - 1 \ge 0$; $(2x-1)(x+1) \ge 0$

   Let $2x - 1 = 0$ $x = 1/2$ $x + 1 = 0$ $x = -1$

   so $\boxed{\{x \mid x \le -1 \text{ or } x \ge 1/2\} \text{ or } (-\infty, -1] \cup [1/2, +\infty)}$

   d) $1 \le 4 - 3x \le 16$

   $-4 \quad -4 \quad -4$

   $-3 \le -3x \le 12$

   Divide by -3; reverse ineq.

   $1 \ge x \ge -4$ (or $-4 \le x \le 1$) or $[-4, 1]$

5. Consider the points $P(5,-1)$ and $Q(3,7)$.

a) What is the slope of the line through the two points? $\frac{7-(-1)}{3-5} = \frac{8}{-2} = \boxed{-4 = m}$

b) What is the equation of the line through the two points, in slope-intercept form?

$7 = -4(3)+b = -12+b$; $b = 19$; so $\boxed{y = -4x+19}$

c) What is the equation of the line **parallel** to the line you found in part b), through the point (0,2)? $\boxed{y = -4x+2.}$

y-intercept

d) What is the equation of the line **perpendicular** to the line you found in part b), through the point (1,1)? $m_\perp = \frac{1}{4}$; $1 = 1/4(1)+b = 1/4+b$; $b = 3/4$. $\boxed{\text{so } y = 1/4\,x + 3/4}$

e) Is the point (-2, 27) on the line you found above? Is the point (1, 16) on the line? Justify your answers.

in part b

Check: $27 \overset{?}{=} (-4)(-2)+19 = 8+19 = 27$ ✓ $\boxed{(-2,27) \text{ is on the line}}$ $16 \overset{?}{=} -4(1)+19 = -4+19 = 15$ $\boxed{\text{No; } (1,16) \text{ is not}}$.

f) What is the distance between the two points? $D = \sqrt{(5-3)^2+(-1-7)^2} = \sqrt{4+64} = \boxed{\sqrt{68}}$

g) What is the equation of the circle, centered at $P(5,-1)$, passing through $Q(3,7)$?

$(x-5)^2+(y-(-1))^2 = \boxed{(x-5)^2+(y+1)^2 = (\sqrt{68})^2 = 68}$

6. Which of the points $C(-4,3)$ or $D(3,1)$ is closer to the point $E(-1,2)$?

$\sqrt{(-4+1)^2+(3-2)^2} = \sqrt{(-3)^2+1^2} = \sqrt{10}$ vs $\sqrt{(3-(-1))^2+(1-2)^2} = \sqrt{16+1} = \sqrt{17}$; $\boxed{(-4,3) \text{ is the closer point}}$

7. Determine whether the equation $x^2+y^2+2x-4y-4=0$ represents a circle, a point, or has no graph. If the equation is that of a circle, find its radius and center.

$x^2+2x+1+y^2-4y+4 = 4+4+1$; $(x+1)^2+(y-2)^2 = 9$; so center is $(-1,2)$. Radius $= \sqrt{9} = 3$.

8. A geologist uses a probe to measure the temperature T (in °C) of the soil at various depths below the surface, and finds that at a depth of x centimeters, the temperature is given by the equation $T = 0.08x - 4$.

a) True or false: The equation that expresses the temperature of the soil is a quadratic equation. (It is a linear equation.)

b) What is the temperature of the soil at a depth of 1 meter (100 centimeters)? $T = 0.08(100)-4 = 8-4 = 4°C$.

c) What do the slope, the x-intercept, and the T-intercept of the graph of this equation represent? Slope = 0.08 represents the fact that the temperature of soil increases by 0.08°C for each additional cm. of depth. y-int: at the surface the temp is −4°C. The soil reaches 0°C at the x-intercept (= 50 cm. deep.)

9. Find the domain of the functions:

a) $f(x) = \sqrt{x^2-16}$

Let $x^2-16 \ge 0$ & solve

$(x+4)(x-4) \ge 0$

$x=-4$ $x=4$

-5 0 5

$x+4$: − + +

$x-4$: − − +

+ −4 − 4 + ← Need +'s; $\boxed{\text{so domain is } \{x \mid x \le -4 \text{ or } x \ge 4\} \text{ or } (-\infty,-4] \cup [4,\infty)}$

b) $g(x) = \dfrac{x}{x^2-5x-6}$ Let $x^2-5x-6=0$

$(x-6)(x+1)=0$ $x=6$, $x=-1$ are excluded values.

$\boxed{\text{Domain: } \{x \mid x \ne 6,\ x \ne -1\}}$

10. Let $g(x) = x^2+4$. Evaluate $g(x)$ at the indicated values:

a) $g(-2) = (-2)^2+4 = 4+4 = 8$

b) $g(1) = 1^2+4 = 5$

c) $g(a) = a^2+4$

d) $g(a+h) = (a+h)^2+4 = a^2+2ah+h^2+4$

e) $g(\frac{1}{x}) = (\frac{1}{x})^2+4 = \boxed{\frac{1}{x^2}+4}$

11. Consider the piecewise defined function below:

$$f(x) = \begin{cases} x^2, & \text{if } x > 2 \\ x - 1, & \text{if } x < 2 \\ 3, & \text{if } x = 2 \end{cases}$$

Find $f(3), f(2), f(-1)$. Be sure to show all work.

$f(3) = 3^2 = 9$

$f(2) = 3$

$f(-1) = -1 - 1 = -2$