NAME (print): Key

Math 114 Fall 2002—Exam I

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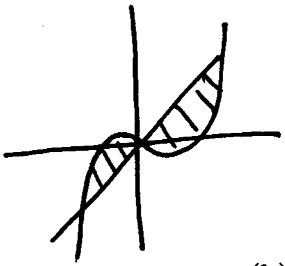
Work carefully and neatly and remember that I cannot grade what I cannot read. You must show all relevant work in the appropriate space. You may receive no credit for a correct answer if there is insufficient supporting work. Notes, books and graphing calculators are NOT ALLOWED. Each problem is worth 10 points.

Here are some formulas you may need:
$\int \tan u du = \ln|\sec u| + C$, $\int \csc u du = \ln|\csc u - \cot u| + C$, $\int \ln u du = u \ln u - u + C$

1. In the this problem, set up, but **do not evaluate** the integral necessary to compute the indicated area or volume. Be sure to sketch the region draw a typical strip. For the volumes indicate method being used.

   (a) The area between the curves $y = x^3 - x$ and $y = 3x$.



$x^3 - x = 3x$

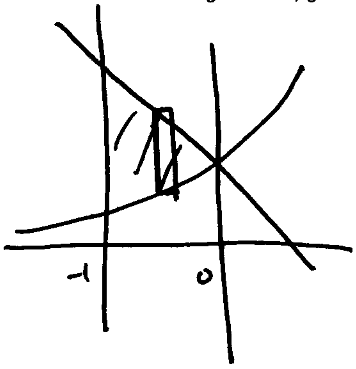
$x^3 - 4x = 0$

$x(x^2-4) = 0$

$x = 0 \quad x = \pm 2$

$$\boxed{\int_{-2}^{0} [(x^3-x)-3x]\,dx + \int_{0}^{2} [3x-(x^3-x)]\,dx}$$

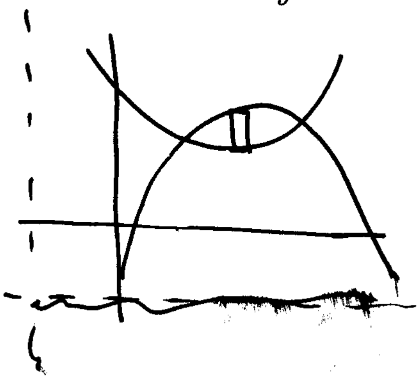
   (b) The volume of the solid obtained by revolving the region bounded by the curves $y = e^x, y = -x + 1, x = -1$ about the x-axis.



Washer method - $\pi r^2 \Delta x$

$$\boxed{\pi \int_{-1}^{0} [(-x+1)^2 - (e^x)^2]\,dx}$$

   (c) The volume of the solid obtained by revolving the region bounded by the curves $y = x^2 - 6x + 10$ and $y = -x^2 + 6x - 6$ about the line $x = -1$.



Shell $\quad 2\pi r h \Delta x$

$x^2 - 6x + 10 = -x^2 + 6x - 6$

$2x^2 - 12x + 16 = 0$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4)$

$$\boxed{2\pi \int_{2}^{4} \underset{\uparrow\, r}{(x+1)} [(x^2-6x+10) - (-x^2+6x-6)]}$$

2. Evaluate the following definite and indefinite integrals.

(a) $\int x \ln x dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{2}x^2 dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\, dx$

$\boxed{= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c}$

(b) $\int x(\sec^2 x)dx$

$u = x \qquad dv = \sec^2 x\, dx$

$du = dx \qquad v = \tan x$

$= x\tan x - \int \tan x\, dx$ (list on first page)

$= x\tan x - \ln|\sec x| + c$

(c) $\int_0^3 \frac{\sqrt{9-x^2}}{x}dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$= \int_{\theta=0}^{\pi/2} \frac{\sqrt{9-9\sin^2\theta}}{3\sin\theta} 3\cos\theta\, d\theta = \int_0^{\pi/2} \frac{3\cos\theta\, \cancel{3}\cos\theta}{\cancel{3}\sin\theta} d\theta = 3\int \frac{\cos^2\theta}{\sin\theta} d\theta$

$= 3\int_0^{\pi/2} \frac{1-\sin^2\theta}{\sin\theta} d\theta = 3\int \Big(\underbrace{\frac{1}{\sin\theta}}_{\csc\theta} - \sin\theta\Big)d\theta = \boxed{3\ln|\csc\theta - \cot\theta| + \cos\theta \Big|_0^{\pi/2}}$

But $\csc \pi/2$ + $\cot \pi/2$ not defined!

(d) $\int e^{2x}\cos x dx$

$u = e^{2x} \qquad dv = \cos x\, dx$

$du = 2e^{2x} \qquad v = \sin x$

$= e^{2x}\sin x - 2\int e^{2x}\sin x\, dx$

$u = e^{2x} \qquad dv = \sin x$

$du = 2e^{2x} \qquad v = -\cos x$

$= e^{2x}\sin x - 2[-e^{2x}\cos x + 2\int e^{2x}\cos x\, dx$

$= e^{2x}\sin x + 2e^{2x}\cos x + 4\int e^{2x}\cos x\, dx$

$5\int e^{2x}\cos x = e^{2x}\sin x + 2e^{2x}\cos x$

$\int e^{2x}\cos x =$

$\frac{1}{5}[e^{2x}\sin x + 2e^{2x}\cos x] + C$

(e) $\int \tan^3\theta \sec\theta d\theta = \int \tan^2\theta \tan\theta \sec\theta \, d\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \int (\sec^2\theta - 1)\tan\theta \sec\theta \, d\theta$ $\quad u = \sec\theta$, $du = \sec\theta\tan\theta \, d\theta$

$= \int (u^2-1)\,du$

$= \frac{1}{3}u^3 - u = \boxed{\frac{1}{3}\sec^3\theta - \sec\theta + C}$

(f) $\int \frac{x+4}{x^2+2x}dx =$

$= \int \frac{x+4}{x(x+2)}dx = \int \frac{A}{x} + \int \frac{B}{x+2}$ $\quad A(x+2) + Bx = x+4$

$A + B = 1$

$2A = 4$

$\begin{cases} A = 2 \\ B = -1 \end{cases}$

$\int \frac{2}{x} - \int \frac{1}{x+2}$

$$\boxed{\begin{aligned} &= 2\ln|x| - \ln|x+2| + C \\ &= \ln\frac{x^2}{|x+2|} + C \end{aligned}}$$

(g) $\int \frac{1}{1+\sqrt[3]{x}}dx$ $\quad u = 1 + \sqrt[3]{x}$ (or $u = \sqrt[3]{x}$)

so $u - 1 = \sqrt[3]{x}$

$(u-1)^3 = x$

Thus $3(u-1)^2 du = dx$

$= \int \frac{3(u-1)^2 du}{u} = 3\int \frac{u^2-2u+1}{u} du$

$= 3\int \frac{u^2}{u} - \frac{2u}{u} + \frac{1}{u}$

$= 3\left[\frac{1}{2}u^2 - 2u + \ln|u|\right] + C$

$= \boxed{3\left[\frac{1}{2}(1+\sqrt[3]{x})^2 - 2(1+\sqrt[3]{x}) + \ln|1+\sqrt[3]{x}|\right] + C}$