High-Fidelity Digital Twins: Detecting and Localizing Weaknesses in Structures

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Outline

- What is a Digital Twin ?
- Why Now ?
- Adjoint-Based Detection of Weaknesses
- Optimal Selection of Forces
- Optimal Placement of Sensors
- Open Questions
- Accounting for Temperature Variations
- Extension to Transient Problems
- Conclusions and Outlook

What Is A Digital Twin ?

AIAA Definition

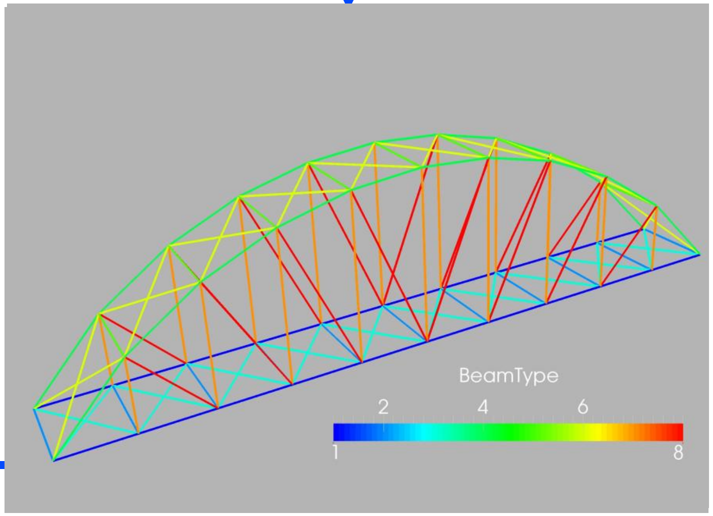
- A Digital Twin is a set of virtual information constructs that mimics the structure, context, and behavior of an individual/unique physical asset, is dynamically updated with data from its physical twin throughout its lifecycle, and informs decisions that realize value.

American Institute of Aeronautics and Astronautics (AIAA),
Digital Engineering Integration Committee. Digital Twin: Definition and Value. AIAA and AIA Position Paper, 2020. https://www.aiaa.org/docs/default-source/uploadedfiles/issues-and-advocacy/ policy-papers/ digital-twin-institute-position-paper-(december-2020).pdf.

What is a Digital Twin ?

- Real `Thing': Object, Process, Patient, …
- Digital Copy/Mirror [Data]
- Sensors

FEM Model, Material Params, …



Compare: As Planned/Built

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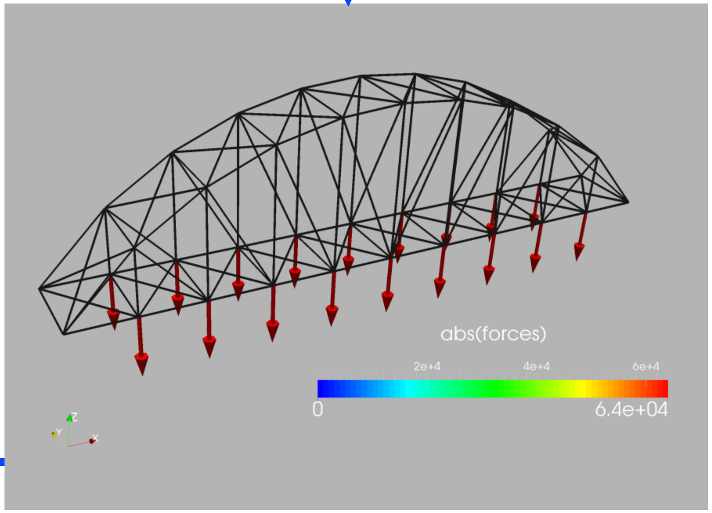
- Sensors

Standard Forces, …



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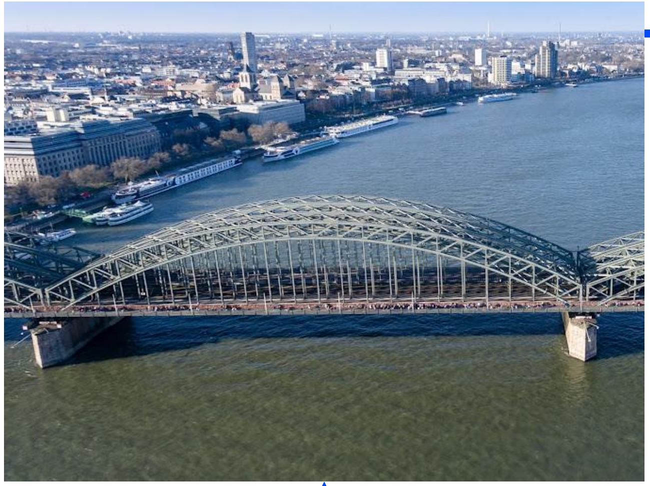
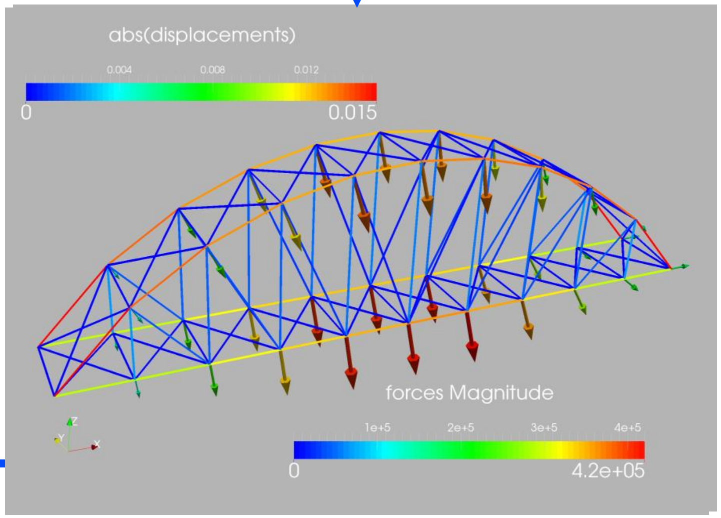
- Sensors

Displacements, Velocities, Accelerations





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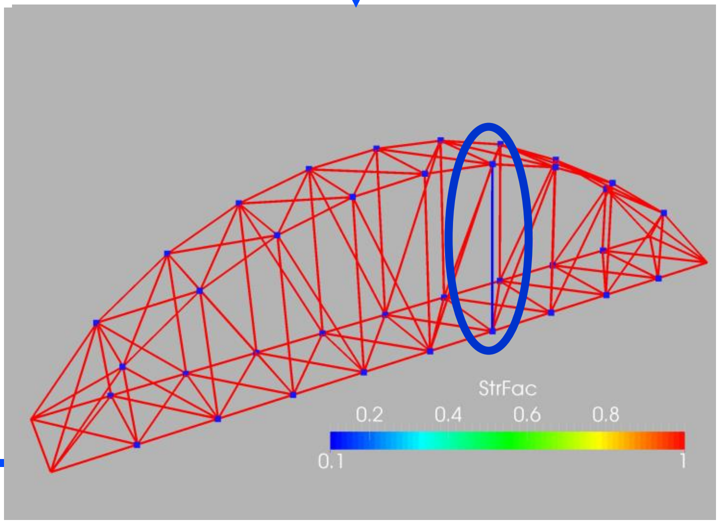
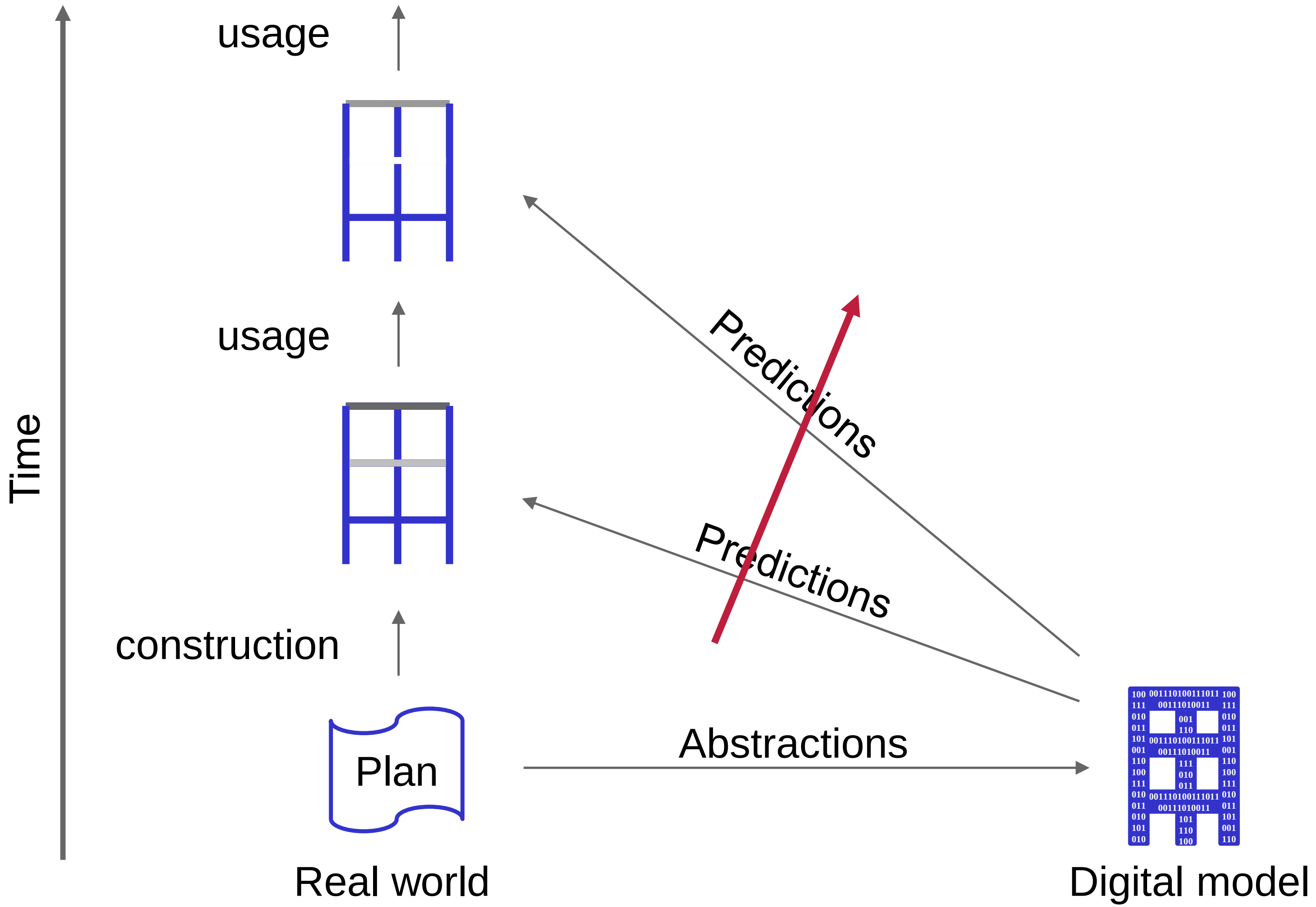
- Sensors

Sensor Data + Model

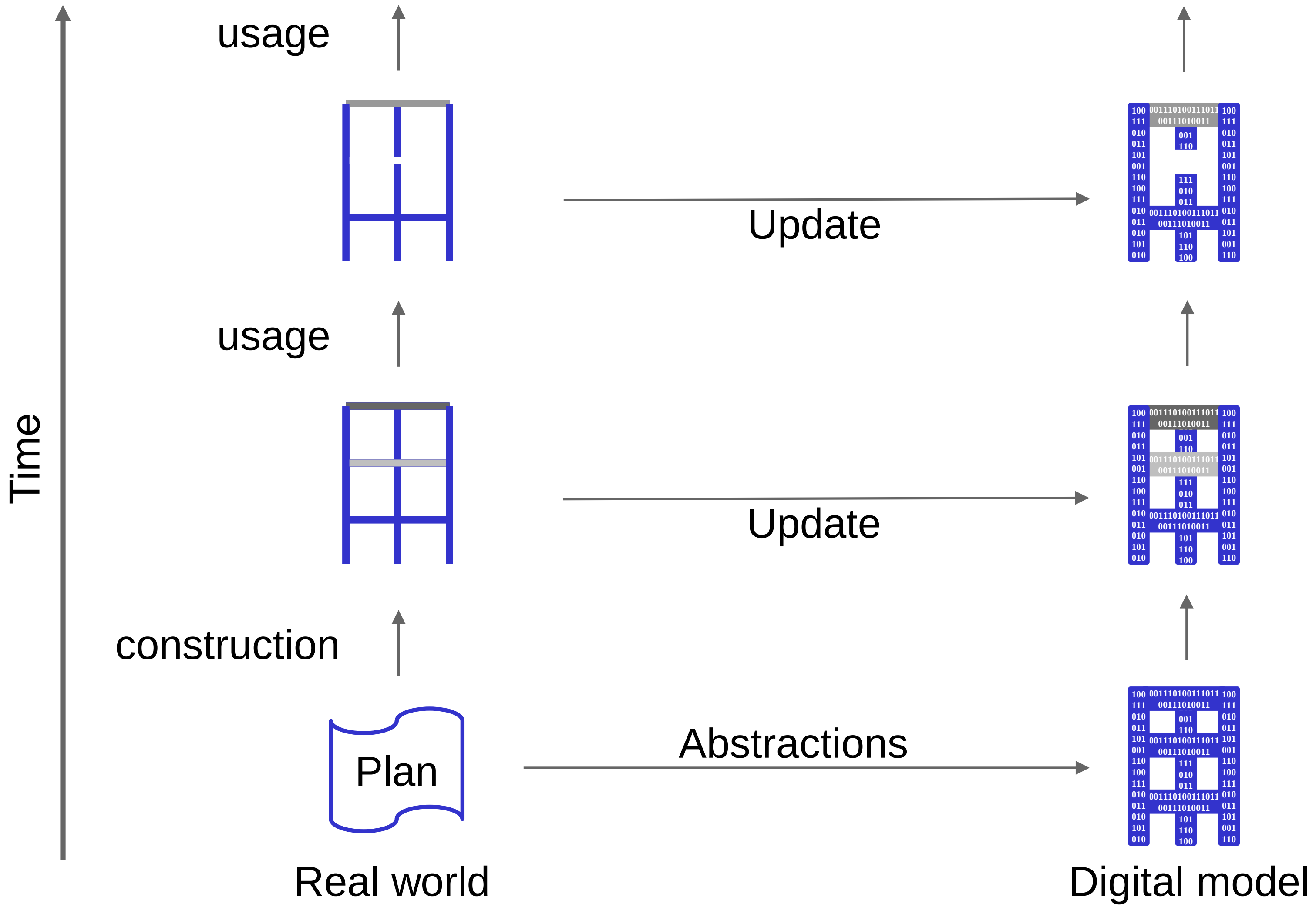


Infer State of System

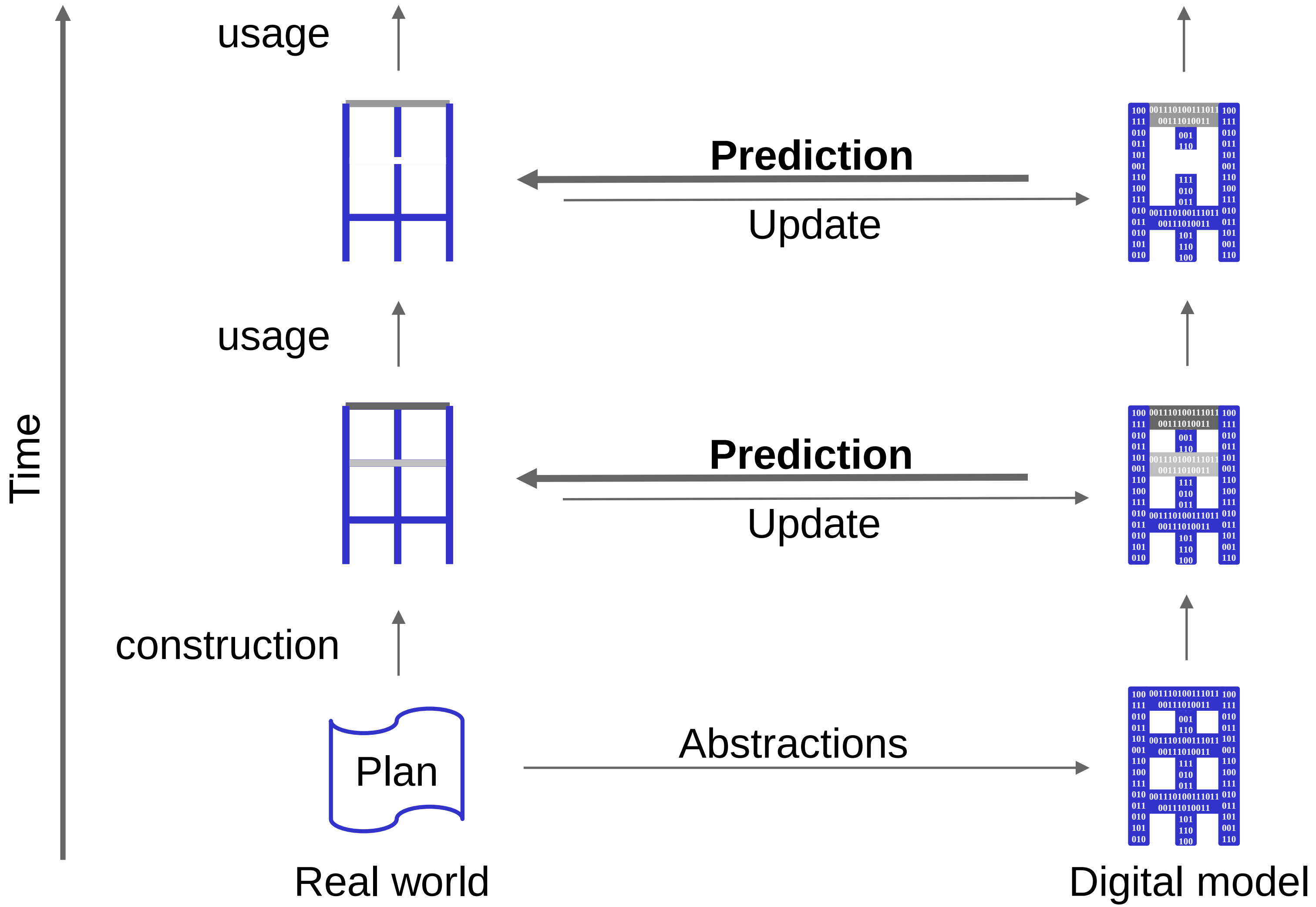
Digital Model



Digital Shadow



Digital Twin



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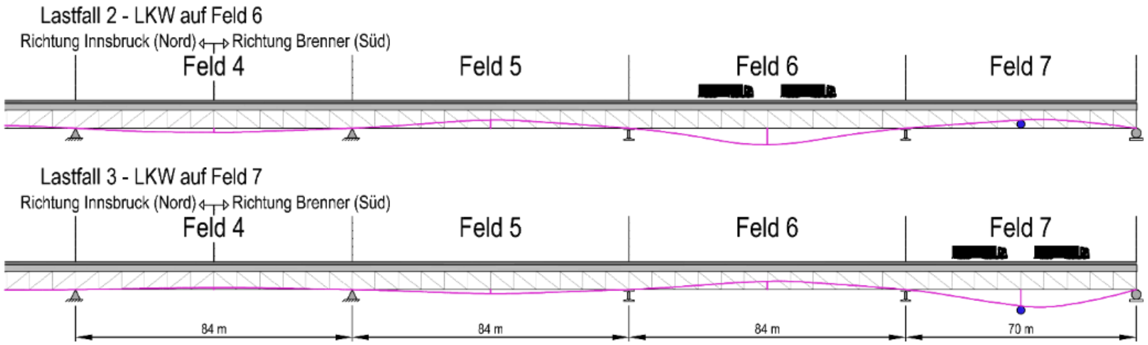
Digital Twins: Product of Megatrends

