

Math 685/CSI 700/OR 682 Homework 4
given 02/22/2010

Problem 1.

Which of the following types of matrices are necessarily orthogonal?

- (a) Permutation
- (b) Symmetric positive definite
- (c) Householder transformation
- (d) Givens rotation
- (e) Nonsingular
- (f) Diagonal

Problem 2.

If Q is a 2×2 orthogonal matrix such that

$$Q \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$

what must be the value of α ?

Problem 3.

What is the Euclidean norm of the minimum residual vector for the following linear least squares problem? What is the solution vector x for this problem?

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \approx \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

Problem 4.

Consider the vector $a = [2 \ 3 \ 4]^T$.

- (a) Specify an elementary elimination matrix that annihilates the 3rd component of a .
- (b) Specify a Householder that annihilates the 3rd component of a .
- (c) Specify a Givens rotation that annihilates the 3rd component of a .
- (d) When annihilating a given nonzero component of any vector, is it ever possible for the corresponding elementary elimination matrix and Householder transformation to be the same? Why?
- (e) When annihilating a given nonzero component of any vector, is it ever possible for the corresponding Givens rotation and Householder transformation to be the same? Why?

Problem 5.

Compute a QR decomposition of the matrix

$$\begin{bmatrix} 3 & 3 \\ 3 & 1 \end{bmatrix}$$

using: (a) a Givens rotation; (b) Gram-Schmidt orthogonalization.