

Math 685.

Lecture 4.

HW9 #5.

$$(A - uv^T)^{-1} = A^{-1} + A^{-1}u(1 - v^T A^{-1}u)^{-1}v^T A^{-1}$$

$(A - uv^T) \times$ both sides

$$I = (A - uv^T)(A^{-1} + A^{-1}u(1 - v^T A^{-1}u)^{-1}v^T A^{-1})$$

$$I = (A - uv^T)A^{-1}(I + u(1 - v^T A^{-1}u)^{-1}v^T A^{-1})$$

$$I = (I - uv^T A^{-1})(I + u(1 - v^T A^{-1}u)^{-1}v^T A^{-1})$$

$$I = I - \underbrace{uv^T A^{-1}} + \underbrace{u(1 - v^T A^{-1}u)^{-1}v^T A^{-1}} - \underbrace{uv^T A^{-1}u(1 - v^T A^{-1}u)^{-1}v^T A^{-1}}$$

$$0 = \underbrace{u(-1 + (1 - v^T A^{-1}u)^{-1} - v^T A^{-1}u(1 - v^T A^{-1}u)^{-1})}_{\times v^T A^{-1}}$$

$$\underbrace{u(-1 + (1 - v^T A^{-1}u)(1 - v^T A^{-1}u)^{-1})}_{1} v^T A^{-1}$$

$0 \equiv 0 \leftarrow$ proof is done.