1) a) $\left(\begin{array}{cccc|c} 0 & 2 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & -3 \\ 0 & 0 & 1 & 3 & -4 \\ -2 & 3 & 2 & 1 & 5 \end{array}\right) \sim \cdots \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 & -5 \end{array}\right)$ Not consistent

b) $\begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ Pivot columns are 1–4.

2) Need to solve $\left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ -2 & -3 & -8 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right) \sim \cdots \sim \left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$x_3$ is free so there are non-trivial solutions
$x_2 = -2x_3$, $x_1 = -2x_2 - 5x_3 = -9x_3$. So solution set is

$x_3 \begin{pmatrix} -9 \\ -2 \\ 1 \end{pmatrix}$ or $t \begin{pmatrix} -9 \\ -2 \\ 1 \end{pmatrix}$ for $-\infty \le t \le \infty$

a) $\{\underline{v}_1, \ldots, \underline{v}_p\}$ are lin. ind. if $c_1\underline{v}_1 + \ldots + c_p\underline{v}_p = \underline{0} \Rightarrow c_1 = c_2 = \ldots = c_p = 0$.

~~Span~~ lin. dep $\Rightarrow \exists\, c_1, \ldots, c_p$ not all $0$ s.t. $c_1\underline{v}_1 + \ldots + c_p\underline{v}_p = \underline{0}$

Let $j$ = largest integer such that $c_j \neq 0$.
Then $c_j\underline{v}_j = -c_1\underline{v}_1 - c_2\underline{v}_2 - \ldots - c_{j-1}\underline{v}_{j-1}$

$c_j \neq 0 \Rightarrow$ can divide by $c_j \Rightarrow \underline{v}_j = -\frac{c_1}{c_j}\underline{v}_1 - \ldots - \frac{c_{j-1}}{c_j}\underline{v}_j$.

$\left(\begin{array}{cccc|c} 1 & 3 & 9 & -2 & 0 \\ 1 & 0 & 3 & -4 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ -2 & 3 & 0 & 5 & 0 \end{array}\right) \sim \cdots \sim \begin{pmatrix} 1 & 3 & 9 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $x_3$ free, $x_4 = 0$, $x_2 = -2x_3$,
$x_1 = -9x_3 - 3x_2 = -3x_3$

So all vectors are $x_3 \begin{pmatrix} -3 \\ -2 \\ 1 \\ 0 \end{pmatrix}$. The CoDomain $\neq$ Range

Inconsistent. It would be $\left(\begin{array}{ccc|c} 1 & 3 & 9 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \leftarrow$

5) a) $h = 26$
b) All $h$.