Evaluate the following definite and indefinite integrals:

1. $\int \frac{1}{(1-3t)^4}dt$

$u = 1-3t$

$du = -3dt$

$= -\frac{1}{3}\int \frac{1}{u^4}dt = -\frac{1}{3}\int u^{-4}dt = -\frac{1}{3}\left(-\frac{1}{3}u^{-3}\right)+C$

$= \frac{1}{9}(1-3t)^{-3}+C$

$= \frac{1}{9(1-3t)^3}+C$

2. $\int \frac{\sin x}{1+\cos^2 x}dx$

$u = \cos x$

$du = -\sin x$

$= -\int \frac{du}{1+u^2} = -\arctan u = -\arctan(\cos x)+C$

3. $\int_0^1 xe^{x^2}dx$

$u = x^2$

$du = 2x\,dx$

$= \frac{1}{2}\int_{x=0}^{x=1} e^u du = \frac{1}{2}e^u\Big|_{x=0}^{x=1}$

$= \frac{1}{2}e^{x^2}\Big|_0^1 = \frac{1}{2}\left[e^1-e^0\right]$

$= \frac{1}{2}[e-1]$