

Math 685.

Lecture 9.

Exam #6: $\tilde{r} - r = (I - A(A^T A + F)^{-1} A^T) A x$

$$(I - A(A^T A + F)^{-1} A^T) A = A - A(A^T A + F)^{-1} A^T A$$

$$= A - A(A^T A + F)^{-1} (A^T A + F - F)$$

$$= A - A \underbrace{(A^T A + F)^{-1} (A^T A + F)}_I + A(A^T A + F)^{-1} F$$

$$= A(A^T A + F)^{-1} F = A \underbrace{(I + \underbrace{(A^T A)^{-1} F}_S)}_S^{-1} \underbrace{(A^T A)^{-1} F}_S$$

$$A^T A + F = A^T A (I + (A^T A)^{-1} F)$$

$$(A^T A + F)^{-1} = (I + (A^T A)^{-1} F)^{-1} (A^T A)^{-1}$$

$$= A (I + S)^{-1} S, \quad \|S\| < 1 \leftarrow \text{since } \|F\| \leq \sigma_n(A)^2$$

use $\|(I + S)^{-1}\| \leq \frac{1}{1 - \|S\|}$

$$\Rightarrow \|\tilde{r} - r\| \leq \|A\| \|(I + S)^{-1}\| \|S\| \|x\|$$

$$\leq 2 \operatorname{cond}(A) \frac{\|F\|}{\|A\|} \|x\|$$