- Pervasive Use of CAD Systems
  - For Every Product/Process/Building/Patient/…
  - ➔ Have Detailed Data of `Real Thing’

- Pre-Compute, Only Then Build/Operate
  - Huge Reduction in Prototyping/Production Costs
  - ➔ Have Detailed Model(s) of `Real Thing’

- Sensors Everywhere
  - Precise, Connected [G4,G5,…], Rugged, Cheap, ….
  - ➔ Can Measure the `Real Thing'

Sensors Everywhere



Courtesy: F. Schill, HS Mainz

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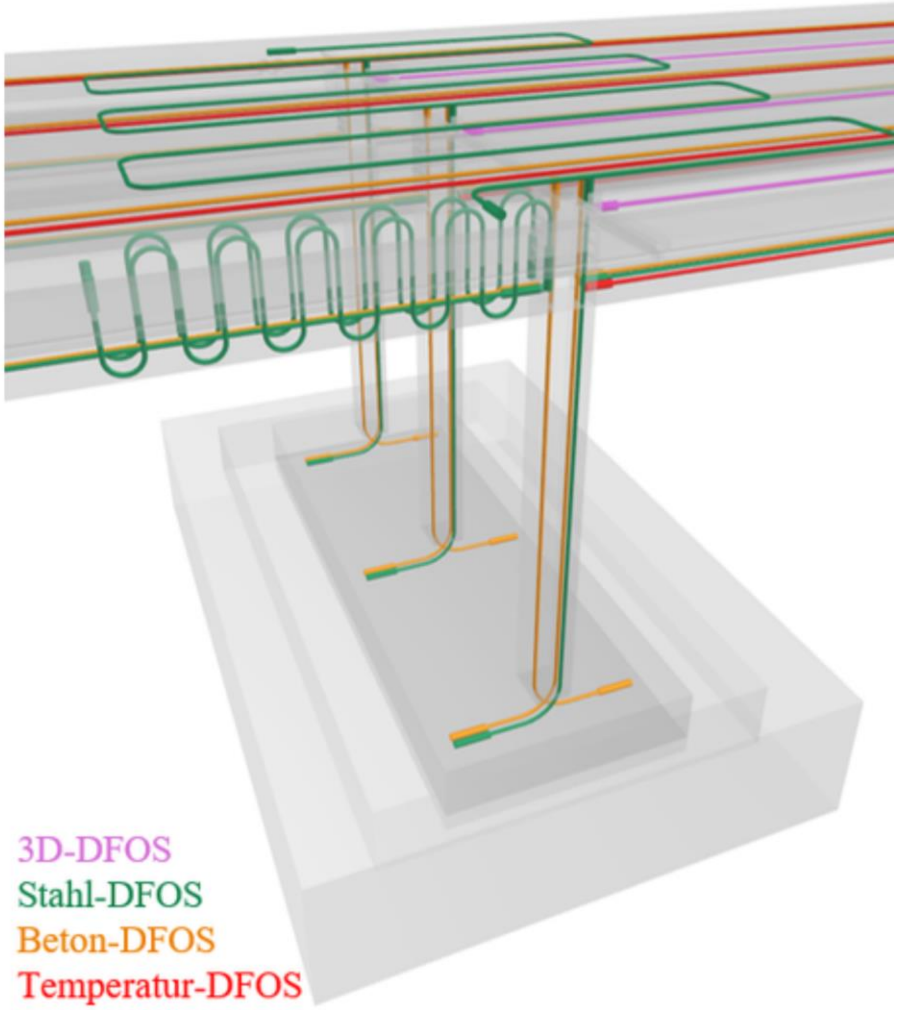
Sensors Everywhere



Digital Twins: Modus Operandi

- Have Digital Copy of Object/Process/Patient/…
- Equip Object/Process/Patient/… With Sensors
- With Data from Sensors (+Models): Infer State
  - Normal, Weakening, Damaged, …
- Update Digital Copy (+Models) Throughout Lifetime

DTs: `Great Expectations’

- Increased Safety
- Increased Comfort
- Longer Life Cycles (Assets, Processes, Humans)
- Optimal Process Control
- Reduced Environmental Footprint
- …

Digital Twin: What Data ?

- Data Levels
  - Description
  - Enumeration of Parts
  - Production/Replacement/Maintenance History
  - Geometry for Display/CAD/Production
  - Geometry/Abstraction for Modeling
    - Needs Proper Data for Each Discipline [CSD, CFD, CEM, …]
    - Needs Proper Mesh/PDE Solver/… for Each Discipline
    - May Involve Extensive `De-Featuring'
  - Computational Mechanics Data
    - Material Data, Mesh, BC, Loads, ROMs, …
  - ….

DT: What Level of Abstraction ?

- **DT** Is Not Reality, Only **Model of Reality**
  - We Are Not Computing Each Atom All The Time
  - Kant: Kritik der Reinen Vernunft
- ➔ **Need Abstraction** Levels
- Partial/Ordinary Differential Equation(s)
  - CFD: Lifting Line/Potential/Euler/RANS/LES/DNS…
  - CSD: Lumped/Beam/Plate/Shell/Solid/…
  - …
- Numerics
  - FEM/FDM/…
- Model Abstraction from Numerics: ROMs/Surrogate Models
  - Modal, POD, PGD,…

DT: Consequences of Abstraction

- Level of Abstraction Determines:
- Digital Twin
  - Data Needed for DT [CAD → DT]
  - Specialized Personnel Needed to Build DT
  - Software That Allows Seamless Updates
- Sensors
  - Type
  - Frequency of Measurements
  - Edge Computing

- ➔ **Determines Possible Type of Monitoring**

DT: Deluge of Data

- DTs for Every Object/Process/Patient/…
- Constant Sensor Data
- Constant Update of DTs
- ➔ Deluge of Data
- Who ?
  - Stores
  - Secures/Insures
  - Manages
  - Retrieves/Compares/Updates
  - Curates
  - …

Digital Twins of All of Us

- Every Click, Every Web Search, Every Call…
- Build Digital Twin of Human Behaviour/Thoughts
- Exploit Economically to the Maximum Extent
  - Directly: Advertising
  - Indirectly: `Time on Subject’
- Examples:
  - Online Merchants: Amazon, Walmart, Groceries, …
  - Web Search Engines
  - Social Media: Facebook, Instagram, TikTok, …
  - Hardware/Software: Apple, Microsoft, …

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  - Social Media: Facebook, Instagram, TikTok, …
  - Hardware/Software: Apple, Microsoft, …
- `We Know More About You Than You’
- `Surveillance Capitalism’ [1789, 1989, …]

Detection of Weaknesses in Structures

IJNMHFF 30, 11, 4837-4863 (2020)
CMAME 417, A, 116471 (2023)
AIAA-2024-2621 (2024)
AIAA-2024-2622 (2024)
IJNME e7568 (2024)
AIAA-2025-0285 (2025)
AIAA-2025-0286 (2025)
FINEL 245, 104316 (2025)
CMAME 438, 117818 (2025)

Motivation

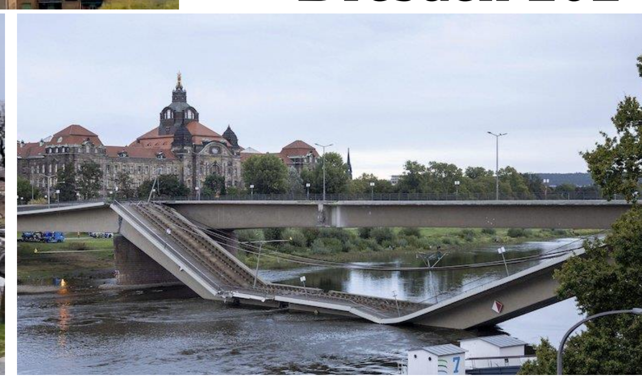
- All Structures Age
- 1st World: Concrete [Bridges, Buildings, …]
  - Life Span: 60-80 Years (Weathering, Cracks, …)
  - Many Bridges and Buildings Nearing That Age
- ➔ Infrastructure Crisis

Recent Bridge Failures


Minneapolis 2007


Genoa 2018

Dresden 2024

Recent Bridge Failures


Bridges Needing Repair/Replacement
US : > 200,000
Germany : > 30,000


Dresden 2024


Motivation

- All Structures Age
- 1st World: Concrete [Bridges, Buildings, …]
  - Life Span: 60-80 Years (Weathering, Cracks, …)
  - Many Bridges and Buildings Nearing That Age
- ➔ Infrastructure Crisis

- Q1: Given Loads and Measurements:
  Can We Infer State of Material (Weakening) ?
- Q2: If We Know/Suspect Weakening:
  Where Should We Reinforce ?

Weakening/Monitoring of Structures

- Problem of High Importance ➔ Considerable Body of Work
- Frequency Domain (Sensors: Accelerometers)
  - Since Mid-70s
  - Large Effort at Sandia National Labs
  - Easy to Detect **That**, But Not **Where**
- Time Domain
  - Several (Some Adjoints in Time-Domain)
- Steady: Adjust/Approximate FEM Model from Measurements
  - Ladeveze et al.
  - Aubry et al. (Adjoints+Patches)

- New Here: DT, Continuous Monitoring, High-Fidelity, …

Assumptions (1)

- Monitoring Via Loads and Measurements:
  - *n* (Standard) Loads **f** Given
  - *n* Displacements/Strains Measured at *m* Locations
- Weakening Can Occur at Any Location
  - Most Conservative
- Weakening Described by Field $0 < \alpha(\mathbf{x}) < 1$
- Deformations and Strains Well Described by FEM:

$$\mathbf{K} \cdot \mathbf{u}_i = \mathbf{f}_i \ , \quad i = 1, n \qquad\qquad \mathbf{K} = \sum_{e=1}^{N_e} \alpha_e \mathbf{K}_e$$

Assumptions (2)

- Sensors Limited by Signal/Noise Ratios ➔

$$|\mathbf{u}^m| > u_0 \quad , \quad |\mathbf{s}^m| > s_0$$

- Forces Used to Monitor Structure Limited by Practical Considerations ➔ Not Arbitrary

Key Idea: Obtain Weakening via Optimization

- Assume:
  - *n* Loads **f** Given
  - *n* Displacements/Strains Measured at *m* Locations
- Then: Determine Strength of Material $\alpha(\mathbf{x})$

$$I(\mathbf{u}_n, \alpha) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}^{md}(\mathbf{u}_{ij}^{md} - \mathbf{I}_{ij}^{d} \cdot \mathbf{u}_i)^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}^{ms}(\mathbf{s}_{ij}^{ms} - \mathbf{I}_{ij}^{s} \cdot \mathbf{s}_i)^2$$

- S.T. FEM Discretization/Digital Twin ➔

$$\mathbf{K} \cdot \mathbf{u}_i = \mathbf{f}_i \ , \quad i = 1, n \qquad\qquad \mathbf{K} = \sum_{e=1}^{N_e} \alpha_e \mathbf{K}_e$$

Optimization Via Adjoints

- Extend to Lagrangian Functional ➔

$$L(\mathbf{u}_n, \alpha, \tilde{\mathbf{u}}_n) = I(\mathbf{u}_n, \alpha) + \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^t \cdot (\mathbf{K} \cdot \mathbf{u}_i - \mathbf{f}_i)$$

$$\frac{dL}{d\tilde{\mathbf{u}}_i} = \mathbf{K}\mathbf{u}_i - \mathbf{f}_i = 0$$

$$\frac{dL}{d\mathbf{u}_i} = \underbrace{\sum_{j=1}^{m} w_{ij}^{md} \mathrm{I}_{ij}^{d} (\mathbf{u}_{ij}^{md} - \mathrm{I}_{ij}^{d} \mathbf{u}_i) + \sum_{j=1}^{m} w_{ij}^{ms} \mathrm{J}_{ij}^{s} (\mathbf{s}_{ij}^{ms} - \mathrm{I}_{ij}^{s} \mathbf{s}_i)}_{I_{,\mathbf{u}_i}} + \mathbf{K}^t \tilde{\mathbf{u}}_i = 0$$

$$\frac{dL}{d\alpha_e} = \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^t \frac{d\mathbf{K}}{d\alpha_e} \mathbf{u}_i = \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^t \mathbf{K}_e \mathbf{u}_i$$

Adjoint Solvers

- Consequences:
  - Gradient of $L_{,I}$ w.r.t. α: n Forward/Adjoint Solves
  - Cost for Evaluation of $L_{,I}$: Independent of the Number of Variables Used for α (!)

- ➔ Can Use Detailed FEM Models ➔ Detailed Digital Twin
  - `Hi-Fi Digital Twin'
  - Based on Algebraic Equations (FEMs), Not PDEs ➔ General

- Most Structural Problems: $\mathbf{K} = \mathbf{K}^t$ ➔
  - Direct Solvers: Cost of Adjoint Negligible
  - Iterative Solver: Preconditioner Can Be Re-Utilized

Optimization Cycle

- For Each Force/Measurement Pair i:
  - With Current α: Obtain Deformations/Strains $\rightarrow \mathbf{u}_i$
  - With Current α, $\mathbf{u}_i$ and $\mathbf{u}^{md}_{ij}$, $\mathbf{s}^{md}_{ij}$: Obtain Adjoints $\rightarrow \mathbf{u}\tilde{}_i$
  - With $\mathbf{u}_i$, $\mathbf{u}\tilde{}_i$: Obtain Gradients $\rightarrow I_{\mathrm{i}\ ,\alpha} = L_{\mathrm{i}\ ,\alpha}$

- Sum Up the Gradients $\rightarrow I_{,\alpha}$

- If Necessary: Smooth Gradients $\rightarrow I^s_{,\alpha}$

- Update $\alpha_{\mathrm{new}} = \alpha_{\mathrm{old}} - \gamma\ I^s_{,\alpha}$
  - Or: BFGS, Quasi-Newton or Newton

Interpolation of Displacements/Strains

- Displacements

$$\mathbf{u}_i(\mathbf{x}_i^m) = \mathbf{I}_i^d(\mathbf{x}_i^m) \cdot \mathbf{u}$$

- Strains

$$\mathbf{s} = \mathbf{D} \cdot \mathbf{u}$$

$$\mathbf{s}_i(\mathbf{x}_i^m) = \mathbf{I}_i^s(\mathbf{x}_i^m) \cdot \mathbf{s} = \mathbf{I}_i^s(\mathbf{x}_i^m) \cdot \mathbf{D} \cdot \mathbf{u}$$

Weights (1)

$$I(\mathbf{u}_n, \alpha) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}^{md}(\mathbf{u}_{ij}^{md} - \mathbf{I}_{ij}^{d}\cdot\mathbf{u}_i)^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}^{ms}(\mathbf{s}_{ij}^{ms} - \mathbf{I}_{ij}^{s}\cdot\mathbf{s}_i)^2$$

- Problem: Dimensionally Inconsistent
- Option 1: Local Weights

$$w_{ij}^{md} = \frac{1}{(\mathbf{u}_{ij}^{md})^2}\ ;\ w_{ij}^{ms} = \frac{1}{(\mathbf{s}_{ij}^{ms})^2}$$

- Option 2: Average Weights

$$u_{av} = \frac{\sum_{j=1}^{m}|\mathbf{u}_{ij}^{md}|}{m}\ ;\ w_{ij}^{md} = \frac{1}{u_{av}^2}\ ;\ s_{av} = \frac{\sum_{j=1}^{m}|\mathbf{s}_{ij}^{ms}|}{m}\ ;\ w_{ij}^{ms} = \frac{1}{s_{av}^2}$$

Weights (3)

- Option 3: Max

$$u_{max} = max(|\mathbf{u}_{ij}^{md}|, j = 1, m) \ ; \ w_{ij}^{md} = \frac{1}{u_{max}^2}$$

$$s_{max} = max(|\mathbf{s}_{ij}^{ms}|, j = 1, m) \ ; \ w_{ij}^{ms} = \frac{1}{s_{max}^2}$$

- Option 4: Limited Local Weights

$$w_{ij}^{md} = \frac{1}{max(\epsilon u_{max}, |\mathbf{u}_{ij}^{md}|))^2} \ ; \ w_{ij}^{ms} = \frac{1}{max(\epsilon s_{max}, |\mathbf{s}_{ij}^{ms}|))^2}$$

$$w_{ij}^{md} = \frac{1}{max(\epsilon u_{max}, |\mathbf{u}_{ij}^{md}|))^2} \ ; \ w_{ij}^{ms} = \frac{1}{max(\epsilon s_{max}, |\mathbf{s}_{ij}^{ms}|))^2}$$

A First Set of Questions

- Does it Work ?
- Can It Resolve Multiple Weakening Regions ?
- Which Set of Forces ?
- Which Set of Sensors ?

