Math 685.

Lecture 10

Midterm:

#2. $\text{cond}\# = \dfrac{\|\nabla f\|\cdot\|x\|_\infty}{\|f\|} = \dfrac{2\max(|x_1|,|x_2|)}{|x_1-x_2|}$

$f = x_1 - x_2$

$\|x\|_\infty = \max(|x_1|,|x_2|)$

$\|f\| = |x_1 - x_2|$

$\|\nabla f\|_\infty = \|(1,-1)\|_\infty = 2$

#3. Gaussian elim. preserves diag. dominance

Suppose $\underset{k\times k}{A} = \left[\begin{array}{c|c} \alpha & w \\ \hline v & \underbrace{B}_{(k-1)\times(k-1)} \end{array}\right]$

To show: after 1 step of GE, the structure is preserved.

$$A^{(0)} = \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{v}{\alpha} & I \end{bmatrix}}_{L} \underbrace{\left[\begin{array}{c|c} \alpha & w \\ \hline 0 & B - \frac{v w}{\alpha} \end{array}\right]}_{U} = A^{(1)}$$

$B - \frac{vw}{\alpha}$ has to be diag. dom.

$$a_{jj}^{(1)} \geq \sum_{\substack{i\geq 2 \\ i\neq j}} |a_{ij}^{(1)}|$$

We know:

$$\sum_{\substack{i\geq 2 \\ i\neq j}} |a_{ij}^{(1)}| = \sum_{\substack{i\geq 2 \\ i\neq j}} \left|b_{ij} - \frac{v_i w_j}{\alpha}\right| \leq \sum_{\substack{i\geq 2 \\ i\neq j}} |b_{ij}| + \frac{|w_j|}{\alpha} \sum_{\substack{i\geq 2 \\ i\neq j}} |v_i|$$

$$\leq |b_{jj}| - |w_j| + \frac{|w_j|}{\alpha}(\alpha - |v_j|)$$

$$= |b_{jj}| - \frac{|w_j|\cdot|v_j|}{\alpha} \leq \left|b_{jj} - \frac{w_j \cdot v_j}{\alpha}\right| = a_{jj}^{(1)}$$

$\blacksquare$

#4. x_1 $\|e_1\|$ – small $\|r_1\|$ – large

x_2 $\|e_2\|$ – large $\|r_2\|$ – small

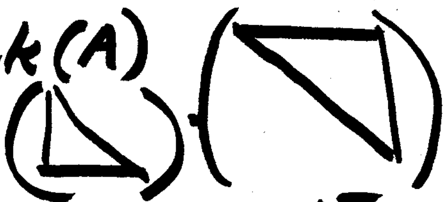
cond $A \sim 10^6$

$$\frac{\|x-\tilde{x}\|}{\|x\|} \le \operatorname{cond}(A)\cdot\frac{\|r\|}{\|A\|\cdot\|\tilde{x}\|}$$

#5. (a) $C = (A^TA)^{-1} = (R^TQ^TQR)^{-1} = (R^TR)^{-1}$

(b) $C = R^{-1}\cdot R^{-T} = L\cdot L^T$

$n = \operatorname{rank}(A)$



L L^T $G = L\cdot L^T$

$c_{kk} = \sum_{l=1}^{k} g_{kl}^2$ ← Cholesky formula for diag. entries