

§6.1 HW Solutions

$$2) \quad \underline{w} \cdot \underline{w} = 35, \quad \underline{x} \cdot \underline{w} = 5 \quad \frac{\underline{x} \cdot \underline{w}}{\underline{w} \cdot \underline{w}} = \frac{5}{35} = \frac{1}{7}$$

$$10) \quad \left\| \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix} \right\| = \sqrt{3^2 + 6^2 + (-3)^2} = \sqrt{54}. \quad \text{So a unit vector is } \frac{1}{\sqrt{54}} \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$14) \quad \underline{u} - \underline{z} = \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} \quad \text{and} \quad \left\| \begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix} \right\| = \sqrt{16 + 16 + 4} = 6$$

$$19) - 28) \quad T T T T T F T F T$$

$$32) \quad \begin{aligned} \|\underline{u} + \underline{v}\|^2 &= (\underline{u} + \underline{v}) \cdot (\underline{u} + \underline{v}) = \underline{u} \cdot \underline{u} + 2\underline{u} \cdot \underline{v} + \underline{v} \cdot \underline{v} = \|\underline{u}\|^2 + 2\underline{u} \cdot \underline{v} + \|\underline{v}\|^2 \\ \|\underline{u} - \underline{v}\|^2 &= (\underline{u} - \underline{v}) \cdot (\underline{u} - \underline{v}) = \underline{u} \cdot \underline{u} - 2\underline{u} \cdot \underline{v} + \underline{v} \cdot \underline{v} = \|\underline{u}\|^2 - 2\underline{u} \cdot \underline{v} + \|\underline{v}\|^2 \end{aligned}$$

Add the two equations.

34) W is the nullspace of the matrix $(5 \ -6 \ 7)$ so by Theorem 2 of Chapter 4 it is a subspace. It is a plane through 0 .

37) Let $\underline{w} = c_1 \underline{v}_1 + \dots + c_p \underline{v}_p$ be any vector in W

$$\text{Then } \underline{w} \cdot \underline{x} = (c_1 \underline{v}_1 + \dots + c_p \underline{v}_p) \cdot \underline{x} = c_1 (\underline{v}_1 \cdot \underline{x}) + \dots + c_p (\underline{v}_p \cdot \underline{x}) = 0$$

Since $\underline{x}$ is orthogonal to every element in the basis.

39) $\underline{x} \in W^\perp \Rightarrow \underline{x}$ is orthogonal to every element in W

$$\text{But } \underline{x} \in W \text{ so } \underline{x} \cdot \underline{x} = 0 \Rightarrow \underline{x} = \underline{0}$$

§6.2 Homework Solutions

1) No (1^{st} and 3^{rd} not orthogonal) 2) Yes

$$7) \underline{u}_1 \cdot \underline{u}_2 = 0 \quad \text{so} \quad \underline{x} = \frac{\underline{x} \cdot \underline{u}_1}{\underline{u}_1 \cdot \underline{u}_1} \underline{u}_1 + \frac{\underline{x} \cdot \underline{u}_2}{\underline{u}_2 \cdot \underline{u}_2} \underline{u}_2 = \frac{39}{13} \underline{u}_1 + \frac{26}{52} \underline{u}_2 \\ = 3\underline{u}_1 + \frac{1}{2}\underline{u}_2$$

$$8) \text{ Same as 7). } \underline{x} = -\frac{9}{10} \underline{u}_1 + \frac{13}{20} \underline{u}_2$$

$$12) \text{ Let } \underline{y} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \underline{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \text{proj}_{\underline{u}} \underline{y} = \frac{\underline{y} \cdot \underline{u}}{\underline{u} \cdot \underline{u}} \underline{u} = \begin{pmatrix} 0.6 \\ -1.2 \end{pmatrix}$$

$$15) \|\underline{y} - \hat{\underline{y}}\| = \left\| \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \frac{\underline{y} \cdot \underline{u}}{\underline{u} \cdot \underline{u}} \underline{u} \right\| = 1$$

$$20) \begin{pmatrix} -2/3 \\ 1/3 \\ 2/3 \end{pmatrix} \cdot \begin{pmatrix} 1/3 \\ 2/3 \\ 0 \end{pmatrix} = 0 \quad \text{so orthogonal. Normalizing} \Rightarrow \begin{pmatrix} -2/3 \\ 1/3 \\ 2/3 \end{pmatrix} \text{ and } \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