Smoothing of Gradients (1)

- Gradient `Lives in H^{-1}' ➔ Need to Smooth
- Starting Point: From Elements to Points/DOFs

$$\alpha_p = \frac{\sum_e \alpha_e V_e}{\sum_e V_e}$$

Smoothing of Gradients (2)

- Option 1: Cycle Between Points and Elements
- In Each Iteration:
- Step 1: From Points/DOFs to Elements

$$\alpha_e = \frac{1}{n_e} \sum_i \alpha_i$$

- Step 2: From Elements to Points/DOFs

$$\alpha_p = \frac{\sum_e \alpha_e V_e}{\sum_e V_e}$$

- Works Surprisingly Well

Smoothing of Gradients (3)

- Option 2: H^1/Weak Laplacian Smoothing

$$\left[1 - \lambda \nabla^2\right] \alpha = \alpha_0 \quad , \quad \alpha_{,n}\big|^{\Gamma} = 0$$

- After FEM Discretization:

$$\left[\mathbf{M}_{\mathrm{c}} + \lambda \mathbf{K}_{\mathrm{d}}\right] \boldsymbol{\alpha} = \mathbf{M}_{\mathrm{p1p0}} \boldsymbol{\alpha}_0$$

- Choice of λ ?

Smoothing of Gradients (4)

- Option 3: Pseudo-Laplacian Smoothing

$$\left[1 - \lambda \nabla h^2 \nabla\right] \alpha = \alpha_0$$

- After FEM Discretization:

$$\left[\mathbf{M}_{\mathrm{c}} + \lambda \mathbf{K}_{\mathrm{d}}\right] \boldsymbol{\alpha} = \mathbf{M}_{\mathrm{p1p0}} \boldsymbol{\alpha}_0$$

- For Linear Elements:

$$\left[\mathbf{M}_{\mathrm{c}} + \lambda \left(\mathbf{M}_{\mathrm{l}} - \mathbf{M}_{\mathrm{c}}\right)\right] \boldsymbol{\alpha} = \mathbf{M}_{\mathrm{p1p0}} \boldsymbol{\alpha}_0$$

- Typical Value: $\lambda=0.05$

Smoothing of Gradients (5)

- Option 4: Convolution Integrals
- Size of Convolution Footprint: O(3-5 h)

- Other Option: Smooth α After Update
  - Works, But Solution Not as `Sharp'
  - ➔ Prefer Gradient Smoothing

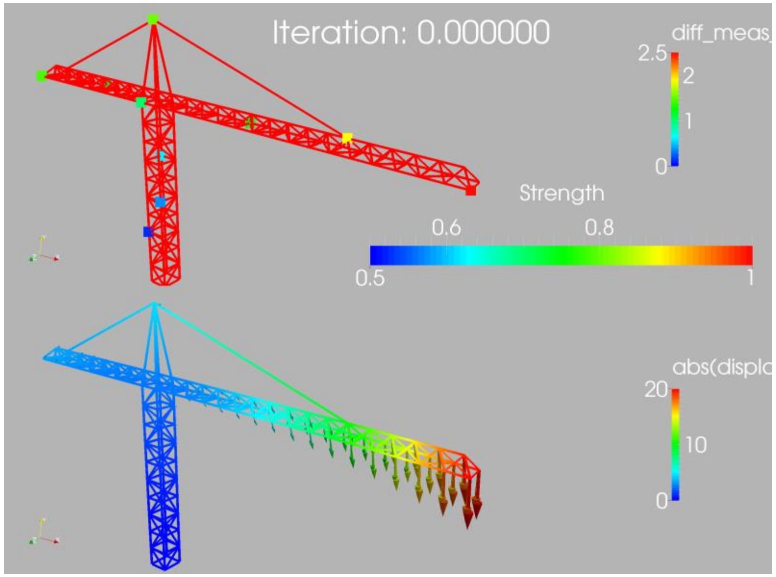
Crane

- Truss Elements
- Forces at Extreme Ends of Arm
- 10 Measurement Points
  - Displacements
  - Strains
- Smoothing: Simple Averaging
  - Element → Point → Element [No Volume Considerations]
  - Separate Smoothing of Gradients and Strength Factors
- FEELAST

Crane: 1 Load; **Displacements**

Weakened Base

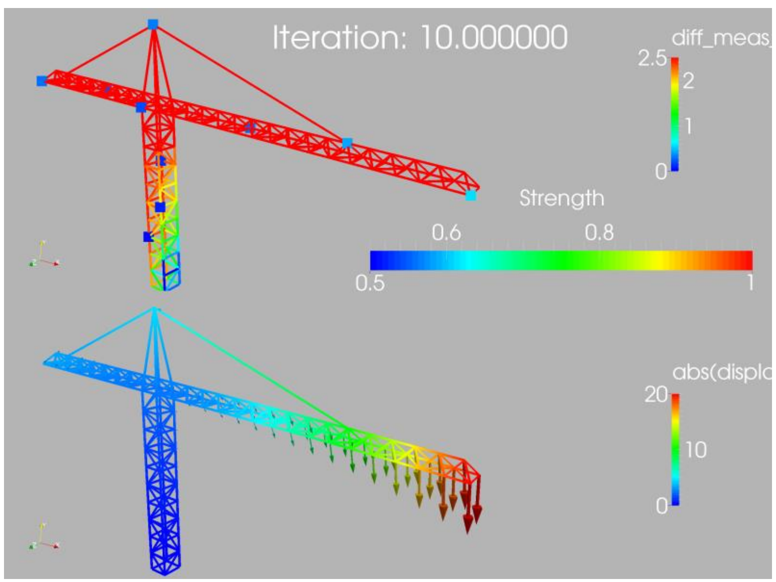
Grad Smoothing, DOF: Base

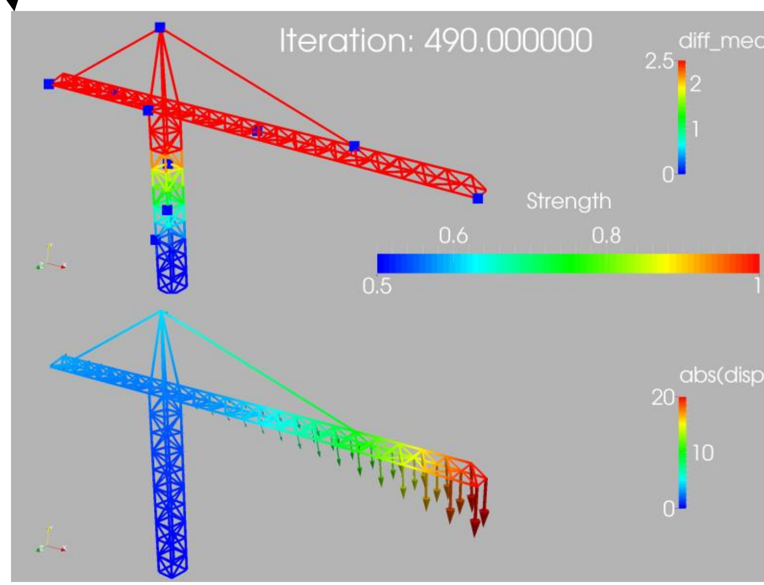


Iteration 0

Iteration 10

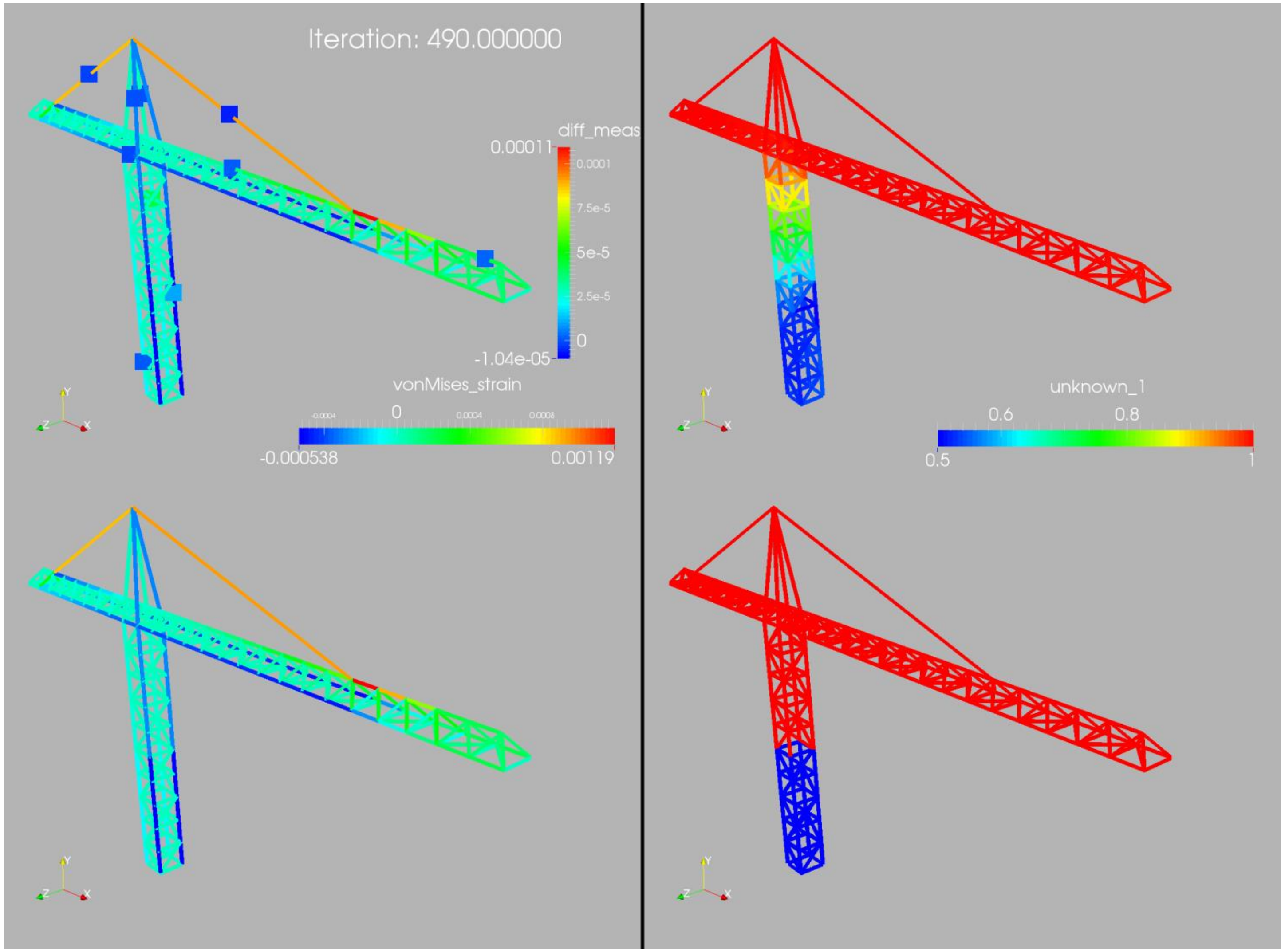
Final





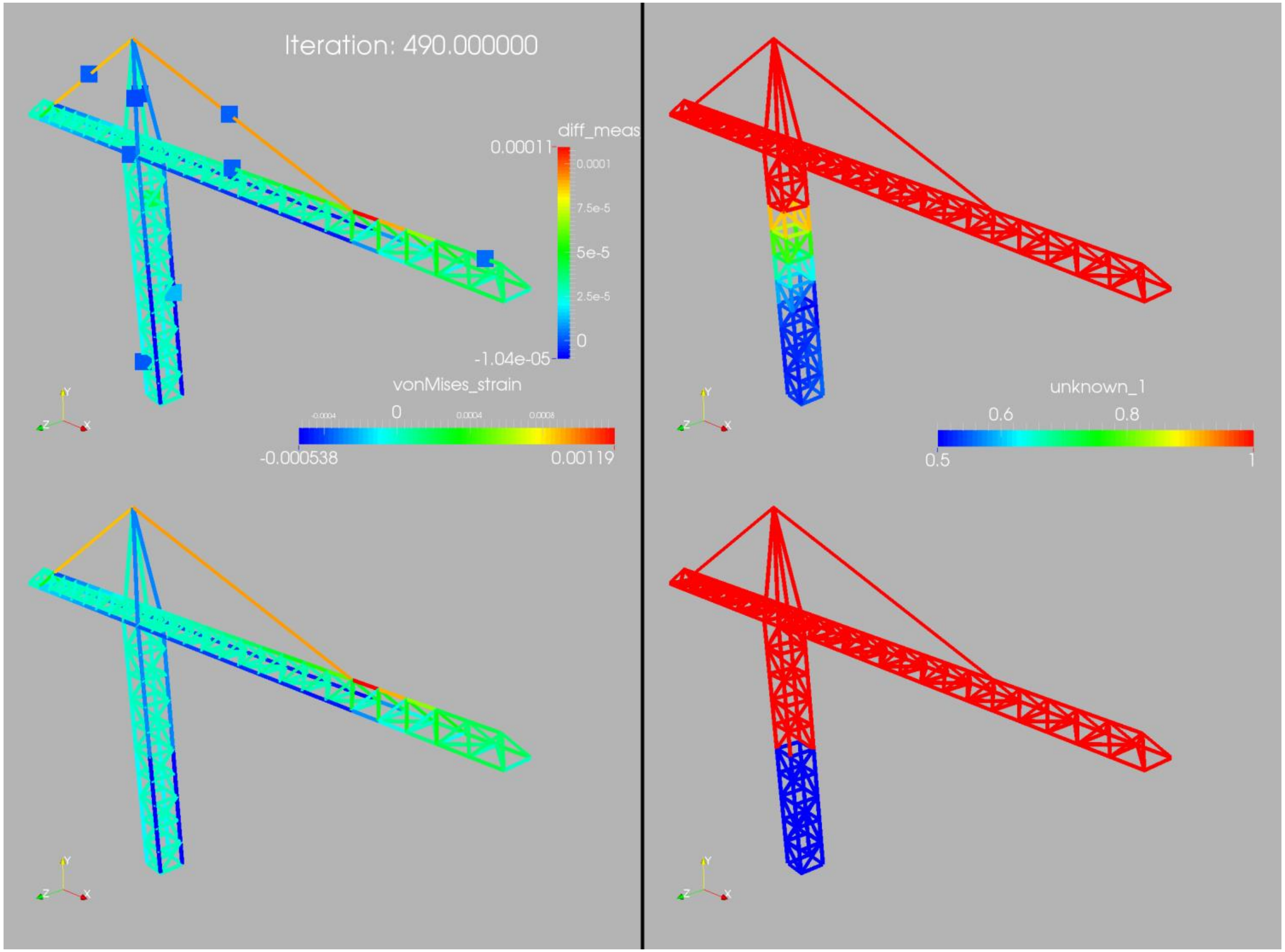
Crane: 1 Load, Strains

Grad Smoothing



Crane: 1 Load, Strains

Grad Smoothing, DOF: Tower Only



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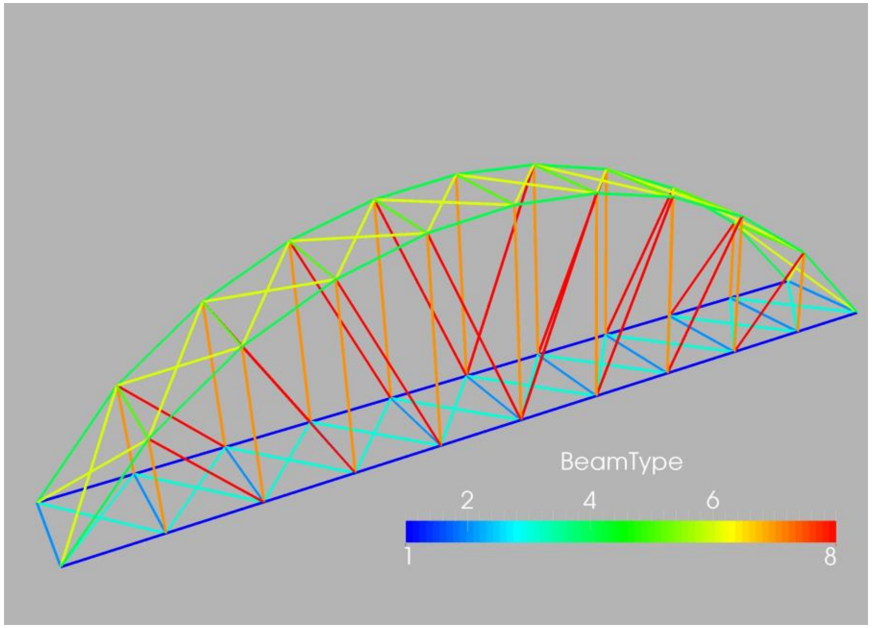
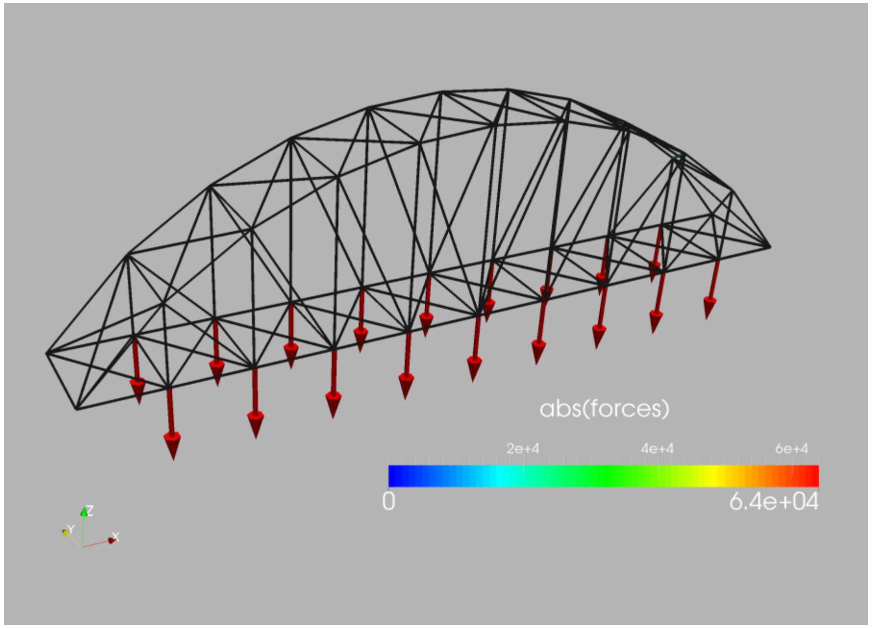
Bridge

