Answers for Homework questions from Section 6.5

Question 1 The normal equations are $(A^TA)\mathbf{x} = A^T\mathbf{b}$ and are given by

$$\begin{pmatrix} 6 & -11 \\ -11 & 22 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -4 \\ 11 \end{pmatrix}.$$

The solution is $\hat{\mathbf{x}} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Question 3 The normal equations are

$$\begin{pmatrix} 6 & 6 \\ 6 & 42 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$$

and their solution is $\hat{\mathbf{x}} = \begin{pmatrix} 4/3 \\ -1/3 \end{pmatrix}$.

True or False
17) True, by the definition of least squares solution.
18) True.
19) True.
20) False. The inequality is reversed.
21) True. See Theorem 13.
22) True. See Theorem 14.
23) False. $A\hat{\mathbf{x}}$, not $\hat{\mathbf{x}}$, is the point in col(A) closest to $\mathbf{b}$.
24) True. This is just what it means to solve the linear system.
25) False. Not true if the normal equations are 'ill-conditoned'.
26) False. Calculating R^{-1} is in general very expensive. It's nearly always cheaper to solve the linear system given in equation 7 by using row operations.