Math 114
Quiz 5

Name (print) Key

Spring 2002

Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work. Graphing calculators are not allowed!

[3] 1. Find the length of the arc of the curve $y = \frac{x^2}{2} - \frac{\ln x}{4}, 2 \le x \le 4$. (Just set up and simplify. Do not perform the integral.)

$$\int_2^4 \sqrt{1+y'^2}\,dx =$$

$$y' = \frac{2x}{2} - \frac{1}{4x} = x - \frac{1}{4x}$$

$$\int_2^4 \sqrt{1+\left(x-\frac{1}{4x}\right)^2}\,dx = \int_2^4 \sqrt{1+\left(x^2-\frac{1}{2}+\frac{1}{16x^2}\right)}\,dx$$

$$= \int_2^4 \sqrt{x^2+\frac{1}{2}+\frac{1}{16x^2}}\,dx$$

[4] 2. A bacteria culture grows with constant growth rate. The count was 500 after 2 hours and 25,000 after 6 hours.

(a) What is the growth rate of the bacteria?

$$P = Ce^{kt}$$

$$\left.\begin{aligned} 500 &= Ce^{2k} \\ 25{,}000 &= Ce^{6k} \end{aligned}\right\}$$

so $\frac{25{,}000}{500} = \frac{Ce^{6k}}{Ce^{2k}} = e^{4k}$

$$50 = e^{4k}$$

$$\ln 50 = 4k$$

$$\boxed{\frac{1}{4}\ln 50 = k}$$

(b) ~~What was the initial population~~? What is the doubling time?

~~500~~ $\frac{1}{4}$

$$t = \frac{7\cancel{0}}{\left(\frac{1}{4}\ln 50\right)10\cancel{0}} = \frac{.7}{\frac{5}{2}\ln 50} = \boxed{\frac{14}{5\ln 50}\text{ hours}}$$

[3] 3. Eliminate the parameter to find a Cartesian equation of the curve: $x = \cancel{\ }\cos t, y = \sin^2 t$.

$$x^2 + y = 1$$

$$y = 1 - x^2$$