Dimensions: 40x5x10m





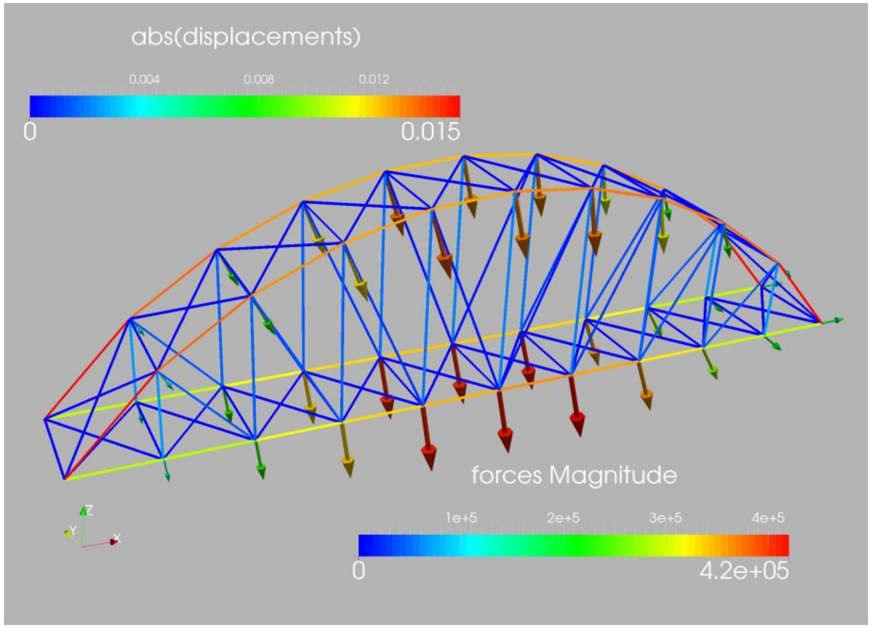
Material: Steel

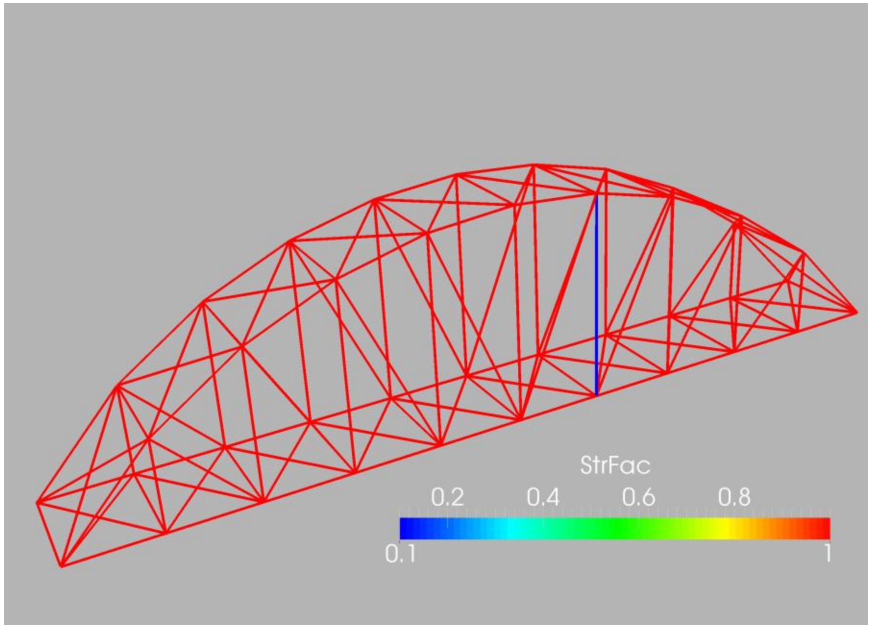
Trusses: A=1-100cm^2

FEELAST

Forces [+Gravity]

Bridge

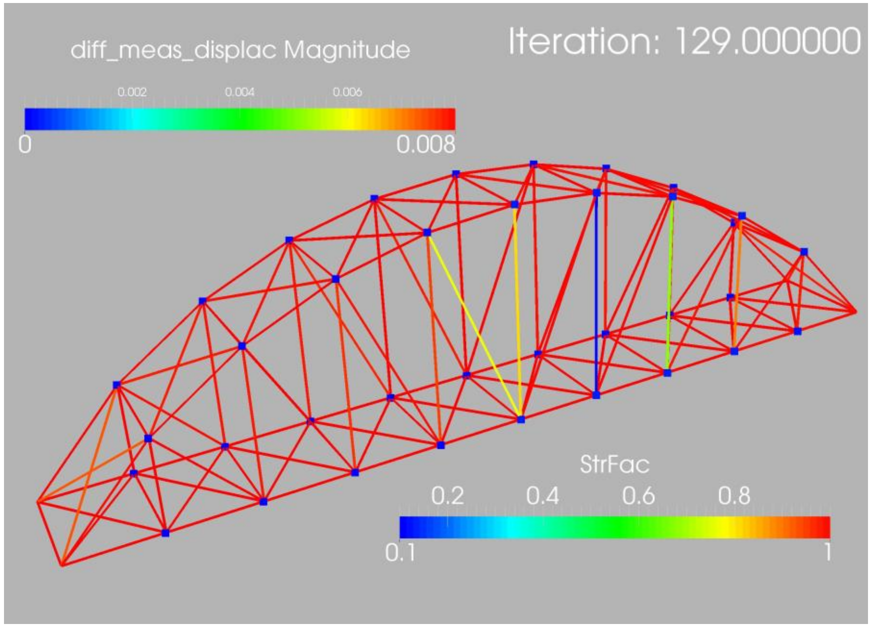


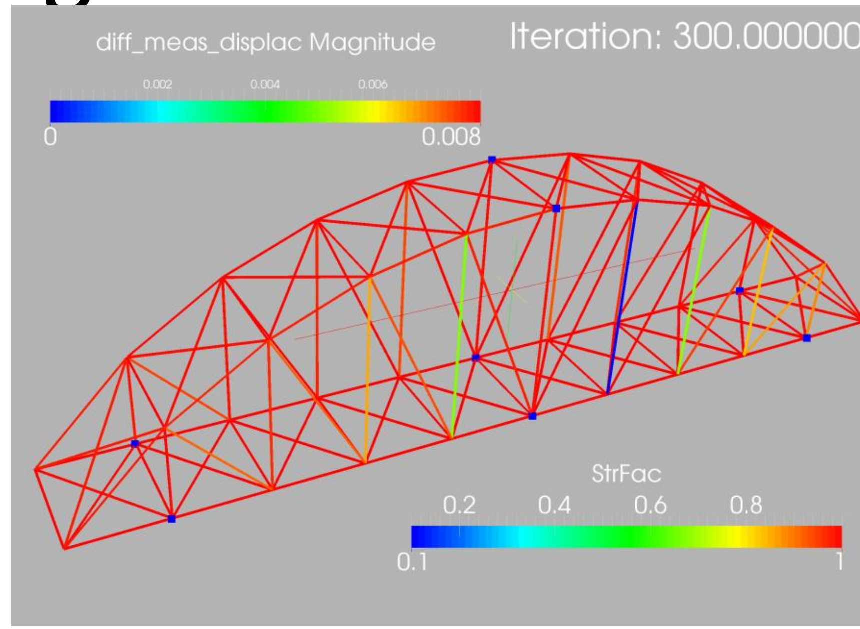


Displacements + Int Forces

Target StrFac

Bridge


diff_meas_displac Magnitude
Iteration: 129.000000
0.002
0.004
0.006
0
0.008
StrFac
0.2
0.4
0.6
0.8
0.1
1


diff_meas_displac Magnitude
Iteration: 300.000000
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0.004
0.006
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0.008
StrFac
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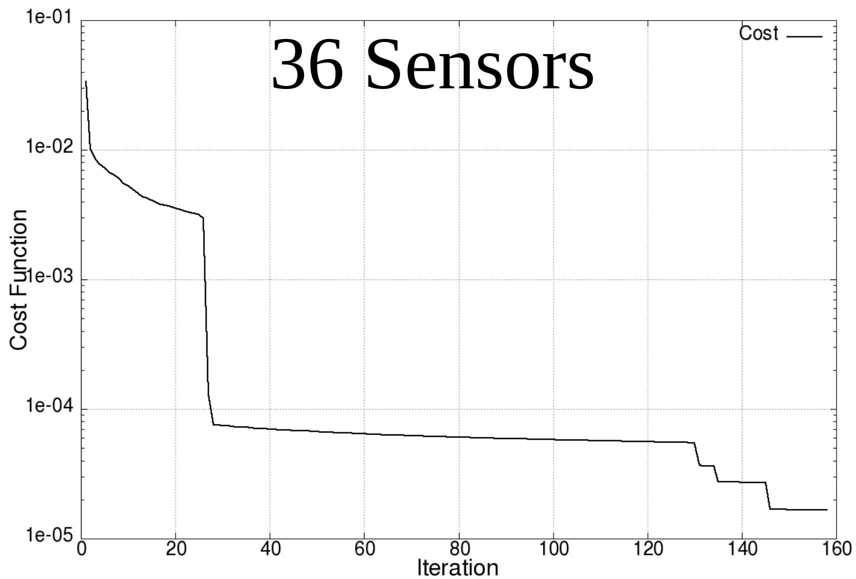
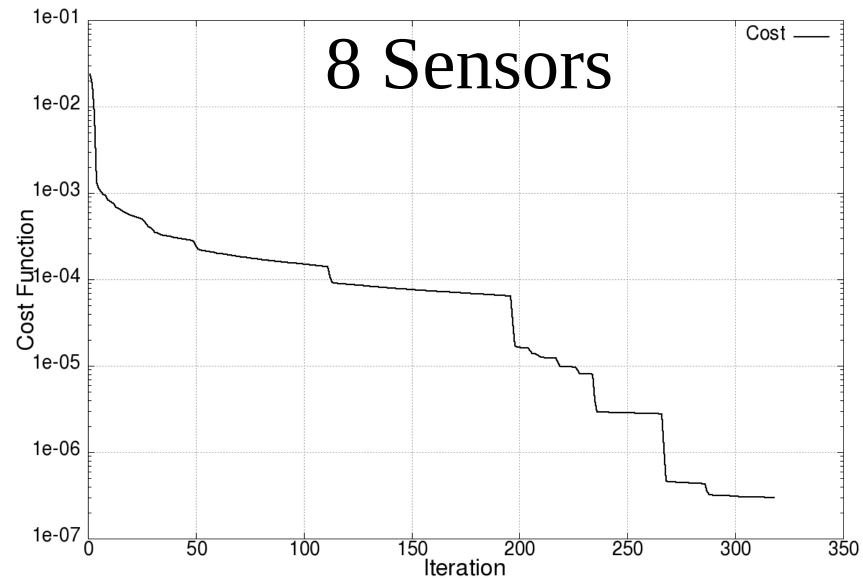
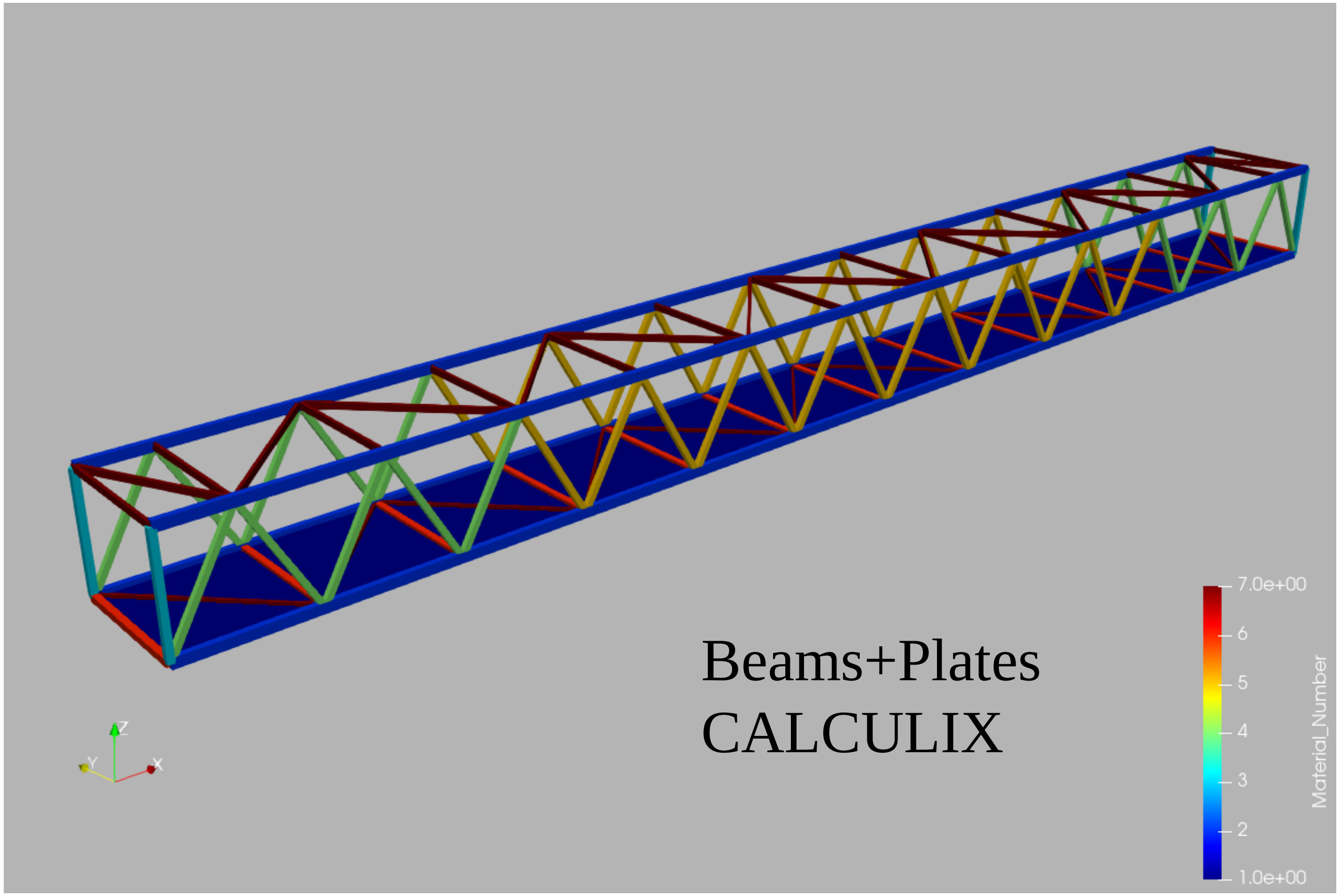
0.1
1

