HW7

1. Find the coefficients $a, b$ and $c$ for the glued transformation:

$$\psi(t) = \begin{cases} \ln(t + 1), & t \geq -0.5 \\ at^2 + bt + c, & t \leq -0.5 \end{cases}$$

so that $\psi(t)$ is twice continuously differentiable.

2. Implement the nonlinear rescaling method with $\psi(t)$ from 1, regularization and the Armijo rule and solve the following problems:

   a) $\min 2x_1 - 3x_2$, s.t. $x_1^2 + x_2^2 \leq 25$.
   b) $\min x_1^2 + 2x_1x_2 + x_2^2$, s.t. $3x_1^2 + x_2^2 \leq 9$.
   c) $\min 3x_1^3 + 2x_2^3 + x_3^3 + x_4^3$, s.t. $x_1^2 + x_2^2 + x_3^2 + x_4^2 \leq 4$, $x_1 + x_2 + 2x_3 + 3x_4 \geq 1$.

For the stopping rule of each unconstrained minimization use the criteria:

$$\|\nabla \Phi_k(x^{s+1}, y^s)\| \leq \max \left\{ 10^{-7}, \frac{1}{k} \| y^s - \Psi \left( kC(x^{s+1}) \right) y^s \| \right\}.$$ We do not need to find an unconstrained minimizer much more accurately than the solution of the whole problem.

For the stopping rule of the nonlinear rescaling method use the criteria:

$$\max \left\{ \| \nabla L(x^s, y^s) \|, \| Y^s c(x^s) \|, \max \left\{ -c_i(x^s) \right\} \right\} \leq 10^{-7}.$$ Perform unconstrained minimizations using Newton’s method. After each unconstrained minimization report $\| \nabla L(x^s, y^s) \|$, $\| Y^s c(x^s) \|$, $\max \left\{ -c_i(x^s) \right\}$, and the number of Newton iterations required for an unconstrained minimization between the Lagrange multipliers updates. Use $k = 100$ and $\eta = 0.1$ (parameter for Armijo rule).