Math 114 Name (print) Key

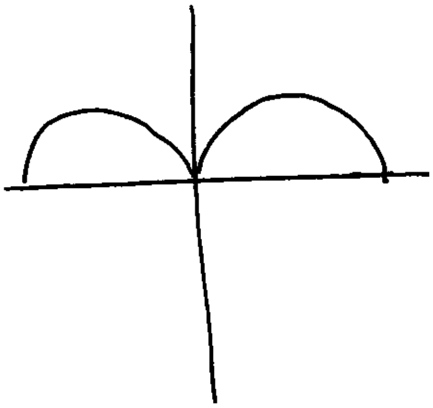
Quiz 6 **Spring 2002**

Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work. Graphing calculators are not allowed!

[6] 1. Consider the polar equation $r = 1 - \sin\theta$ for $0 \le \theta \le \pi$.

(a) graph the function

θ	r
0	1
$\pi/4$	$1-\sqrt{2}/2$
$\pi/2$	0
$\frac{3\pi}{4}$	$1-\sqrt{2}/2$
π	1



(b) determine where the function has a horizontal tangent line.

$y = r\sin\theta = (1-\sin\theta)\sin\theta = \sin\theta - \sin^2\theta$

$x = r\cos\theta = (1-\sin\theta)\cos\theta = \cos\theta - \sin\theta\cos\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta - 2\sin\theta\cos\theta}{-\sin\theta \cancel{- (\sin^2\theta} + \cos^2\theta)}$$

$= -\sin\theta + \sin^2\theta - \cos^2\theta$

$\cos\theta - 2\sin\theta\cos\theta = 0$

$\cos\theta(1 - 2\sin\theta) = 0$

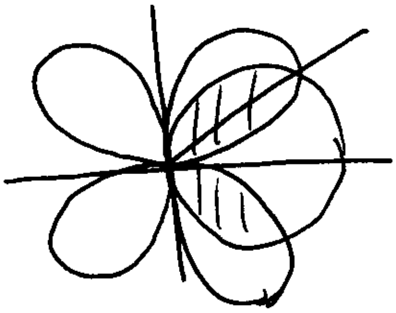
$\cos\theta = 0 \qquad 1 = 2\sin\theta$

$\theta = \frac{\pi}{2} \qquad \frac{1}{2} = \sin\theta$

$\boxed{\theta = \pi/6,\ \frac{5\pi}{6}}$

at $\theta = \pi/2$ tan. line is vertical

[4] 2. Find the area that lies inside both of the curves $r = \sin 2\theta$ and $r = \cos\theta$



$\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta = \cos\theta$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\theta = \pi/2 \qquad \theta = \pi/6$

$$\boxed{2\left[\frac{1}{2}\int_0^{\pi/6} \sin^2 2\theta \, d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} \cos^2\theta \, d\theta\right]}$$