Math 114 Name (print) Key

Quiz 9 **Spring 2002**

Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work.

[3] 1. Determine if the series $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$ converges. You must indicate which test you are using!

Ratio Test

$$\frac{a_{n+1}}{a_n} = \frac{2^{n+1}}{(2n+3)!} \cdot \frac{(2n+1)!}{2^n} = \frac{2 \, (2n+1)(2n)(2n-1)\cdots}{(2n+3)(2n+2)(2n+1)\cdots}$$

$$= \frac{2}{(2n+3)(2n+2)} \longrightarrow 0 < 1$$

$\boxed{\text{So series converges}}$

[7] 2. Find the radius of convergence and the interval of convergence of the following series:

(a) $\sum_{n=0}^{\infty} \frac{n^2 x^n}{10^n}$.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{10^{n+1}} x^{n+1} \cdot \frac{10^n}{n^2 \cdot x^n} = \left(\frac{n+1}{n}\right)^2 \frac{1}{10} x$$

$$\left|\left(\frac{n+1}{n}\right)^2 \frac{1}{10} x\right| < 1$$

$$\downarrow$$

$$\frac{1}{10}|x| < 1$$

$$\sim |x| < 10$$

So $\boxed{R = 1}$

$(-10, 10)$

check endpoints

at $x = -10$ have

$\sum \frac{(-10)^n}{10^n} \cdot n^2 = \sum (-1)^n n^2$ - Diverges clearly

at $x = 10$ have

$\sum \frac{10^n}{10^n} \cdot n^2 = \sum n^2$ - Diverges

$\boxed{\text{Interval } (-10, 10)}$

(b) $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(-2)^{n+1}}{\sqrt{n+1}} (x+3)^{n+1} \cdot \frac{\sqrt{n}}{(-2)^n (x+3)^n}$$

$$= \left|(-2)\sqrt{\frac{n}{n+1}} \cdot (x+3)\right| < 1$$

$$\downarrow$$

$$|x+3| < \frac{1}{2}$$

$\boxed{\text{So } R = \frac{1}{2}}$

End points

$x = -3\frac{1}{2} = -\frac{7}{2}$

$\sum \frac{(-2)^n (-\frac{1}{2})^n}{\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$

diverges (compare to $\frac{1}{n}$)

at $x = -2\frac{1}{2}$

$\sum \frac{(-2)^n (\frac{1}{2})^n}{\sqrt{n}} = \sum (-1)^n \frac{1}{\sqrt{n}}$

converges by AST.

$\boxed{\text{Interval } (-7/2, -\frac{5}{2}]}$