§4.3 Solutions

1) A basis (3 pivots)

3) Not a basis. Not lin ind and does not span $\mathbb{R}^3$

6) Not a basis but are lin. ind.

8) Not a basis but does span $\mathbb{R}^3$

12) $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

15) The pivots are in columns 1, 2, 4 so basis is $\{\underline{v}_1, \underline{v}_2, \underline{v}_4\}$

21) F (could be $\underline{0}$) 22) F must also span H

23) F (must also be lin. ind.) 24) T (spanning set theorem)

25) T 26) T 27) F (as small as possible) 28) F

29) F 30) F (pivot columns of A) 31) F (the c_i change)

32) T 33) $A = (\underline{v}_1\ \underline{v}_2\ \underline{v}_3\ \underline{v}_4)$ has a pivot in every column $\Rightarrow$ lin. ind.

39) Theorem 4. Not a pivot in every row $\Rightarrow$ does not span.

§4.4

1) $P_B = \begin{pmatrix} 3 & -4 \\ -5 & 6 \end{pmatrix}$ & so $\underset{\sim}{x} = P_B [\underset{\sim}{x}]_B = \begin{pmatrix} 3 & -4 \\ -5 & 6 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$

2) Do same as in 1). Answer is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

5) $P_B = \begin{pmatrix} 1 & 2 \\ -3 & -5 \end{pmatrix}$ and so $P_B^{-1} = \begin{pmatrix} -5 & -2 \\ 3 & 1 \end{pmatrix}$. Then $[\underset{\sim}{x}]_B = P_B^{-1} \underset{\sim}{x} = \begin{pmatrix} -5 & -2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}$

6) Do same as in 5). Answer is $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$.

9) $P_B = \begin{pmatrix} 2 & 1 \\ -9 & 8 \end{pmatrix}$

13) $B = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right\}$ $\quad \underset{\sim}{p} = \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}$. Solve $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 4 \\ 1 & 1 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 \end{array}\right)$. So $[\underset{\sim}{p}]_B = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$.

14) Do same as 13). Answer is $\begin{pmatrix} 7 \\ -3 \\ -2 \end{pmatrix}$

15) a) True $\quad$ 17) False $\left([\underset{\sim}{x}]_B = P_B^{-1} \underset{\sim}{x}\right)$ $\quad$ 19) False $\mathbb{P}_3$ is isomorphic to $\mathbb{R}^4$.

16) a) True $\quad$ 18) False (should be $\underset{\sim}{x} \mapsto [\underset{\sim}{x}]_B$) $\quad$ 20) True (if plane passes through $\underset{\sim}{0}$).

§4.5

1) $s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ has dimension 2 and a ~~possible~~ basis is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \right\}$.

3) $a\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + b\begin{pmatrix} 0 \\ -1 \\ 1 \\ 2 \end{pmatrix} + c\begin{pmatrix} 2 \\ 0 \\ -3 \\ 0 \end{pmatrix}$ has dimension 3 and a basis is $\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -3 \\ 0 \end{pmatrix} \right\}$

8) Solving the linear system $\begin{pmatrix} 1 & -3 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = 0$ gives the general solution as
$\text{span}\left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

So dimension = 3 and these vectors form a basis.

§4.5 ctd

10) $\begin{pmatrix} 1 & -3 & -8 & -3 \\ -2 & 4 & 6 & 0 \\ 0 & 1 & 5 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & -8 & -3 \\ 0 & -2 & -10 & -6 \\ 0 & 0 & 0 & 4 \end{pmatrix}$ There are 3 pivot columns so dim = 3 (and vectors 1, 2, 4 form a basis)

12) $\dim(\mathrm{col}(A)) = 3$ since there are 3 pivots. $\dim(\mathrm{nul}(A)) = 6 - 3 = 3$

17) T 18) F ~~(must pass through 0)~~ 19) F (must pass through $\underline{0}$)

20) F (it is 5) 21) F (but we didn't cover this) 22) T 23) F (of $\cancel{B}$ A not $\cancel{B}$ B)

24) T 25) T 26) F (must be a lin. ind. infinite set)

27) Written as vectors in the standard basis for $\mathbb{P}_3$ they are
$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\0\end{pmatrix}, \begin{pmatrix}-2\\0\\4\\0\end{pmatrix}, \begin{pmatrix}0\\-12\\0\\8\end{pmatrix}$. They are clearly lin. ind and spanning

29) $[p]_B = P_B^{-1}[p]$. So solve $\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 7 \\ 0 & 2 & 0 & -12 & -12 \\ 0 & 0 & 4 & 0 & -8 \\ 0 & 0 & 0 & 8 & 12 \end{array}\right)$ giving $[p]_B = \begin{pmatrix} 3 \\ 3 \\ -2 \\ 3/2 \end{pmatrix}$

~~35) Y Y 36) 2 2 N~~

35) YN (see back of book) 36) 2 & N (col A is isomorphic to $\mathbb{R}^4$)

37) 2 (6 columns − 4 pivot columns) 39) 5 and 5

43) T 44) F 45) T 46) T 47) T 48) F

§4.6

1) a) $\underset{C \leftarrow B}{P} = \begin{pmatrix} 6 & 9 \\ -2 & -4 \end{pmatrix}$ b) $[\underline{x}]_C = \underset{C \leftarrow B}{P}[\underline{x}_B] = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

7) $\underset{C \leftarrow B}{P} = P_C^{-1} P_B = \begin{pmatrix} -3 & 1 \\ -5 & 2 \end{pmatrix}$, $\underset{B \leftarrow C}{P} = P_B^{-1} P_C = \underset{C \leftarrow B}{P}^{-1} = \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix}$

8) $\underset{C \leftarrow B}{P} = P_C^{-1} P_B = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$, $\underset{B \leftarrow C}{P} = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$

11) F (wrong way round, see Example 1) 12) T 13) T

14) F (it satisfies $[\underline{x}]_C = P[\underline{x}]_B$.