Answers to Spring 2003 Final exam.

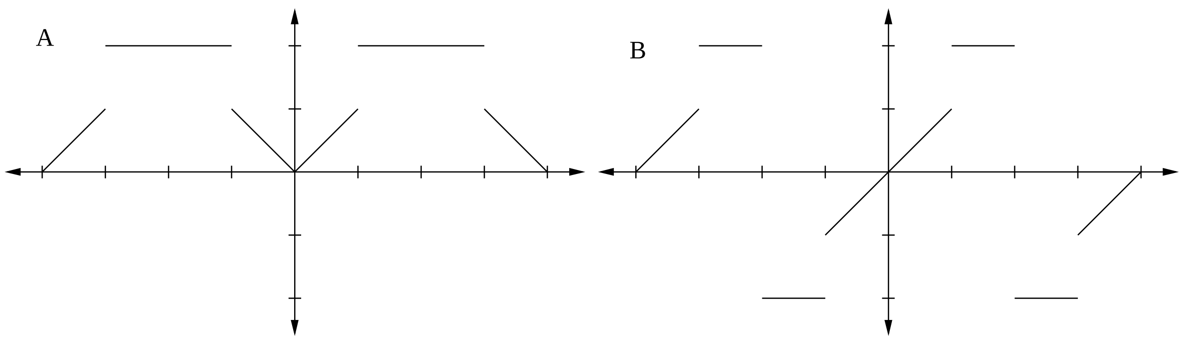
Q#:1 Answer D.
Q#:2 Answer C.
Q#:3 Answer C.
Q#:4 Answer B.
Q#:5 Answer B.
Q#:6 Answer A.
Q#:7 Answer A.
Q#:8 Answer C.
Q#:9

- **A** Solution exists on $(0, \infty)$.
- **B** The solution is $y(t) = c_1 t + c_2 \frac{1}{t}$.

Q#:10

- **A** The linearized matrix is $\begin{bmatrix} y-3 & x \\ 2y & 2x+2 \end{bmatrix}$.
- **B** The critical points are $P = (0,0)$ and $Q = (-1,3)$.
- **C** Point P is a saddle point, which is unstable. Point Q is a centre, which is stable.

Q#:11



- **C** Part (a) has a cosine series and part (b) has a sine series.
- **D** $\tilde{f}(-2) = 0$, $\tilde{f}(\frac{1}{2}) = \frac{1}{2}$ and $\tilde{f}(3) = -\frac{3}{2}$.

Q#:12 Denoting $u(x,y) = F(x)G(y)$,

- **A** $G''(y) + 2G'(y) - \lambda y^2 G(y) = 0$ and $F''(x) - \lambda F(x) = 0$ are the two ordinary differential equations.
- **B** The boundary conditions become $F(0) = F(L) = 0$.

Q#:13

$$\lambda_n = \frac{(2n-1)^2}{4} \qquad f_n(t) = \sin\left(\frac{(2n-1)t}{2}\right) \qquad n \in \mathbb{N}$$

Q#:14

- **A** $u_t = 2u_{xx}$ $\quad u_x(0,t) = u_x(10,t) = 0$ $\quad u(x,0) = 3t\cos(\frac{\pi x}{5}) - 5\cos(\frac{\pi}{2})$.
- **B** $u(x,t) = 3 + \exp(\frac{-2\pi^2 t}{25})\cos(\frac{\pi x}{5})$.
- **C** The steady state emperature is 3.

Q#:15

- **A** The general solution is $X(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-t} \right)$.
- **B** The specific solution is $X(t) = -\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + 3 \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-t} \right)$.