Homework # 6

For now these are all to be considered additional problems. They may appear later.

1. Let m be an orientation reversing motion. Prove that m^2 is a translation.
2. Prove that $\mathcal{O}$ the group of all orthogonal operators is <u>not</u> a normal subgroup of M.
3. Prove the a linear operator is a glide reflection if and only if its eigenvalues are 1 and -1 and the corresponding eigenvectors are orthogonal.
4. Prove that the map $M \longrightarrow \{1, r\}$ defined by $t_a\rho_\theta \mapsto 1$ and $t_a\rho_\theta r \mapsto r$ is a group homomorphism.

Here are some relations in M you can use. You should convince yourself that they are true:

$t_a t_b = t_{a+b}$
$\rho_\theta t_a = t_{a'}\rho_\theta$ where $a' = \rho_\theta(a)$
$r t_a = t_{a'} r$ where $r' = r(a)$
$r\rho_\theta = \rho_{-\theta} r$