Math 677. Fall 2009.
Homework #3.
Due Thursday 10/08/09 in class.

Solutions should represent individual work, with all necessary details. Only facts discussed in class or given in the main textbook can be used without proof. Only selected problems will be graded. No homework will be accepted after the due date has passed.

Part I. Complete the following exercises from "Diffential Equations and Dynamical Systems" by Perko, 3rd edition.

Chapter 2, Problem Set 1: # 4, 6
Chapter 2, Problem Set 3: # 1, 2(a)
Chapter 2, Problem Set 4: # 2(c), 5

Part II.
(a) Show that the solution of the initial value problem

$$y' = \sqrt{|y|}, y(t_0) = 0$$

exists, but is not unique by producing two linearly independent solutions. Does this contradict the Fundamental Existence and Uniqueness theorem? What can you say about the following perturbations of the above system:

$$(i)\, y' = \sqrt{|y|} + \epsilon, y(t_0) = 0, \qquad (ii)\, y' = \frac{y^2}{y^2 + \epsilon^2}\sqrt{|y|}, y(t_0) = 0,$$

where $\epsilon > 0$.

(b) Picard map for the IVP $\dot{x} = f(t, x), x(t_0) = x_0$ is given by $(Ax)(t) = x_0 + \int_{t_0}^{t} f(\tau, x(\tau))d\tau$ and the corresponding sequence $x, Ax, A^2x, \ldots$ is the sequence of Picard approximations. Use these approximations to solve the IVP $\dot{x} = x, x(0) = 1$.

(c) Show that contraction implies Lipschitz condition, which in turn implies continuity.