Math 685/CSI 700/OR 682 Project 1.2
given 02/16/10, due in class 03/01/10

Each group should submit a report, which should include a short 2-5 page account on the implementation and solution, plus any necessary supplements in the form of graphs and/or tables, in class on the date listed above. Matlab codes should be sent to me via email (in archived form if necessary). Late submissions will bear a penalty.

Problem 2.

A Bezier cubic curve is defined by the parametric equations

$$x(t) = a_x t^3 + b_x t^2 + c_x t + x_1$$
$$y(t) = a_y t^3 + b_y t^2 + c_y t + y_1$$

where $0 \le t \le 1$. The curve goes from $(x(0), y(0)) = (x_1, y_1)$ to $(x(1), y(1)) = (x_4, y_4)$ and is tangent to the lines $(x_1, y_1) - (x_2, y_2)$ and $(x_3, y_3) - (x_4, y_4)$. The Bezier control points are given by the relations

$$\begin{array}{ll} x_2 = x_1 + c_x/3 & y_2 = y_1 + c_y/3 \\ x_3 = x_2 + (c_x + b_x)/3 & y_3 = y_2 + (c_y + b_y)/3 \\ x_4 = x_1 + c_x + b_x + a_x & y_4 = y_1 + c_y + b_y + a_y \end{array}$$

(a) Reformulate the problem as a linear system $Ca = f$ for some matrix C. Write a program to draw a Bezier curve, given control points $(x_1, y_1) \ldots (x_4, y_4)$.

(b) Estimate $||C||$ via column sums and $||C^{-1}||$ via choosing f so that the ratio $||a||/||f||$ is large, where a is the solution to $Ca = f$ above. Compute the condition number using these estimates and compare with the value given by MATLAB condition number estimator `cond`.

(c) Analyze the problem for accuracy and stability. How many accurate digits can you expect in each calculation? Can you say something about the forward and backward errors in this calculation?

(d) Draw the curve with control points $(0, 0), (2, 1), (-1, 1), (1, 0)$.