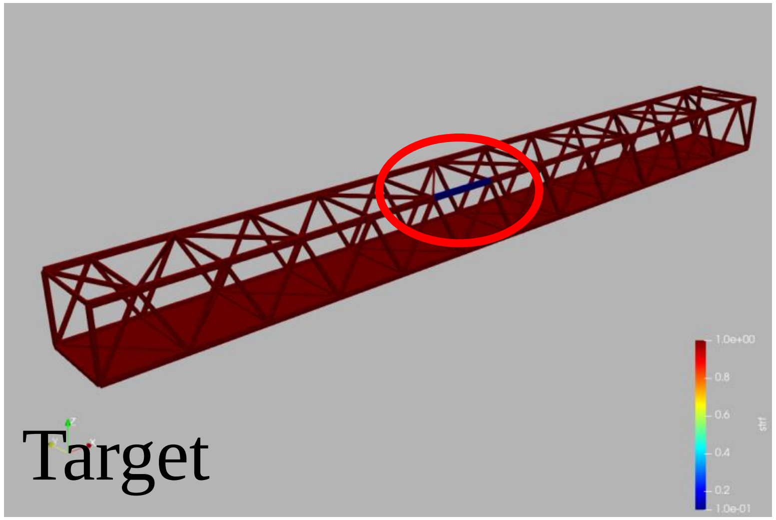

36 Sensors
Cost
Cost Function
Iteration
1e-01
1e-02
1e-03
1e-04
1e-05
0
20
40
60
80
100
120
140
160

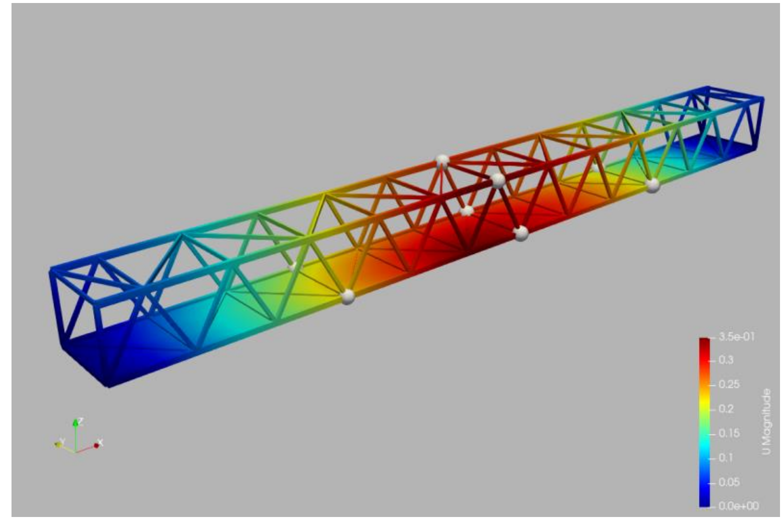

8 Sensors
Cost
Cost Function
Iteration
1e-01
1e-02
1e-03
1e-04
1e-05
1e-06
1e-07
0
50
100
150
200
250
300
350

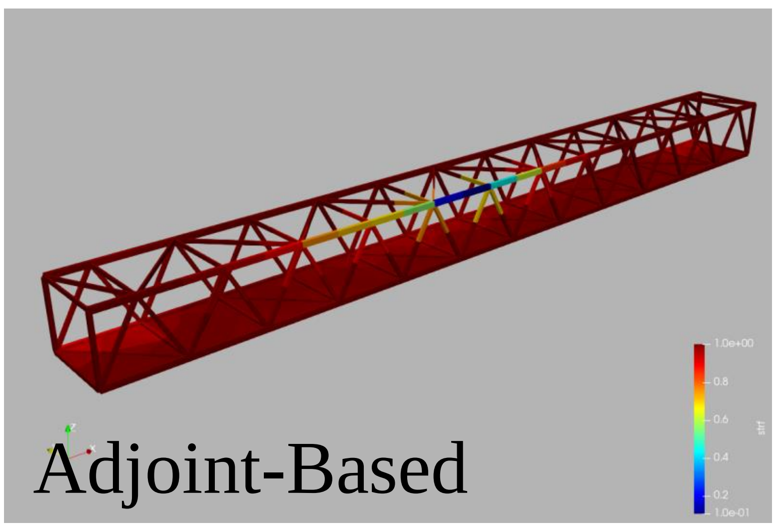
Footbridge

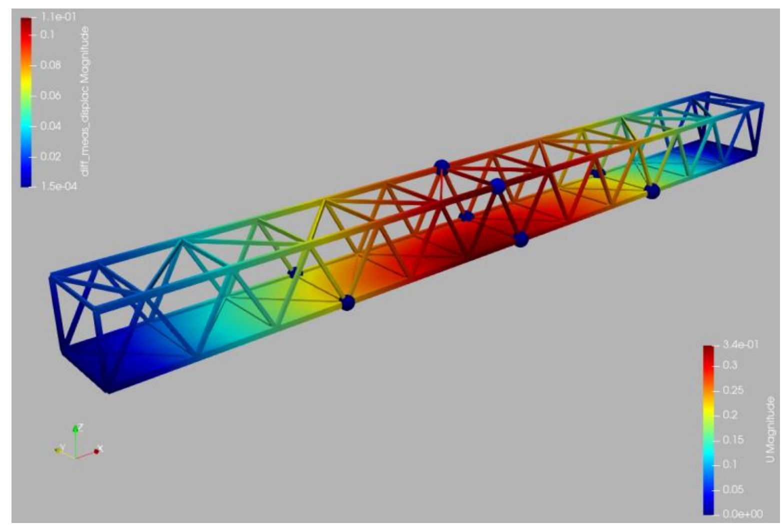

Beams+Plates
CALCULIX
7.0e+00
6
5
4
3
2
1.0e+00
Material_Number
Z
Y
X

Footbridge


Target
1.0e+00
0.8
0.6
0.4
0.2
1.0e-01


3.5e-01
0.3
0.25
0.2
0.15
0.1
0.05
0.0e+00
U Magnitude


Adjoint-Based
1.0e+00
0.8
0.6
0.4
0.2
1.0e-01


1.1e-01
0.1
0.08
0.06
0.04
0.02
1.5e-04
diff_meas_displac Magnitude
3.4e-01
0.3
0.25
0.2
0.15
0.1
0.05
0.0e+00
U Magnitude

Footbridge

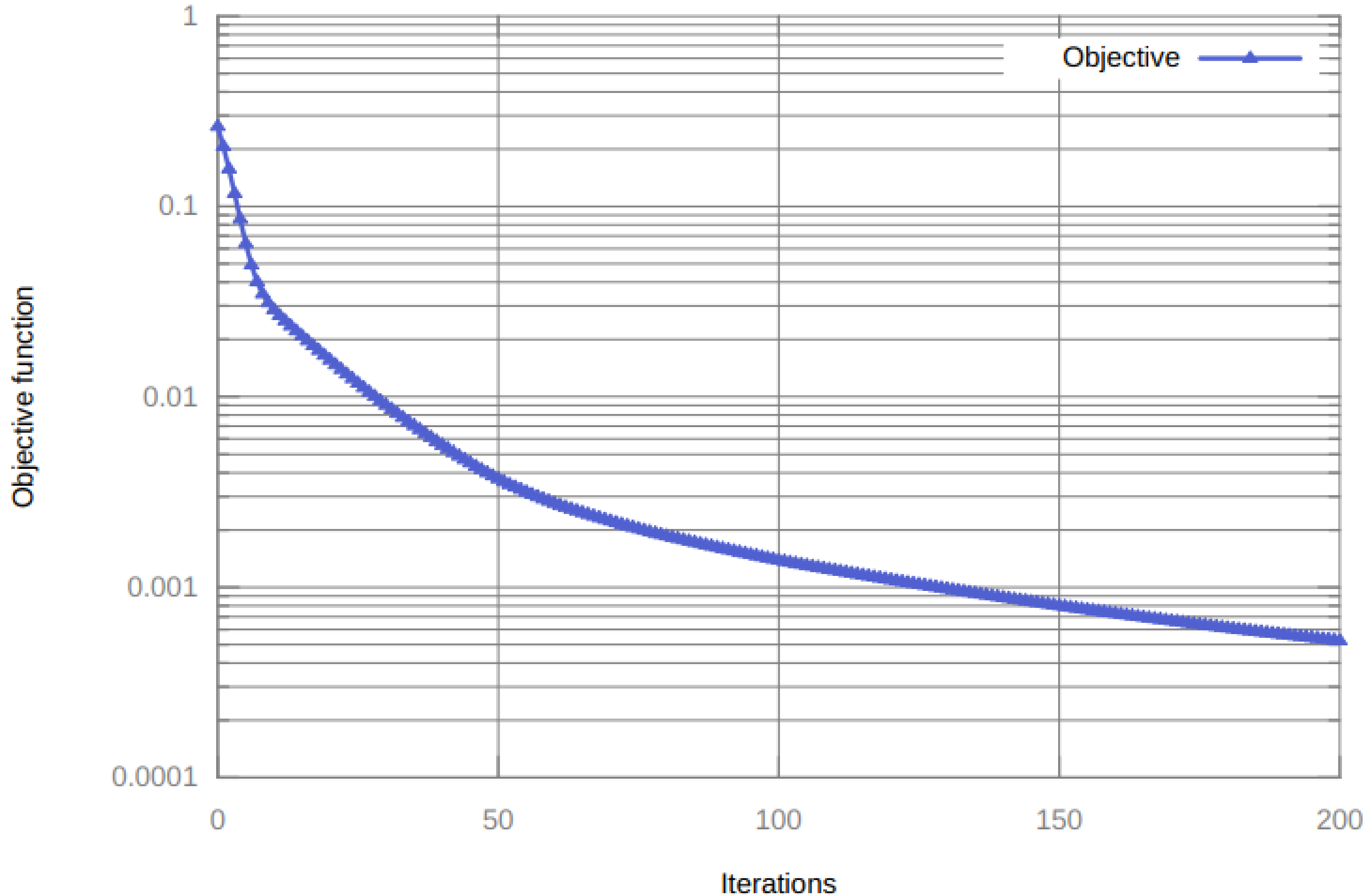


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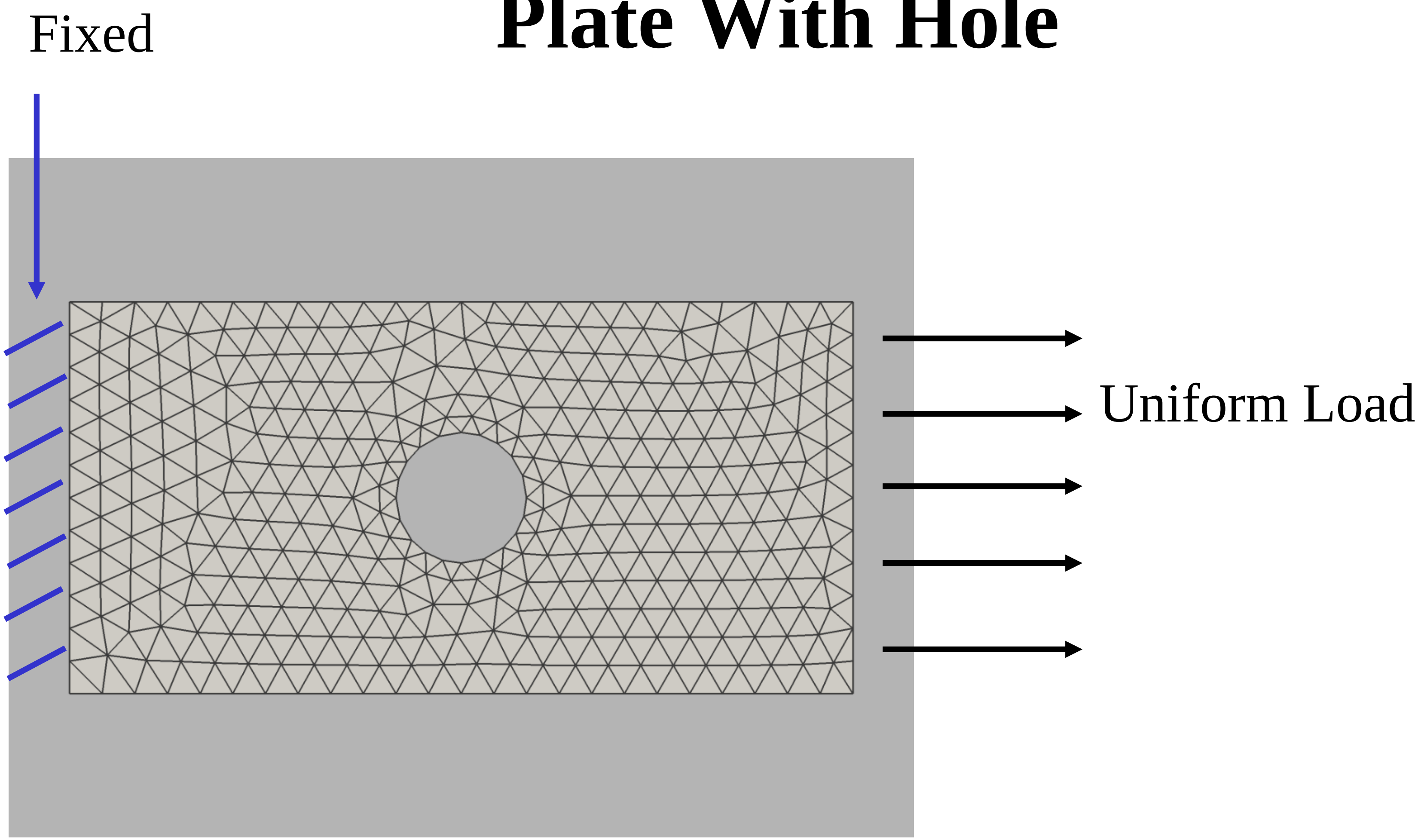
Plate With Hole



