Math 114 Name (print) Key

Quiz 7 **Spring 2002**

Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work.

[3] 1. Determine if the sequence $\left\{\frac{n+1}{3n-1}\right\}$ is convergent. If it is convergent, find its limit.

$$\lim_{n\to\infty} \frac{n+1}{3n-1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{1+\frac{1}{n}}{3-\frac{1}{n}} \to \frac{1+0}{3-0} = \frac{1}{3}$$

$\boxed{\text{converges to } \frac{1}{3}}$

[4] 2. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ converges. If it is convergent, find its sum.

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

So $Bn + A(n+2) = 1$

n: $A + B = 0$

$2A = 1$

$A = 1/2$

$B = -1/2$

$$\sum \frac{1}{n(n+2)} = \frac{1}{2}\left[\left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \dots\right.$$

$$S_n = \frac{1}{2}\left[\left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) \dots \frac{1}{n} - \frac{1}{n+2}\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2} - \frac{1}{n+2}\right] = \frac{1}{2}\left[\frac{3}{2} - \frac{1}{n+2}\right] \to \frac{1}{2}\left[\frac{3}{2}\right]$$

$= \frac{3}{4}$ converges

[3] 3. Use the integral test to determine if the series $\sum_{i=1}^{\infty} e^{-n}$ is convergent.

$$\int_1^\infty e^{-x}\,dx \qquad u = -x, \quad du = -dx$$

$$= -e^{-x}\Big|_1^\infty$$

$$= -\frac{1}{e^\infty} - (-e^{-1}) \to 0 + \frac{1}{e} \text{ - converges}$$

But we don't know what the series converges to.