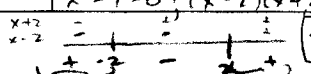
Math 105 Precalculus
Quiz 5, Section 2.7

Name ANSWER KEY A
October 22, 2009

- For problems 1 and 2 record your answers **neatly** in the tables provided.
- Use of calculators is not permitted on this quiz.
- You do not need to simplify your answers.

Domain of f: $\mathbb{R}$
Domain of g: $x \geq 0$

1. Let $f(x) = x^2 - 4$ and let $g(x) = \sqrt{x}$. Find the following combined functions and their domains:

	Function	Domain
$\frac{g}{f}$	$\frac{\sqrt{x}}{x^2-4} = \frac{g}{f}$	(issue: x^2-4 may not $= 0$. Let $x^2-4=0$, $x = \pm 2$) Final domain $\{x \mid x \geq 0, x \neq 2\}$ (from f and from denominator)
$f \circ g$	$f(\sqrt{x}) = (\sqrt{x})^2 + 4 = f \circ g$	Domain: $\{x \mid x \geq 0\}$ Reason: domain of composed function may be no larger than domain of input.
$g \circ f$	$g(x^2-4) = \sqrt{x^2-4} = g \circ f$	Restriction: $x^2-4 \geq 0$. Solve the inequality: $x^2-4=0$; $(x-2)(x+2)=0$; $x=2$, $x=-2$  Domain: $\{x \mid x \leq -2 \text{ or } x \geq 2\}$

Domain of f: $x \neq 0$
Domain of g: $x \neq -1$

2. Let $f(x) = \dfrac{x+2}{x}$ and let $g(x) = \dfrac{1}{2(x+1)}$. Find the following combined functions and their domains.

	Function	Domain
$f+g =$	$\frac{x+2}{x} + \frac{1}{2(x+1)}$	$\{x \mid x \neq 0, x \neq -1\}$
$f \cdot g =$	$\left(\frac{x+2}{x}\right)\left(\frac{1}{2(x+1)}\right)$	$\{x \mid x \neq 0, x \neq -1\}$
$f/g =$	$\frac{\frac{x+2}{x}}{\frac{1}{2(x+1)}}$ ← omit zero denom from non-simplified function	$\{x \mid x \neq 0, x \neq -1\}$ (same zero denominators)
$g \circ f =$	$g\left(\frac{x+2}{x}\right) = \frac{1}{2\left(\frac{x+2}{x}+1\right)} =$ (omit 0, for zero denom. here) (Let this $= 0$ → $\frac{x+2}{x}+1=0 = \frac{x+2+x}{x}=0$ $\frac{2x+2}{x}=0$, $x=-1$)	Domain: $\{x \mid x \neq 0, x \neq -1\}$
$f \circ f =$	$f\left(\frac{x+2}{x}\right) = \frac{\frac{x+2}{x}+2}{\frac{x+2}{x}}$ } omit zero denominators.	Domain: $\{x \mid x \neq 0, x \neq -2\}$

3. Find functions f and g such that the function $F(x) = \sqrt[3]{x+4}$ can be expressed in the form $f \circ g$.

Let $g = x+4$
$f = \sqrt[3]{x}$

then $f \circ g = f(x+4) = \sqrt[3]{x+4}$ ✓

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Domain: $x \geq 0$ Domain $\mathbb{R}$

1. Let $f(x) = \sqrt{x}$ and let $g(x) = x^2 - 9$. Find the following combined functions and their domains:

	Function	Domain
$\frac{f}{g}$	$\frac{\sqrt{x}}{x^2-9} = \frac{f}{g}$	$\{x \mid x \geq 0, x \neq 3\}$ $[0,3) \cup (3, \infty)$ Issue: domain may be no larger than $x \geq 0$ + omit zero denominators (Let $x^2-9=0$, $x = \pm 3$)
$f \circ g =$	$f(x^2-9) = \sqrt{x^2-9}$	$\{x \mid x \leq -3 \text{ or } x \geq 3\}$ $(-\infty, -3] \cup [3, \infty)$ (Solve inequality $x^2-9 \geq 0$)
$g \circ f =$	$g(\sqrt{x}) = (\sqrt{x})^2 - 9 = x - 9$	$\{x \mid x \geq 0\}$ or $[0, \infty)$ (Domain may be no larger than domain of f, of "input" function)

Domain: $x \neq 1$ Domain: $x \neq 0$.

2. Let $f(x) = \frac{1}{3(x-1)}$ and let $g(x) = \frac{x+5}{x}$. Find the following combined functions and their domains.

	Function	Domain
$f - g =$	$\frac{1}{3(x-1)} - \frac{x+5}{x}$	$\{x \mid x \neq 1, x \neq 0\}$ (omit zero denominators)
$f \cdot g =$	$\frac{1}{3(x-1)} \cdot \frac{x+5}{x}$	$\{x \mid x \neq 1, x \neq 0\}$ (omit zero denominators)
$g/f =$	$\frac{\frac{x+5}{x}}{\frac{1}{3(x-1)}}$	$\{x \mid x \neq 0, x \neq 1\}$ (omit zero denominators both places.)
$f \circ g =$	$f\left(\frac{x+5}{x}\right) = \frac{1}{3\left(\frac{x+5}{x} - 1\right)}$ Check for 0-denominator: Let $\frac{x+5}{x} - 1 = 0 = \frac{x+5-x}{x} = 0$ $\frac{5}{x} = 0$ (No solutions)	$\{x \mid x \neq 0\}$ (omit original zero denominator, omit any others: none found) ↑ only problem
$g \circ g =$	$g\left(\frac{x+5}{x}\right) = \frac{\frac{x+5}{x} + 5}{\frac{x+5}{x}}$ } omit all zero denominators	$\{x \mid x \neq 0, x \neq -5\}$

3. Find functions f and g such that the function $F(x) = \sqrt[6]{x-7}$ can be expressed in the form $f \circ g$.

Let $g = x - 7$
$f = \sqrt[6]{x}$

Then $f \circ g = f(x-7) = \sqrt[6]{x-7}$ ✓