## Math 677. Fall 2009. Exam I Review. Material: Perko, Chapters 1.1-2.7; lectures 1-12.

## 1. Linear systems $\dot{x} = Ax$ :

- Diagonalization and Jordan representation
  - (a) Case of distinct roots
  - (b) Case of complex roots
  - (c) Case of repeated roots
  - (d) Jordan canonical form (general case)
- Properties of operator exponentials
- Solution to IVP  $\dot{x} = Ax, x(0) = x_0$ 
  - (a) Fundamental theorem for linear systems
  - (b) Formula for solution given Jordan matrix representation
- Stability theory
  - (a) Stable and unstable subspaces  $E^s$  and  $E^u$ , center subspace  $E^c$ .
  - (b) Characterization of sinks and sources
- Fundamental matrices and nonhomogeneous systems
- **2.** Nonlinear systems  $\dot{x} = f(x, t)$ :
- Fundamental Existence-Uniqueness theorem
  - (a) Continuity, Lipschitz condition, contraction mapping property
  - (b) Picard iteration

(c) Necessary and sufficient conditions on f to guarantee  $\exists$  solution, uniqueness of solution

- Dependence on initial data and parameters (a) Gronwall Lemma
  - (b) Theorem on continuous dependence
- Maximal interval of existence
- Stability theory
  - (a) Flow, linearization, linear stability
  - (b) Stable Manifold Theorem