FEELAST

Plate With Hole:
Small Damaged Region

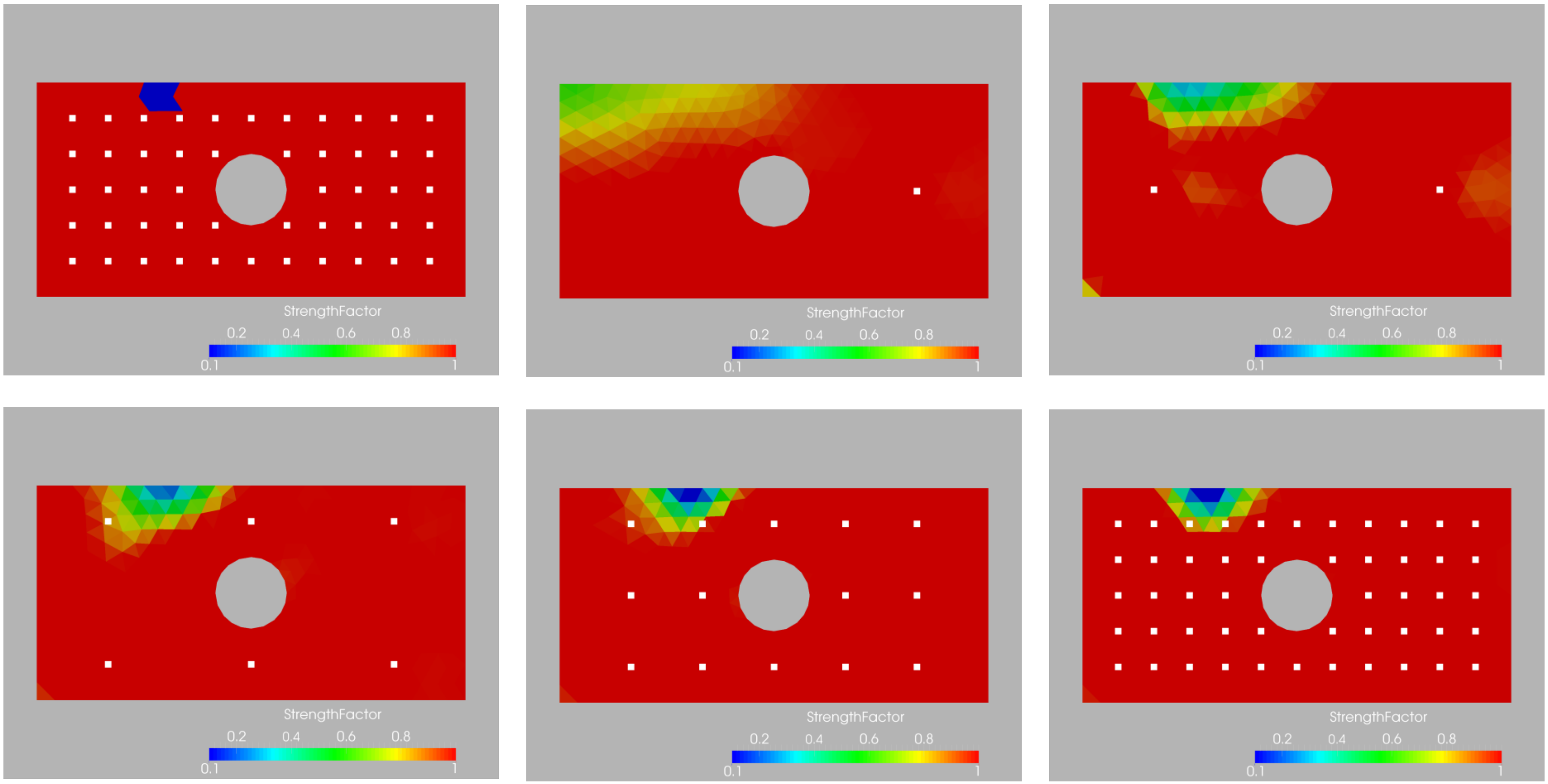


Plate With Hole

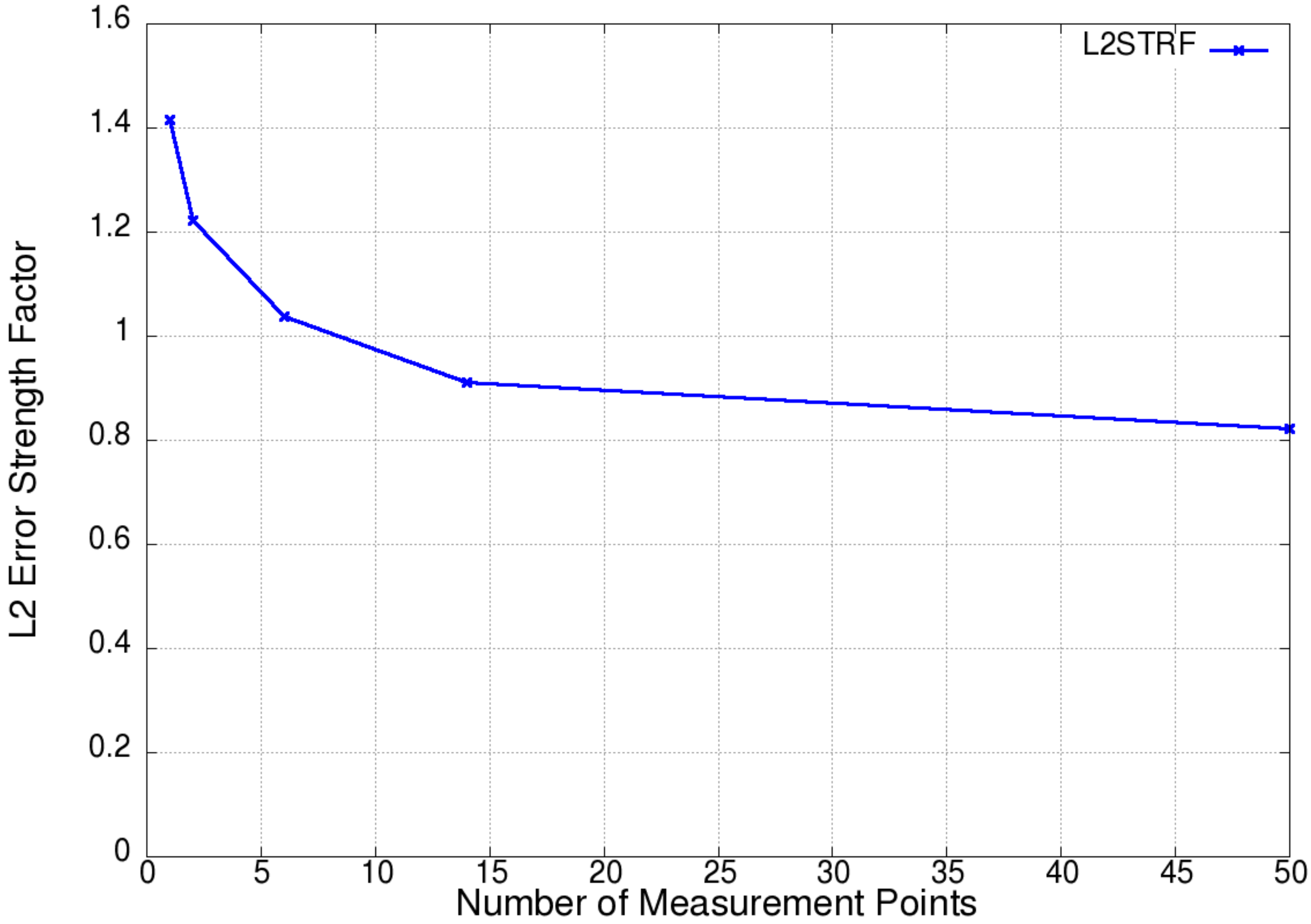


Plate With Hole 3D

- 16Kels
- 120Kels
- X=0: Clamped
- X=Xmax: Fx
- FEELAST

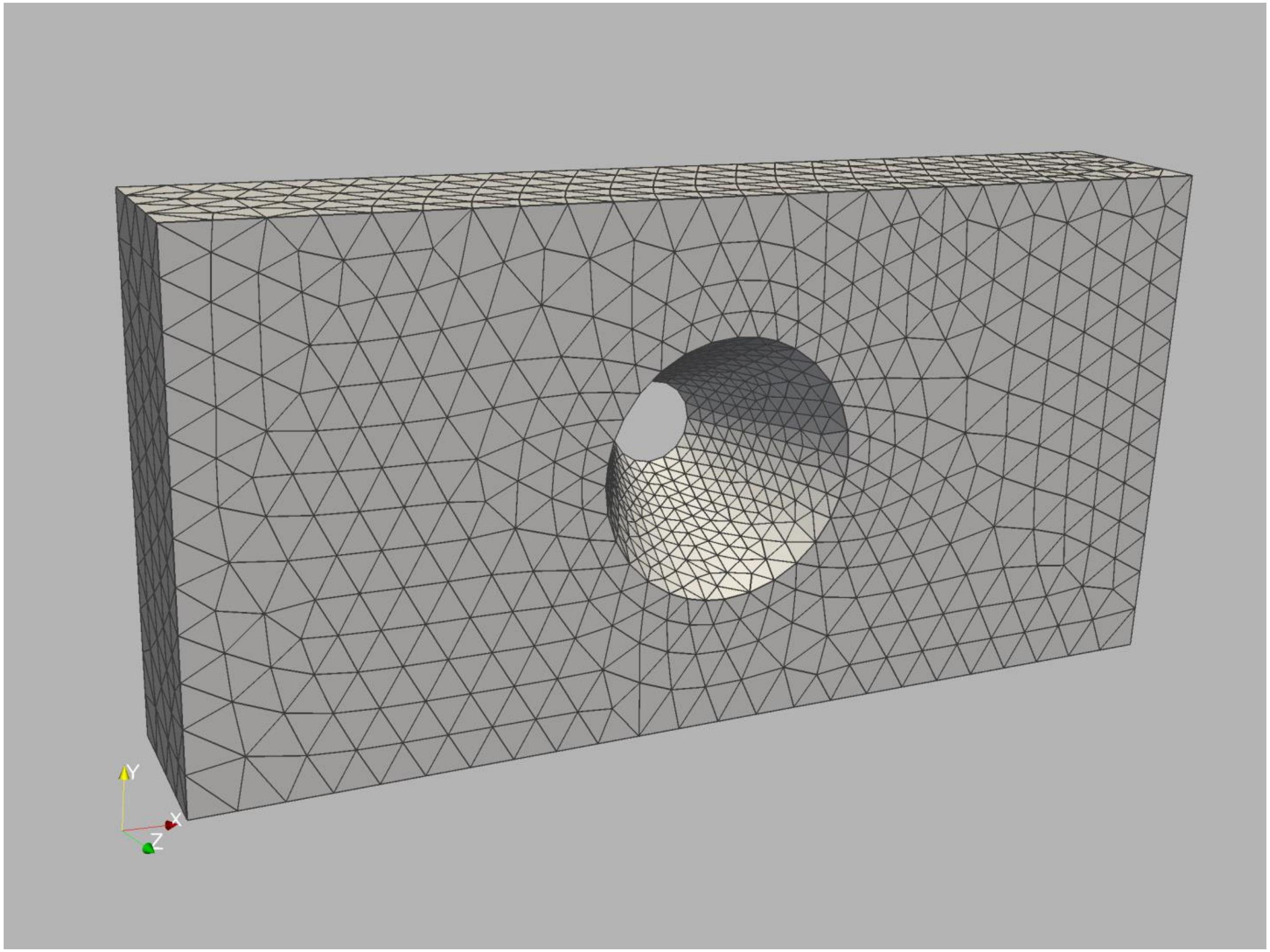


Plate With Hole 3D

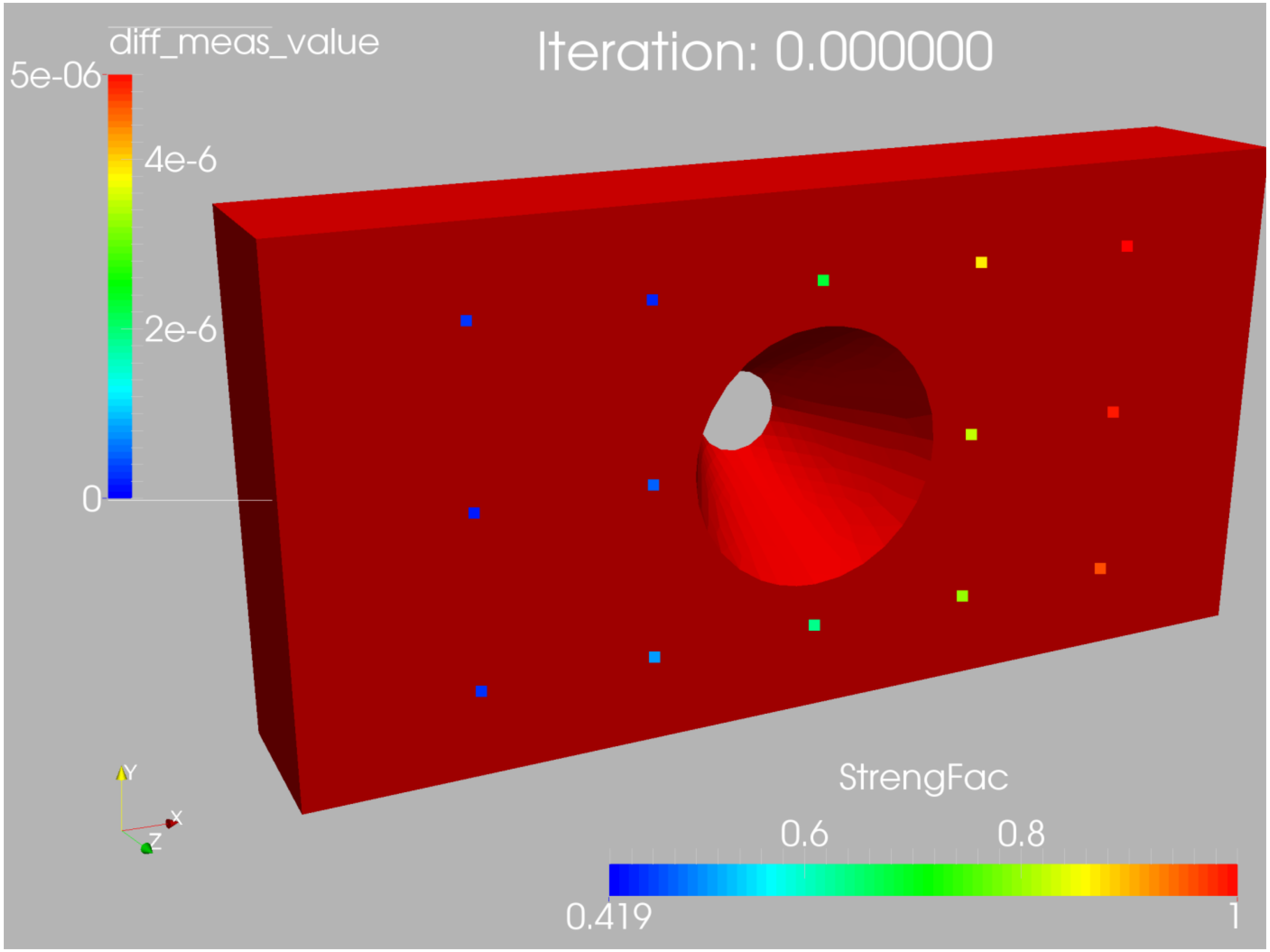
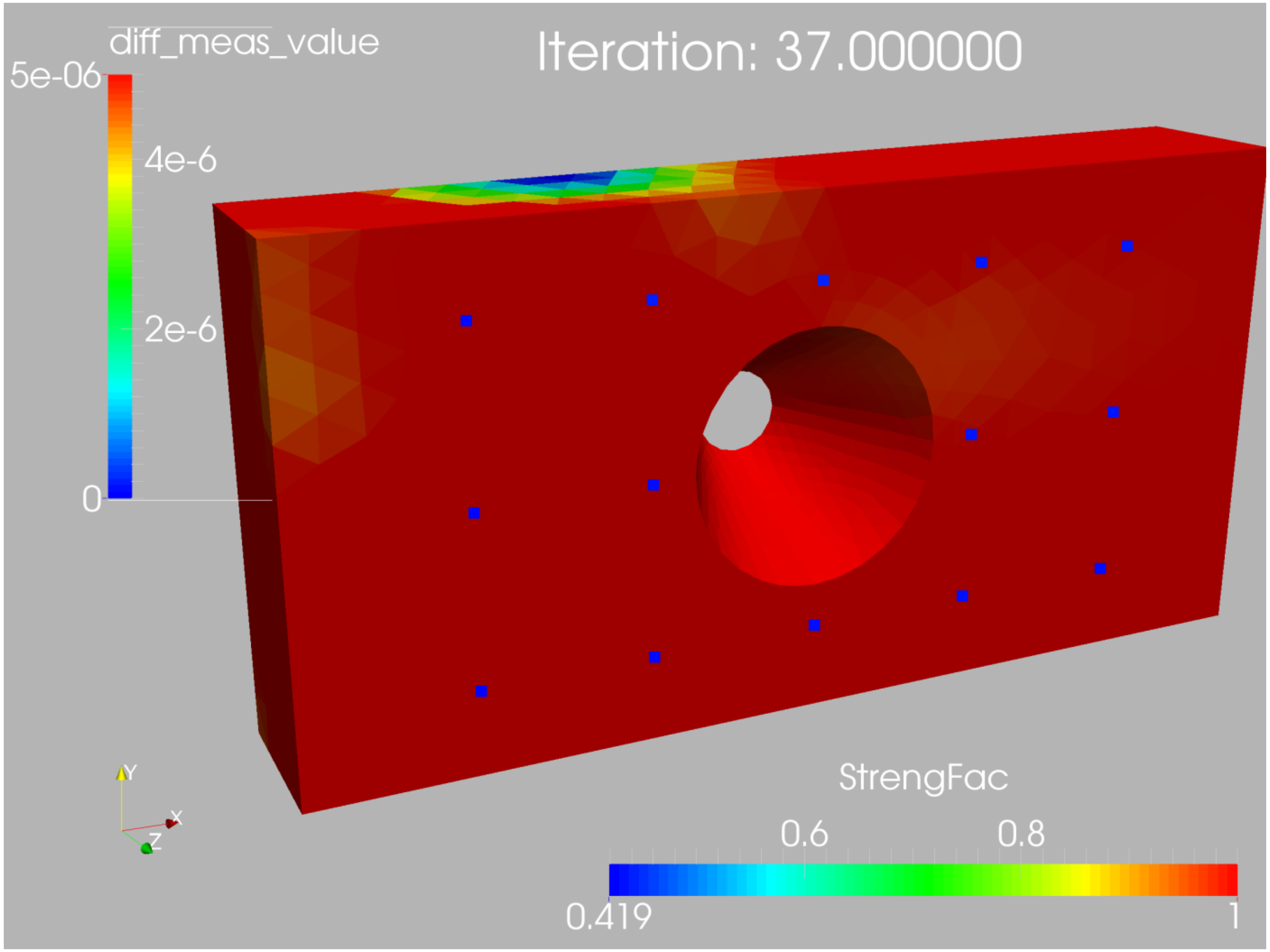


Plate With Hole 3D



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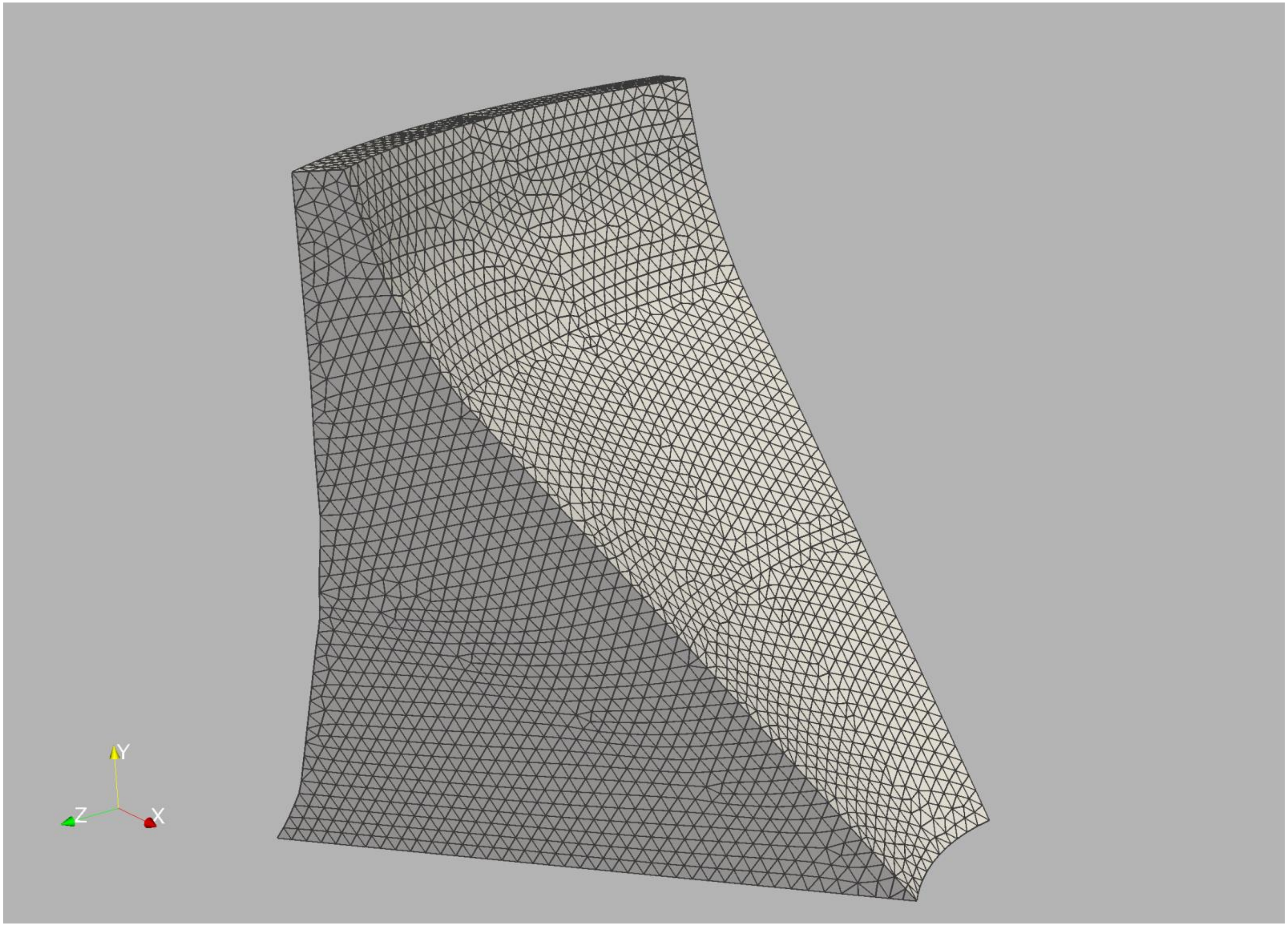
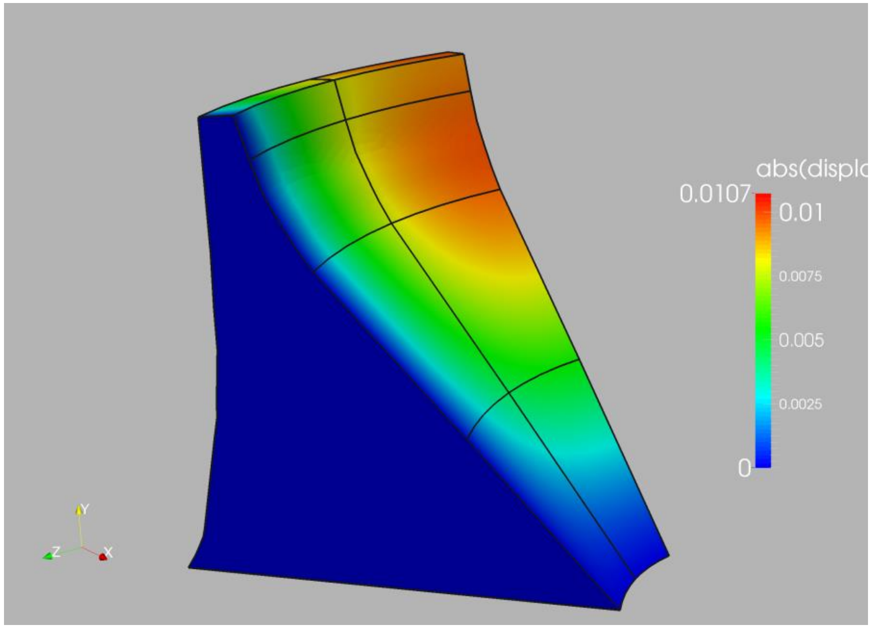
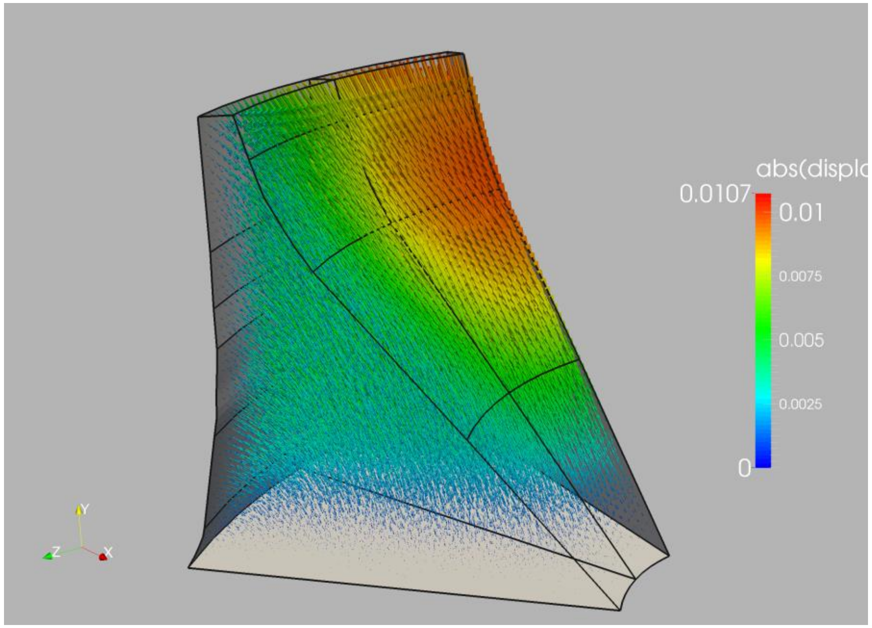
Hoover Dam

