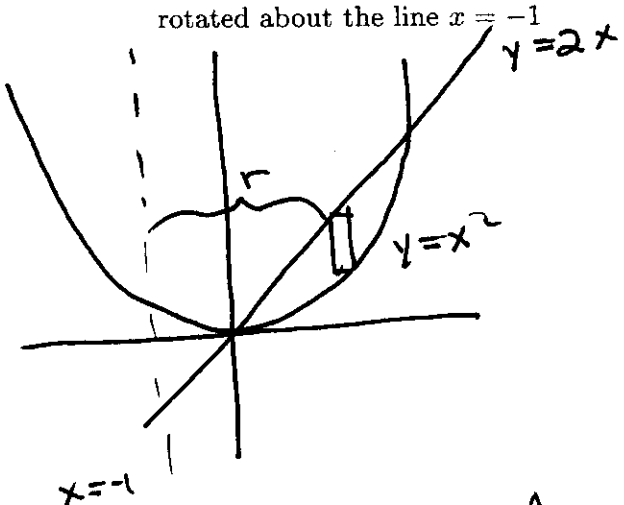
Math 114
Quiz 2

Name (print) Key

Spring 2002

Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work. Graphing calculators are not allowed!

[4] 1. Sketch the region bounded by the curves $y = 2x$ and $y = x^2$. Then, using the method of cylindrical shells, set up, but do not evaluate the integral for the volume when this region is rotated about the line $x = -1$



$2x = x^2$

$0 = x^2 - 2x$

$0 = x(x-2)$

$x = 0, \quad x = 2$

Vol of shell

$= 2\pi r h \Delta x$

$= 2\pi (x+1)(y_1 - y_2)\Delta x$

$y_1 = 2x, \quad y_2 = x^2$

$= 2\pi (x+1)(2x - x^2)\Delta x$

$$\text{Ans} = 2\pi \int_0^2 (x+1)(2x - x^2)\,dx$$

[3] 2. Evaluate $\int_1^2 t \ln t\, dt$

$u = \ln t \qquad dv = t\,dt$

$du = \frac{1}{t}dt \qquad v = \frac{1}{2}t^2$

$= uv - \int v\,du = \frac{1}{2}t^2 \ln t - \frac{1}{2}\int t\,dt$

$= \frac{1}{2}t^2 \ln t - \frac{1}{4}t^2 \Big|_1^2 = (2\ln 2 - 1) - (0 - \frac{1}{4})$

$= 2\ln 2 - \frac{3}{4}$

[3] 3. Evaluate $\int \sin^3 x \cos^2 x\, dx$

$= \int \sin x (\sin^2 x) \cos^2 x\, dx$

$= \int \sin x (1 - \cos^2 x) \cos^2 x\, dx$

$u = \cos x \qquad = -\int (1 - u^2) u^2\, du = -\int u^2 - u^4\, du$

$du = -\sin x\, dx \qquad = \int (u^4 - u^2)\, du = \frac{1}{5}u^5 - \frac{1}{3}u^3$

$= \frac{1}{5}(\cos^5 x) - \frac{1}{3}\cos^3 x + C$