Math 677. Fall 2009. Homework #1. Due Thursday 09/24/09 in class.

Solutions should represent individual work, with all necessary details. Only facts discussed in class or given in the main textbook can be used without proof. Only selected problems will be graded. No homework will be accepted after the due date has passed.

Part I. Complete the following exercises from "Differial Equations and Dynamical Systems" by Perko, 3rd edition.

Problem Set 6: # 4 Problem Set 7: # 2(c), 4(c) Problem Set 8: # 3, 6(h), 12 Problem Set 9: # 3, 6 Problem Set 10: # 2

Part II.

(a) Prove that if α is a real number and $A \in \mathbb{R}^{n \times n}$ such that $\langle Av, v \rangle \leq \alpha ||v||^2$ for all $v \in \mathbb{R}^n$, then $||e^{tA}|| \leq e^{\alpha t}$ for all t > 0. *Hint: consider the equation* $\dot{x} = Ax$ and the inner product $\langle \dot{x}, x \rangle$.

(b) Suppose that $t \to A(t)$ and $t \to B(t)$ are smooth $n \times n$ -valued functions defined on \mathbb{R} such that $\langle A(t)v, v \rangle \leq \alpha(t) ||v||^2$ and $\langle B(t)v, v \rangle \leq 0$ for all $t \geq 0$ and all $v \in \mathbb{R}^n$. If $t \to x(t)$ is a solution of the differential equation $\dot{x} = A(t)x + B(t)x$, then

$$||x(t)|| \le e^{\int_0^t \alpha(s)ds} ||x(0)||, \forall t > 0$$