Hoover Dam



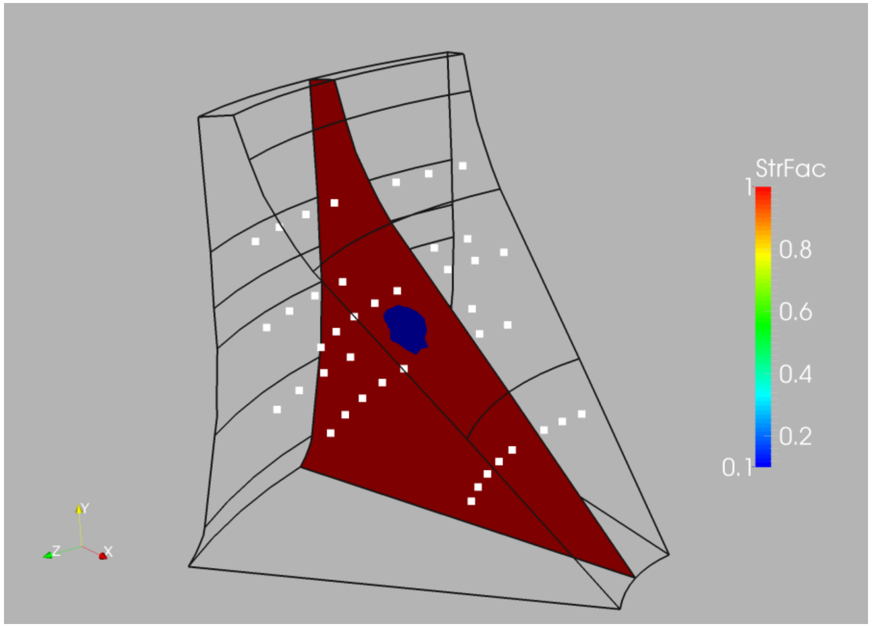
Hoover Dam: 51 Sensors

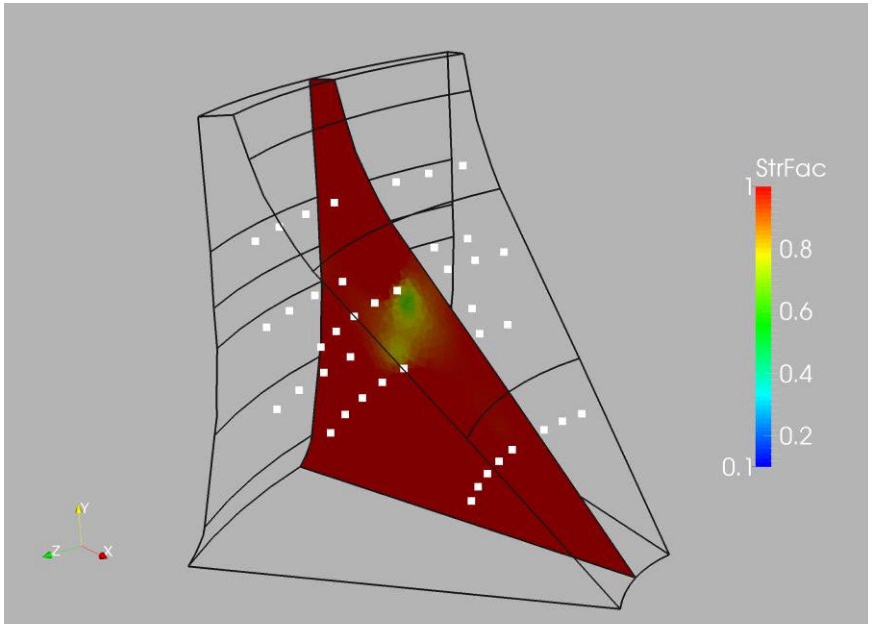




FEELAST

Hoover Dam: 51 Sensors

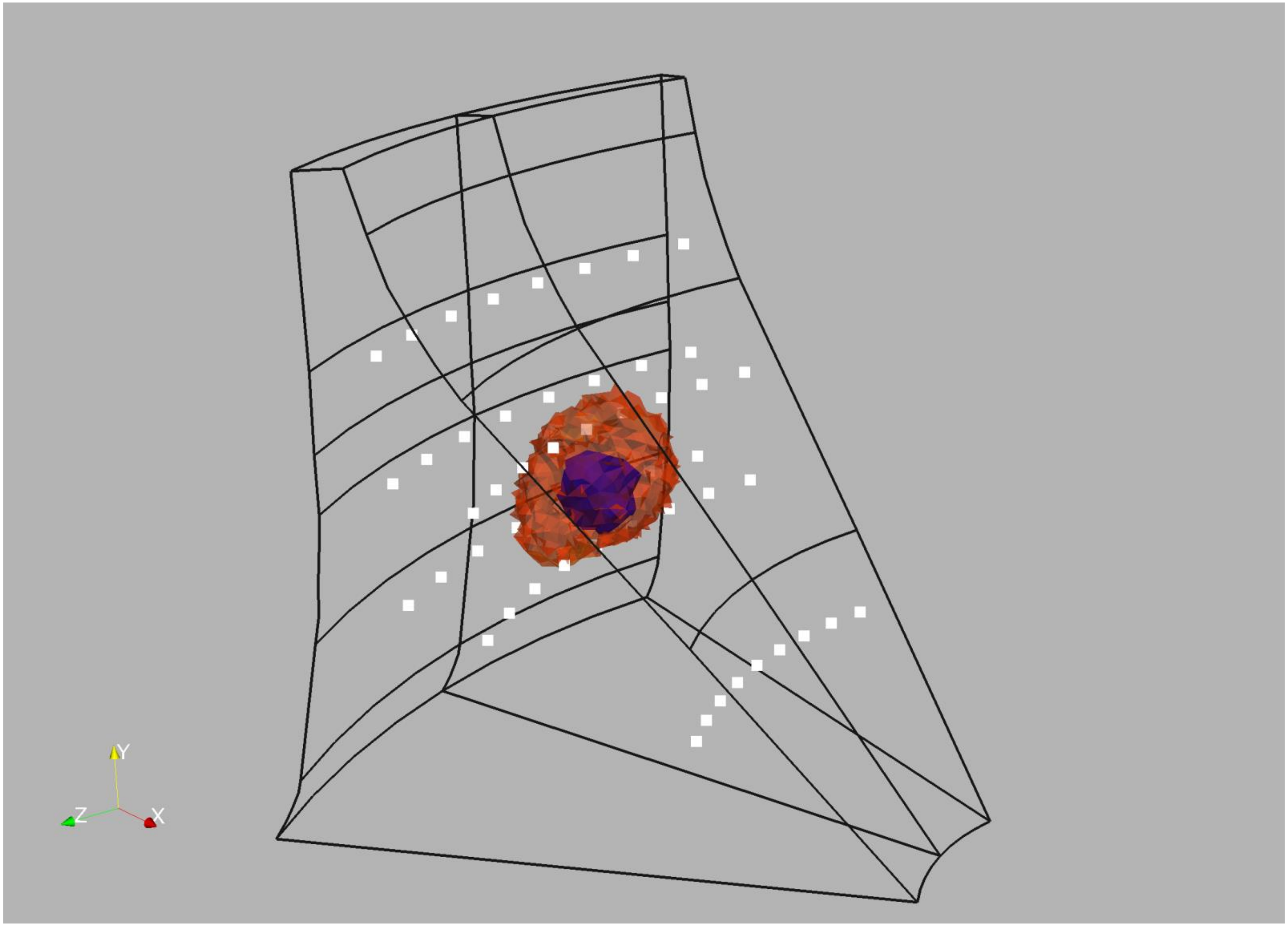




Target

Detected/Recovered

Hoover Dam: 51 Sensors



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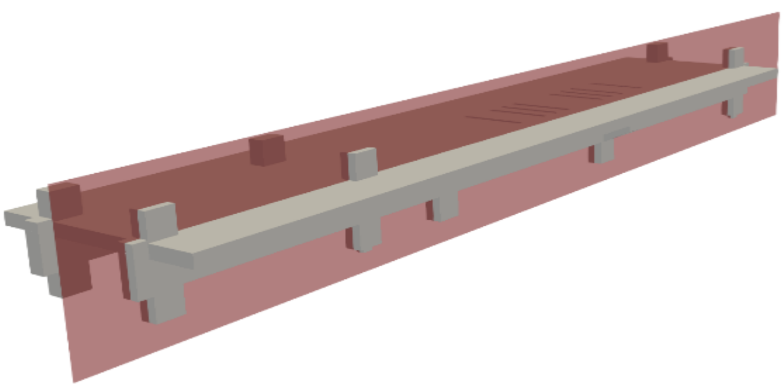
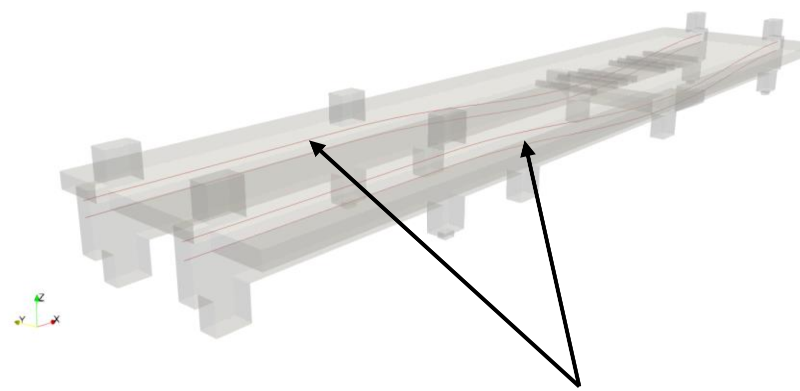
Concerto Bridge

- Built in 2005 by iBMB – TU Braunschweig for Testing Measurement Technologies
- Loads Can be Applied Using Hydraulic Presses and External Tendons
- Constructed with Prestressed, Post-Tensioned Concrete

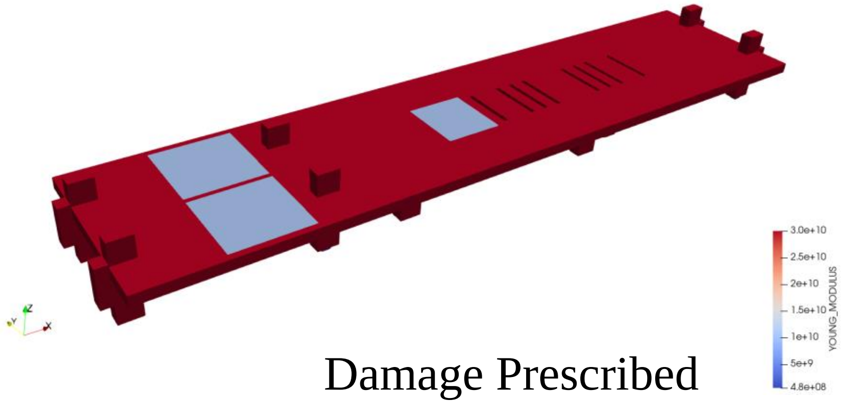




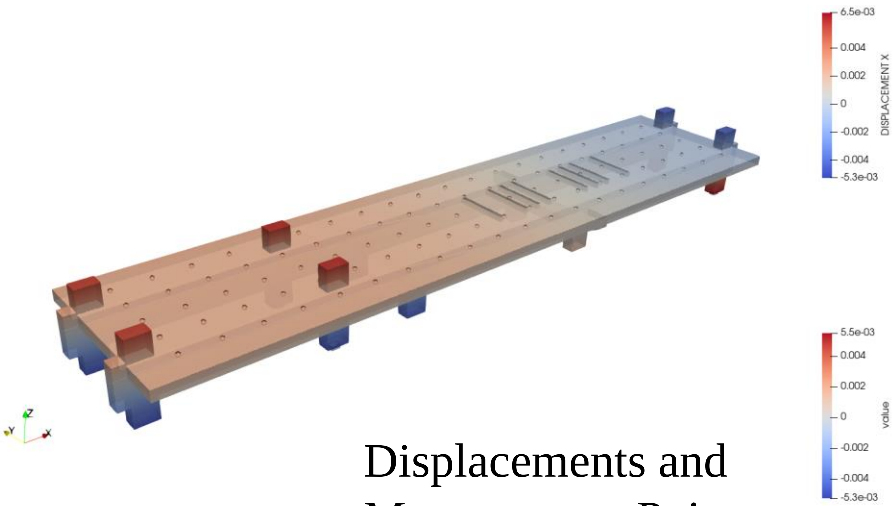
Concerto Bridge

Configuration: KRATOS

- Concrete: 77k Small Displacement Hex Elements
- Tendons: 800 Truss Elements
- 100 Potential Displacement (x) Sensors



Damage Prescribed



Displacements and Measurement Points

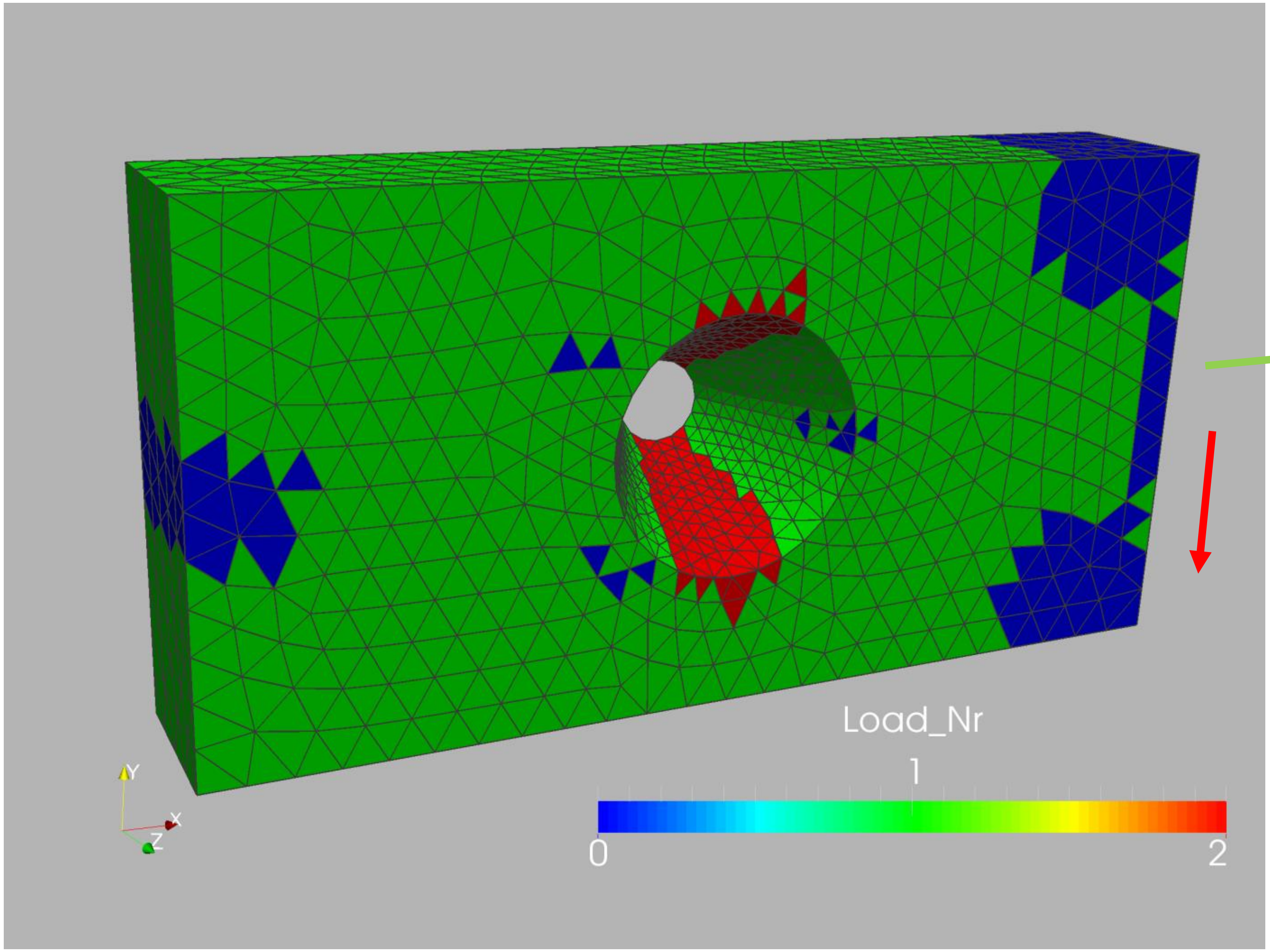
Damage Found

Which Set of Forces ?

Which Set of Forces ?

- Key Idea: Avoid Regions With No Strains
  - No Strain → No Effect of Weakening → No Signal
- Recursive Algorithm:
  - Compute The Nr. Of Elements/Volume With Measurable Strains for Each Force
- Until No More Elements Are Available/Sensed:
  - Select Force With Highest Nr. Of Elements/Volume
  - Remove Elements Marked So Far

Plate With Hole 3D



Load 1

Load 2

Which Set of Sensors ?

Which Set of Sensors ?

- Key Idea: Try to Detect Largest Possible Weakening Region
- Recursive Algorithm:
  - Compute The Nr. Of Sensors With Measurable Strains for Each Weakening Region (Min: 1 Element)
- Until No More Sensors Are Available Elements Sensed:
  - Select Sensor With Highest Nr. Of Weakening Regions Sensed
  - Remove Elements Marked So Far

Which Set of Sensors ?

