

Math 677. Fall 2009.
Homework #4.
Due Thursday 10/23/09 in class.

Solutions should represent individual work, with all necessary details. Only facts discussed in class or given in the main textbook can be used without proof. Only selected problems will be graded. No homework will be accepted after the due date has passed.

Part I. Complete the following exercises from "Differential Equations and Dynamical Systems" by Perko, 3rd edition.

Chapter 2, Problem Set 5: # 2, 5

Chapter 2, Problem Set 6: # 2, 3

Chapter 2, Problem Set 7: # 4, 5

Part II.

(a) Consider a Lotka-Volterra two species competition model

$$\begin{aligned}\dot{x}_1 &= \gamma_1 x_1 \left(1 - \frac{x_1}{K_1}\right) - \alpha_1 x_1 x_2 \\ \dot{x}_2 &= \gamma_2 x_2 \left(1 - \frac{x_2}{K_2}\right) - \alpha_2 x_1 x_2\end{aligned}$$

Let $\frac{\gamma_1}{\alpha_1} < K_1$, $\frac{\gamma_2}{\alpha_2} < K_2$. Classify the equilibria, sketch the phase portrait and determine the stable and unstable manifolds around each saddle point. What can you say about the global asymptotic behavior of the nonlinear system?

(b) Consider the system

$$\begin{aligned}\dot{x}_1 &= x_1 + e^{x_2} \\ \dot{x}_2 &= -x_2\end{aligned}$$

Find all equilibria, classify them and approximate stable and unstable manifolds for each of them.

(c) Prove that the system

$$\begin{aligned}\dot{x}_1 &= -x_1^3 \\ \dot{x}_2 &= -x_2\end{aligned}$$

has a continuum family of center manifolds at the origin.