Math 677. Fall 2009.
Homework #6.
Due Thursday 11/19/09 in class.

Solutions should represent individual work, with all necessary details. Only facts discussed in class or given in the main textbook can be used without proof. Only selected problems will be graded. No homework will be accepted after the due date has passed.

Part I. Complete the following exercises from "Diffential Equations and Dynamical Systems" by Perko, 3rd edition.

Chapter 2, Problem Set 10: # 4
Chapter 2, Problem Set 11: # 2(a,b,c), #3(a,b,c)
Chapter 2, Problem Set 12: # 1, 2

Part II.
(1) Find a center manifold for the system

$$\dot{x} = -xy$$
$$\dot{y} = -y + x^2 - 2y^2$$

for the rest point at the origin, and find a differential equation for the dynamics (the flow) on the center manifold.

(2) For the ordinary differential equation

$$\dot{x}_1 = -x_2 + x_1 y$$
$$\dot{x}_2 = x_1 + x_2 y$$
$$\dot{y} = -y - x_1^2 - x_2^2 + y^2$$

describe the flow near the equilibrium (0, 0, 0) as follows:
(a) Determine the stable/center/unstable subspaces for the linearization at the origin.
(b) Find an expansion for the center manifold at the origin.
(c) Find a differential equation which describes the flow on the center manifold at the origin.
(d) Sketch the phase portrait of the system near the origin. (Hint: Convert the ODE derived in (c) to polar coordinates.)
You can use Matlab, Maple, Mathematica or other packages to help you with tedious calculations when necessary.