Math 114 Name (print) Key

Quiz 4 **Spring 2002**

Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work. Graphing calculators are not allowed!

[3] 1. Set up, but do not do the final sum to compute $\int_{-1}^{1} \sqrt{1+x^3}dx$ using the Trapezoidal Rule with $n = 4$.

$x_0 = -1,\ x_4 = 1 \quad \Delta x = \frac{2}{4} = 1/2$

$x_1 = -1/2,\ x_2 = 0,\ x_3 = 1/2$

$$\int_{-1}^{1} \sqrt{1+x^3}\,dx \approx \frac{\frac{1}{2}}{2}\left[\sqrt{0} + 2\sqrt{1-\tfrac{1}{8}} + 2\sqrt{1-0} + 2\sqrt{1+\tfrac{1}{8}} + \sqrt{2}\right]$$

$$= \boxed{\frac{1}{4}\left[2\sqrt{7/8} + 2 + 2\sqrt{9/8} + \sqrt{2}\right]}$$

[7] 2. Determine whether or not each integral is convergent or divergent. Evaluate those that are convergent.

[3] (a) $\int_{2}^{\infty} \frac{1}{(x+3)^{3/2}}dx$

$u = x+3$

$du = dx$

$$= \int u^{-3/2}du = -2u^{-1/2}\Big|_2^{\infty} = \frac{-2}{\sqrt{x+3}}\Big|_2^{\infty} = \frac{-2}{\infty} - \left(\frac{-2}{\sqrt{5}}\right)$$

$\boxed{\text{converges to } \frac{2}{\sqrt{5}}}$

[4] (b) $\int_{0}^{\infty} xe^{-x^2}dx$

$u = -x^2$

$du = -2x\,dx$

$$= -\frac{1}{2}\int_{x=0}^{\infty} e^u du = -\frac{1}{2}e^u\Big|_{x=0}^{\infty} = -\frac{1}{2}e^{-x^2}\Big|_0^{\infty}$$

$$= \frac{-1}{2e^{\infty}} - \left(-\frac{1}{2}\frac{1}{e^0}\right)$$

$\boxed{= \frac{1}{2} - \text{convergent}}$