Math 685/CSI 700/OR 682 Project 2
given 02/22/10, due in class 03/15/10

Each group should submit a report, which should include a short 2-5 page account on the implementation and solution, plus any necessary supplements in the form of graphs and/or tables, in class on the date listed above. Matlab codes should be sent to me via email (in archived form if necessary). Late submissions will bear a penalty.

Problem 1.

Suppose you are fitting a straight line through the three data points

$$(0,1),(1,2),(3,3).$$

Set up an overdetermined linear system for the least squares problem and find the corresponding normal equations. Perform Cholesky factorization of the resulting system by hand and come up with the least squares solution. Compare your calculation by solving the system of normal equations in MATLAB.

Problem 2.
We have seen that if

$$A = \begin{pmatrix} 1 & 1 \\ \epsilon & 0 \\ 0 & \epsilon \end{pmatrix},$$

where $0 < \epsilon < \sqrt{\epsilon_{mach}}$ in a floating-point system with machine precision constant ϵ_{mach}, then the product $A^T A$ is singular in such floating-point arithmetic. Show that if $A = QR$ is the reduced QR factorization for this matrix, then R is not singular in the same floating point arithmetic.

Problem 3.
(a) Pick one of the datasets provided on the course website. Form a linear least squares problem based on the list of observations provided (i.e. choose $f = a+bt$ as your model function). This should leave you with an overdetermined system of equations of the form $Ax \approx b$.

(b) Solve this least squares problem using each of the following methods and compare the solutions you obtain.
- normal equations (you can use `slash` command in MATLAB)
- augmented system method
- QR method (you can use `qr` command in MATLAB)
- SVD method (you can use `svd` command in MATLAB)

(c) Perturb the initial data slightly by adding to each data point a random number uniformly distributed on the interval $[-10^{-3}, 10^{-3}]$ multiplied by a maximal value in the column and apply each of the above least squares solvers to the perturbed system. Compare the values for the parameters with those previously obtained - what effect does the perturbation have on the plot of the least squares solution? Which solution would you recommend - one which fits the data more closely or one that is less sensitive to perturbations? Explain.

Bonus research question: try fitting the model to another polynomial, say quadratic or cubic, and see whether you can get a better goodness of fit estimation in terms of the residual $r = b - Ax$. Is there another model, say logarithmic or exponential, that can provide an even better fit based on your observations?