Answers to Summer 2003 Final exam.

Q#:1

A A D.E. that satisfies these requirements is $y' = (y-1)(y-2)(y-3)$.
B A D.E. that satisfies these requirements is $y'' + 4y' + 4y = 3\cos t - 4\sin t$.
C A D.E. that satisfies these requirements is $2xy + e^y + (x^2 + xe^y)\frac{dx}{dy} = 0$.

Q#:2 The answer, for both methods, is $y(t) = \frac{t}{2}e^t - \frac{1}{4}e^t + \frac{5}{4}e^{-t}$

Q#:3

A There is an equilibrium solution at $v = 15$, which is stable, and another at $v = -15$, which is unstable.
B The solution is $t + c = \frac{1}{120}[\ln(30+2v) - \ln(30-2v)]$.

Q#:4

A The D.E. is $y'' + 7y' + 10y = \cos t - u_{3\pi}(t)\cos t$, with initial conditions $y(0) = 0$ and $y'(0) = 2$.
B $y(t) = \frac{9}{130}\cos t + \frac{7}{130}\sin t + \frac{8}{15}e^{-2t} - \frac{47}{78}e^{-5t} + u_{3\pi}(t)\left[-\frac{9}{130}\cos t - \frac{7}{130}\sin t - \frac{2}{15}e^{-2(t-3\pi)} + \frac{5}{78}e^{-5(t-3\pi)}\right]$.
C $y(\pi) = -\frac{9}{130} + \frac{8}{15}e^{-2\pi} - \frac{57}{78}e^{-5\pi}$ and $y(4\pi) = \frac{8}{15}e^{-8\pi} - \frac{57}{78}e^{-20\pi} - \frac{2}{15}e^{-2\pi} + \frac{5}{78}e^{-5\pi}$
D The natural frequency is $\omega_o = \sqrt{10}\frac{rad}{s}$

Q#:5

A The solution is $X(t) = -e^{-t}\begin{bmatrix} 5 \\ -3 \end{bmatrix} + e^{-3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.
B This critical point, the only one, is a stable node. It is assymptotically stable.

Q#:6

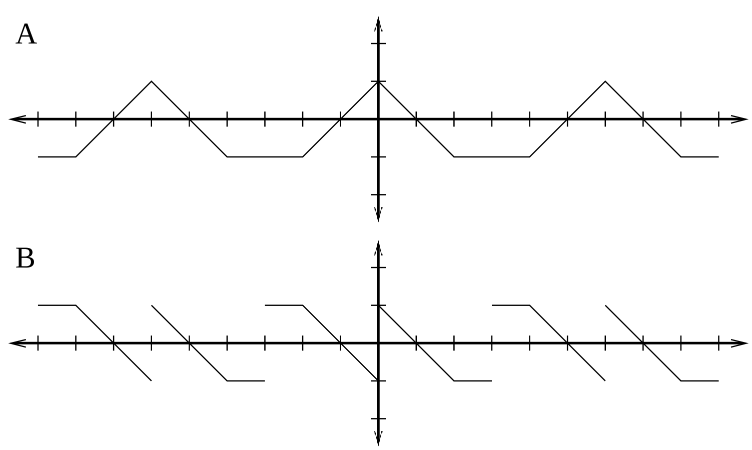
A The critical points are $P = (1,-1)$, $Q - (-1,-1)$, $R = (2,-2)$ and $S = (2,2)$.
B The linearized matrix is $A(x,y) = \begin{bmatrix} 2x & 2y \\ y+1 & x-2 \end{bmatrix}$.
C

- Point P: $X' = \begin{bmatrix} 2 & -2 \\ 0 & -1 \end{bmatrix} X$. It is a saddle point, which is unstable.
- Point Q: $X' = \begin{bmatrix} -2 & -2 \\ 0 & -3 \end{bmatrix} X$. It is a stable node, which is assymptotically stable.
- Point R: $X' = \begin{bmatrix} 4 & -4 \\ -1 & 0 \end{bmatrix} X$. It is an unstable node.
- Point S: $X' = \begin{bmatrix} 4 & 4 \\ 3 & 0 \end{bmatrix} X$. It is a saddle point, which is unstable.

Q#:7

A The eigenvalues are $\lambda_n = \frac{(2n-1)^2}{4}$, where $n \in \mathbb{N}$.
B The eigenfunctions are $f_n = \cos(\frac{(2n-1)t}{2})$, where $n \in \mathbb{N}$.

Q#:8



C The coefficients for the odd series are given by $a_n = \frac{1}{3}\int_{-3}^{3} f(x)\sin(\frac{n\pi x}{3})\,dx$.
D $\tilde{f}(-3) = 0$, $\tilde{f}(0) = 0$ and $\tilde{f}(2) = -1$.

Q#:9

A The two ordinary D.E.s are $t^2T''(t) - \lambda T'(t) = 0$ and $xX'(x) - \lambda X(x) = 0$.
B The boundary conditions become $X(0) = X(1) = 1$.

Q#:10

$$u(x,t) = 20 + 10x + 5e^{-10\pi^2 t}\sin \pi x - 10\exp\left(\frac{-45\pi^2 t}{2}\right)\sin\left(\frac{3\pi x}{2}\right)$$