Math 413, Fall 2010
Practice questions - Exam I.

Q1. The frequency of a sinusoidal deep-water wave is related only to its wavelength λ and the acceleration due to gravity g. What does dimensional analysis tell you about this relation?

Q2. Show that it is possible to nondimensionalize the logistic growth problem

$$\frac{dy}{dt} = r(1 - \frac{y}{K})y, y(0) = y_0$$

and the resulting system looks like

$$\frac{du}{dt} = (1 - u)u, u(0) = u_0$$

Solve the simplified system and convert your solution back to dimensional variables.

Q3. The equation for concentration c on an interval of length l is given by

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} + \mu c, c(x, 0) = 0, x(0, t) = c_0, c(l, t) = 0.$$

(a) Find the dimensions of D, c_0, μ
(b) Nondimensionalize the problem and find the corresponding boundary conditions.

Q4. Solve by perturbation (obtain 3 term approximations) for

$$x^2 - 4 = \epsilon e^x$$

Q5. Consider the perturbed initial-value problem

$$y'' + y = 4\epsilon y(y')^2, \epsilon \ll 1, y(0) = 1, y'(0) = 0.$$

Find a 2-term asymptotic approximation using regular perturbation theory. [You will get partial credit if you leave the second term as a solution to a clearly specified IVP.]