Math 685/CSI 700/OR 682 Homework 1
given 01/25/10

The following are some practice problems I would like you to do. Solutions will be discussed in class on 02/01/10.

Problem 1.
What do the following pieces of Octave/Matlab code accomplish?
(a) $\texttt{x} = (\texttt{0} : \texttt{40})./\texttt{40};$
(b) $\texttt{a} = 2; \texttt{b} = 5; \texttt{x} = \texttt{a} + (\texttt{b} - \texttt{a}). * (\texttt{0} : \texttt{40})./\texttt{40};$
(c) $\texttt{x} = \texttt{a} + (\texttt{b} - \texttt{a}). * (\texttt{0} : \texttt{40})./\texttt{40}; \texttt{y} = \sin(\texttt{x}); \texttt{plot}(\texttt{x}, \texttt{y});$

Problem 2.
Write the code to implement the factorial function for integers:

$$\texttt{function}[\texttt{nfact}] = \texttt{factorial}(\texttt{n})$$

where n factorial is equal to $1 \cdot 2 \cdot 3 \ldots (n-1) \cdot n$. Either use a 'for' loop, or write the function to recursively call itself.

Problem 3.
(a) Write a program to compute an approximate value for the derivative of a function using the finite-difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Test your program using the function $\tan(x)$ for $x = 1$. Determine the error by comparing with the square of the built-in function $\sec(x)$. Plot the magnitude of the error as a function of h, for $10^{-k}, k = 0, \ldots, 16$. You should use a log scale for h and for the magnitude of the error. Is there a minimum value for the magnitude of the error? How does the corresponding value of h compare with the rule of thumb $h \approx \sqrt{\epsilon_{mach}}$? (MATLAB command for ϵ_{mach} is `eps`)

(b) Repeat the exercise using the centered difference approximation

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Do you observe any changes compared to the previous case? Explain the behavior of the total error from the point of view of truncation and roundoff errors.

Problem 4.
(a) Give an example of floating-point numbers x, y, z for which addition is not associative, i.e. $(x+y)+z \neq x+(y+z)$.
(b) Find another example for the multiplication, to show that $(xy)z \neq x(yz)$.
(c) Finally, find x, y, z such that multiplication does not distribute over addition: $x(y+z) \neq xy + xz$.
Avoid using expressions evaluating to NaN or Inf in all of your examples.