23) T 24) T (one of the vectors might be $\underline{0}$) 25) T

26) F (also need $\underline{u}_i \cdot \underline{u}_i = 1$) 27) F 28) T

29) F (must be square) 30) T 31) F ($\underline{y} - \hat{\underline{y}}$ does) 32) T

§6.3 Homework Solutions

1) See 2) and answer in back of book.

2) $\underline{v} \cdot \underline{u}_1 = 14$ so $\underline{v} = 14\underline{u}_1 + \underbrace{(\underline{v} - 14\underline{u}_1)}_{\text{in span } \{\underline{u}_2, \underline{u}_3, \underline{u}_4\}}$

4) $\underline{u}_1 \cdot \underline{u}_2 = 0 \checkmark$ $\text{proj}_W \underline{y} = \frac{\underline{y} \cdot \underline{u}_1}{\underline{u}_1 \cdot \underline{u}_1} \underline{u}_1 + \frac{\underline{y} \cdot \underline{u}_2}{\underline{u}_2 \cdot \underline{u}_2} \underline{u}_2$
 $= \frac{24}{25} \underline{u}_1 + \frac{7}{25} \underline{u}_2 = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$ [or spot that span $\{\underline{u}_1, \underline{u}_2\}$ is the x - y plane!]

8) Since $\underline{u}_1 \cdot \underline{u}_2 = 0$ $\hat{\underline{y}} = \text{proj}_W \underline{y} = \frac{\underline{y} \cdot \underline{u}_1}{\underline{u}_1 \cdot \underline{u}_1} + \frac{\underline{y} \cdot \underline{u}_2}{\underline{u}_2 \cdot \underline{u}_2} = \begin{pmatrix} 3/2 \\ 7/2 \\ 1 \end{pmatrix}$

$\underline{z} \in W^\perp$ is given by $\underline{y} - \hat{\underline{y}} = \begin{pmatrix} -5/2 \\ 1/2 \\ 2 \end{pmatrix}$

14) $\hat{\underline{z}} = \frac{7}{14} \underline{u}_1 + 0 \underline{u}_2 = \frac{1}{2} \underline{u}_1 = \begin{pmatrix} 1 \\ 0 \\ -3/2 \end{pmatrix}$

19) ~~$\underline{u}_3$~~ $\underline{u}_3 - \text{proj}_W \underline{u}_3 = \begin{pmatrix} 0 \\ 2/5 \\ 1/5 \end{pmatrix}$ is such a vector.

21) T 22) T 23) F 24) T 25) F (it is $\text{proj}_W \underline{y}$)

26) T 27) T 28) T 29) T 30) F (unless $n=p$)

§6.4 Homework Solutions

$$3) \quad \underline{v}_1 = \underline{x}_1, \quad \underline{v}_2 = \underline{x}_2 - \frac{\underline{x}_2 \cdot \underline{v}_1}{\underline{v}_1 \cdot \underline{v}_1} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3/2 \\ 3/2 \end{pmatrix} \quad \left(\text{or } \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right)$$

$$7) \quad \frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} \quad \text{and} \quad \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$10) \quad \underline{v}_1 = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}, \quad \underline{v}_2 = \begin{pmatrix} 6 \\ -8 \\ -2 \\ 4 \end{pmatrix} - \frac{-36}{12} \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\underline{v}_3 = \begin{pmatrix} 6 \\ 3 \\ 6 \\ -3 \end{pmatrix} - \frac{6}{12} \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} - \frac{30}{12} \begin{pmatrix} 3 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -3 \\ 1 \end{pmatrix}$$

17) F (c might be 0) 18) F (they must not be 0)

19) T 20) T 21) T (see Example 4) 22) T (Theorem 12)

§6.5 Solutions

1)