

Ok, here is the complete solution to Ch 7, Problem 56. My method of proof was correct but I made a stupid slip in the algebra before class.

Let Y be the number of stops made by the elevator and let X_i be the indicator function of the event that the elevator stops at the i^{th} floor. Then $Y = X_1 + \dots + X_n$ and $E[Y] = E[X_1] + \dots + E[X_n] = NE[X_1]$ since the X_i are all iid. Thus it only remains to calculate $E[X_i]$ which is the probability that the elevator stops at the i^{th} floor.

Consider $P(X_1 = 0)$, i.e. the probability that the elevator doesn't stop at the i^{th} floor and let the number of people in the lift be i . Then

$$P(X_1 = 0) = \sum_{i=0}^{\infty} P(X_1 = 0|i)P(i) = \sum_{i=0}^{\infty} \left(\frac{N-1}{N}\right)^i \frac{e^{-10}10^i}{i!}$$

. Now after a little rearranging (done correctly) we get

$$= e^{-10} \sum_{i=0}^{\infty} (10 - 10/N)^i / i! = e^{-10} e^{10-10/N}$$

since the sum is just the power series of $e^{10-10/N}$. Thus we get $P(X_1 = 0) = e^{-10/N}$ and $P(X_1 = 1) = 1 - e^{-10/N}$ and so $E[Y] = N(1 - e^{-10/N})$.