

Math 685/CSI 700/OR 682 Project 1.1
given 04/16/10, due in class 03/01/10

Each group should submit a report, which should include a short 2-5 page account on the implementation and solution, plus any necessary supplements in the form of graphs and/or tables, in class on the date listed above. Matlab codes should be sent to me via email (in archived form if necessary). Late submissions will bear a penalty.

Problem 1.

For a set of data points (x_i, y_i) with $i = 1, \dots, N$ one can find the polynomial $p(x) = a_1 + a_2x + \dots + a_Nx^{N-1}$ that passes through these points (i.e. $p(x_i) = y_i$) by solving the linear system $Va = y$ of the form:

$$\begin{pmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{N-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_N & x_N^2 & \dots & x_N^{N-1} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

Matrix V is the famous Vandermonde matrix.

(a) Write a program to compute the solution using MATLAB built-in Gaussian elimination routine and taking $x_i = i$ (evenly spaced data).

(b) Plot the error estimated as $E = \max |D_{ij}|$ versus N , where $D \equiv VV^{-1} - I$. What do you observe?

(c) For $N = 10$, estimate $\|V\|$ via column sums and $\|V^{-1}\|$ via choosing y so that the ratio $\|a\|/\|y\|$ is large, where a is the solution to $Va = y$ above. Compute the condition number using these estimates and compare with the value given by MATLAB condition number estimator `cond`.

(d) Analyze the problem for accuracy and stability. How many accurate digits can you expect in each calculation? Can you say something about the forward and backward errors in this calculation based on given error estimator?

(e) Graph the initial dataset and the polynomial approximation for $N = 10$ and $N = 1000$. Comment on your results.