- `Forward-Based': Change in Element/Region ➔

$$(\mathbf{K} + \Delta\mathbf{K}) \cdot (\mathbf{u} + \Delta\mathbf{u}) = \mathbf{f}$$

- With Original Balance Equation:

$$\mathbf{K} \cdot \Delta\mathbf{u} = -\Delta\mathbf{K} \cdot (\mathbf{u} + \Delta\mathbf{u})$$

- Iterative Solution:

$$\mathbf{K} \cdot \Delta\mathbf{u}^{i+1} = -\Delta\mathbf{K} \cdot (\mathbf{u} + \Delta\mathbf{u}^{i}) \quad , \quad i = 1, k$$

$$\Delta\mathbf{u}^{0} = 0$$

- For Each Element/Region: Effect/Measurement at Sensors

Which Set of Sensors ?

- Assume: **K = L·U** Given (Needed Anyhow)
- If Effect of Weakening Each Element Desired:
  - CPU: $N_{el}^2 \cdot N_{bandwidth}$
  - Storage: $m \cdot N_{el}$
- Workarounds
  - Grouping of Elements CPU: $N_{group} \cdot N_{el} \cdot N_{bandwidth}$
  - Power 2 Storage of Active/Inactive

- Other Possible Improvements
  - Different Levels of Thresholding
  - Limiting Distance Between Sensors
  - Limiting Influence Distance of Sensors

Which Set of Sensors ?

- \`Adjoint-Based': Have Function (e.g. Displacements)

$$J = u(bc, loads, \alpha, \mathbf{x})$$

- Desire:

$$\frac{dJ}{d\alpha} = \frac{\partial J}{\partial \alpha} + \frac{\partial J}{\partial u}\frac{\partial u}{\partial \alpha}$$

- Augmented Lagrangian:

$$L^J = u(bc, loads, \alpha, \mathbf{x}) + \tilde{\mathbf{u}} \cdot (\mathbf{K} \cdot \mathbf{u} - \mathbf{f})$$

Which Set of Sensors ?

- Derivatives:

$$L^J_{,\tilde{\mathbf{u}}} = \mathbf{K} \cdot \mathbf{u} - \mathbf{f} = 0 \quad ,$$

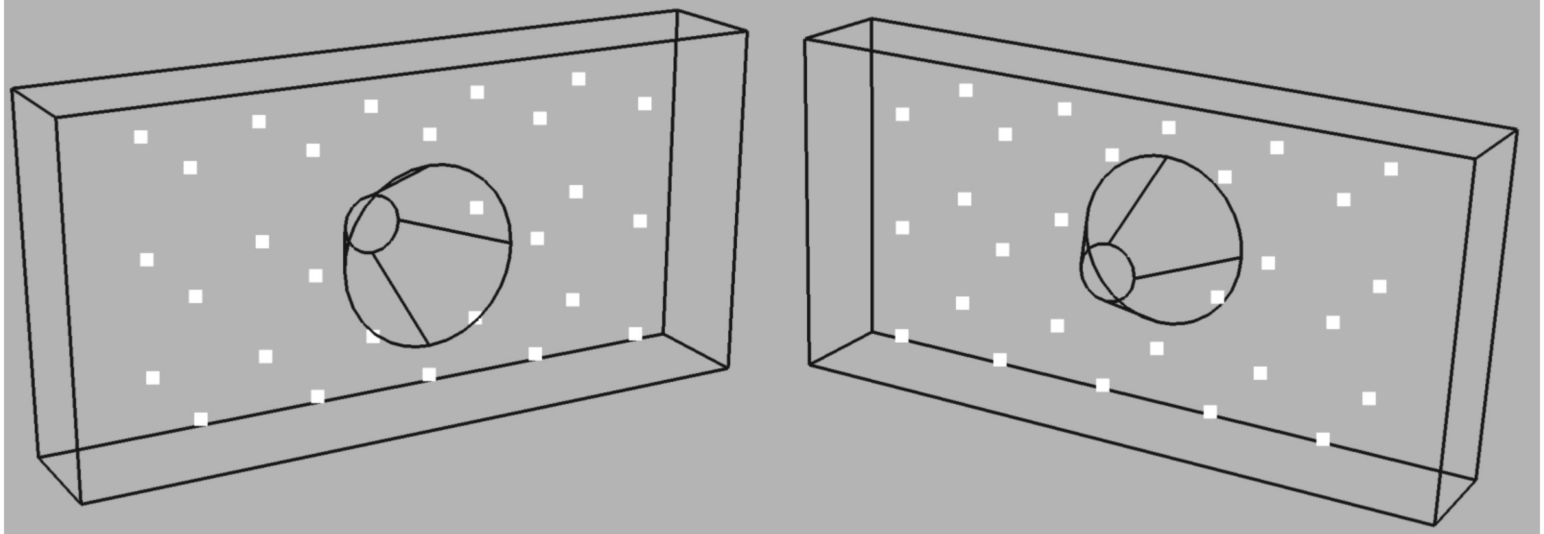
$$L^J_{,\alpha_e} = \tilde{\mathbf{u}} \cdot \mathbf{K}_e \cdot \mathbf{u} \quad ,$$

$$L^J_{,\mathbf{u}} = \frac{\partial u(bc, loads, \alpha, \mathbf{x})}{\partial \mathbf{u}} + \tilde{\mathbf{u}} \cdot \mathbf{K} = 0$$

- For Each Sensor: Sensitivity for all Elements ➔ Reverse

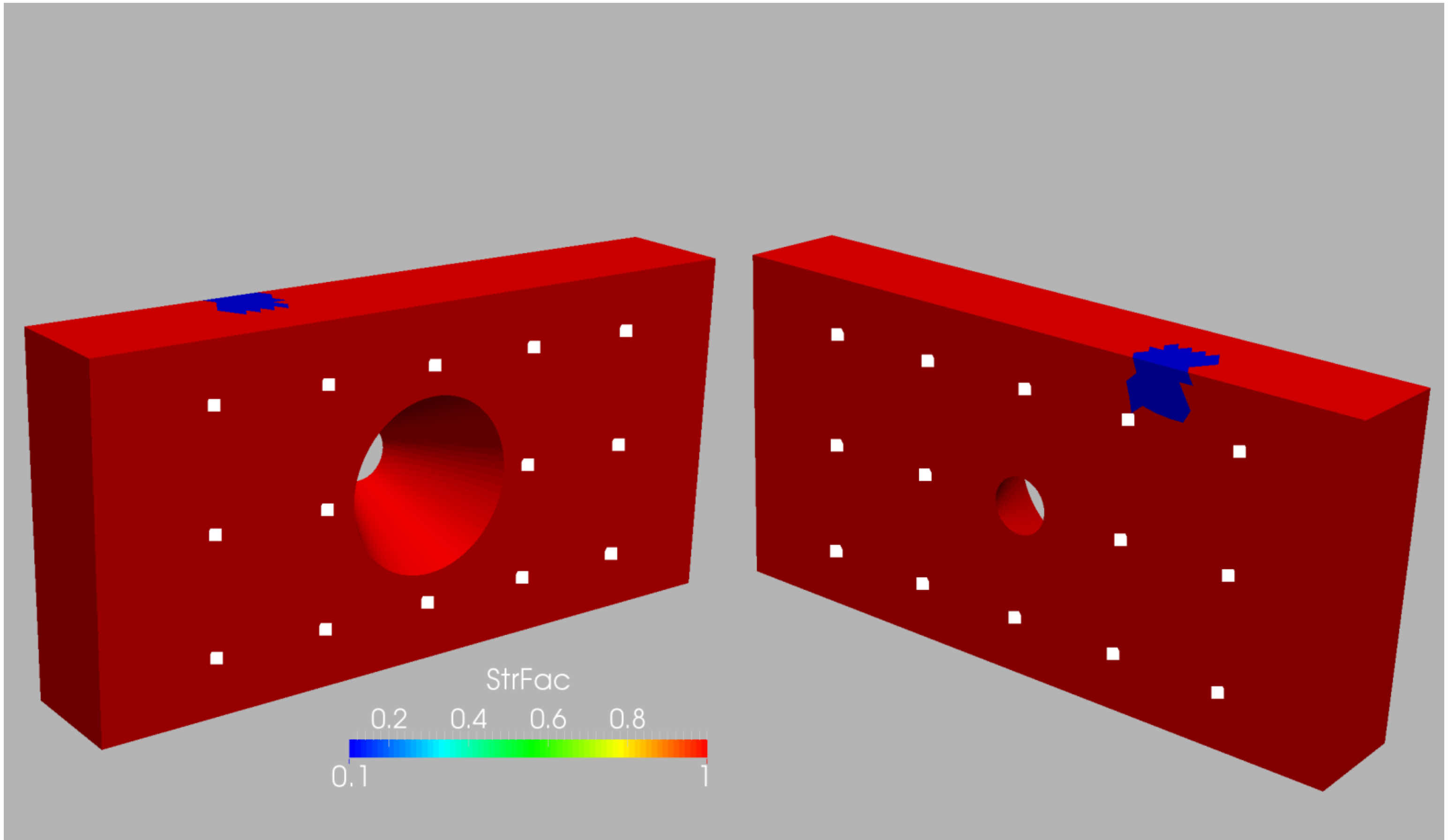
- Assume: **K = L·U** Given (Needed Anyhow)
- If Effect of Each Sensor Desired:
  - CPU: $m \cdot N_{el} \cdot N_{bandwidth}$
  - Storage: $m \cdot N_{el}$

Plate With Hole 3D



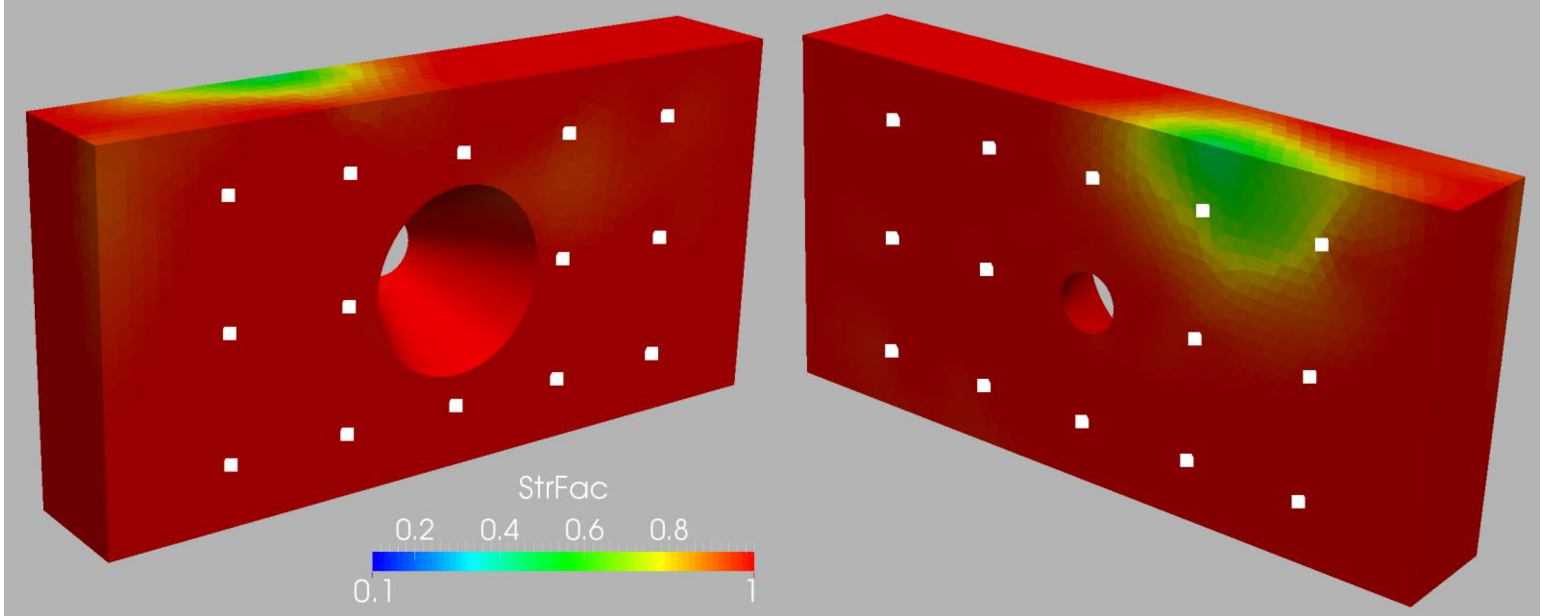
28 Possible Sensor Locations

Plate With Hole 3D



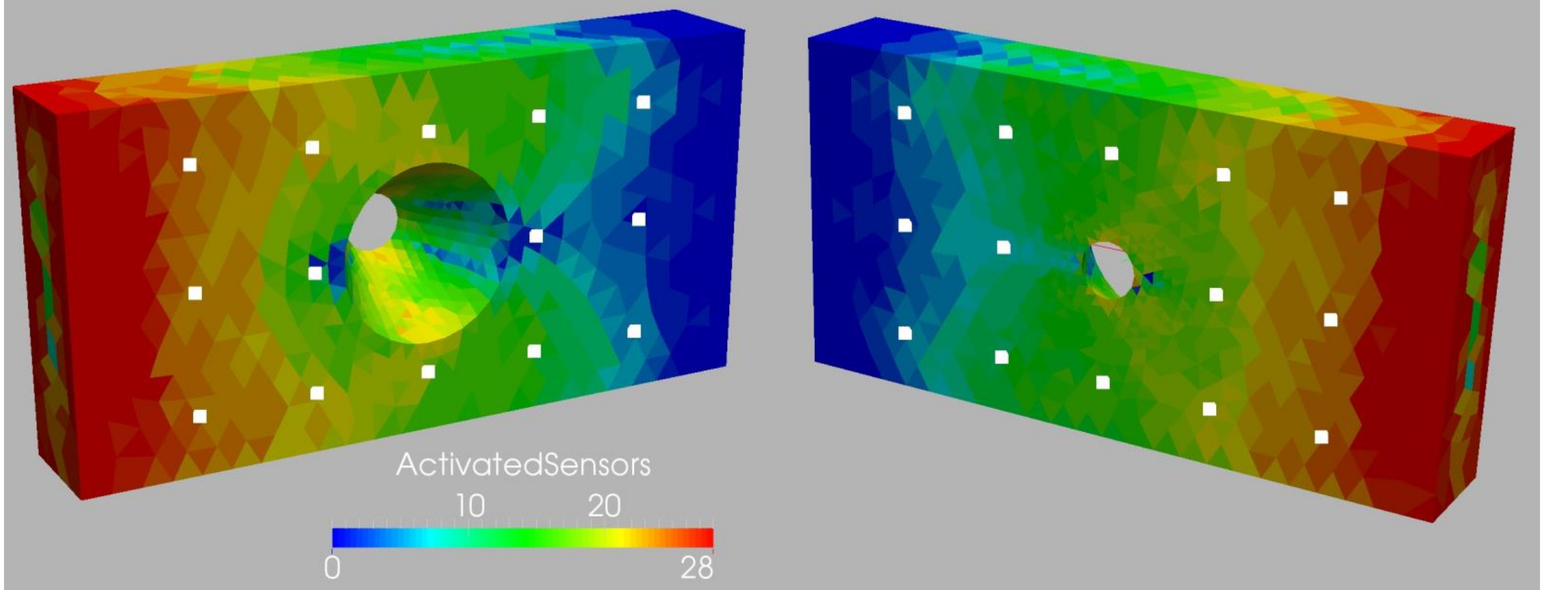
Target Strength Factor (120Kels)

Plate With Hole 3D



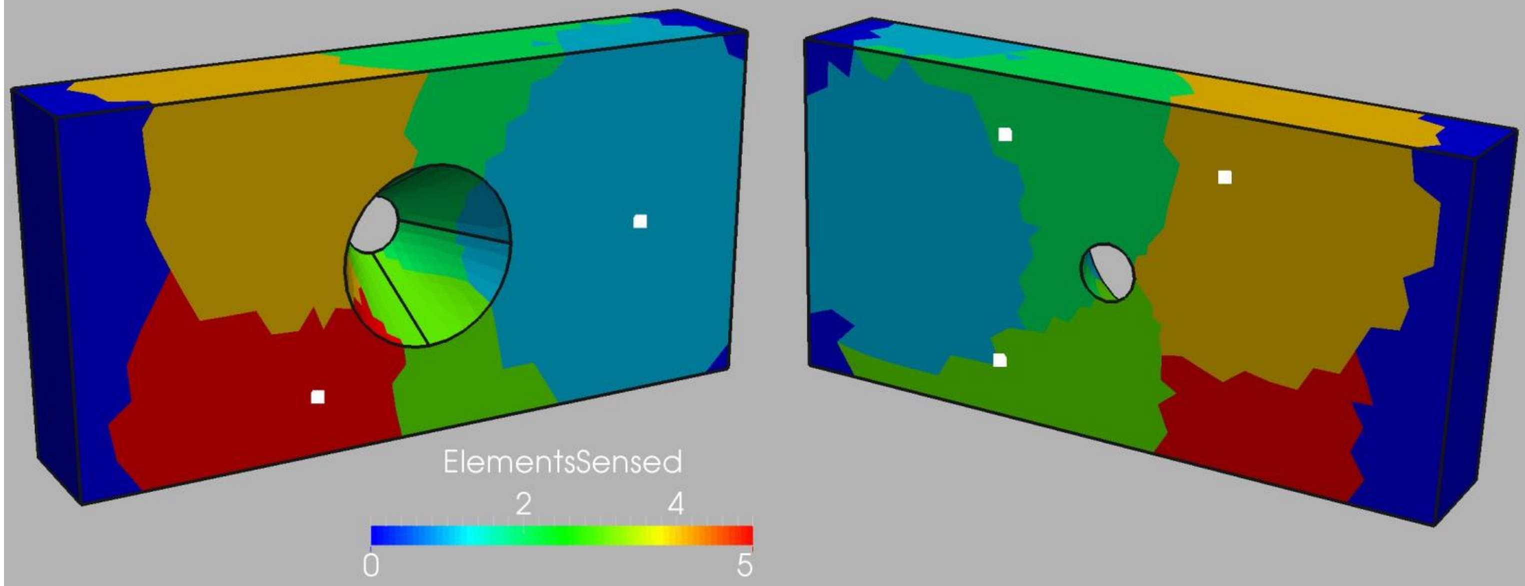
Computed Strength Factor (120Kels)

Plate With Hole 3D



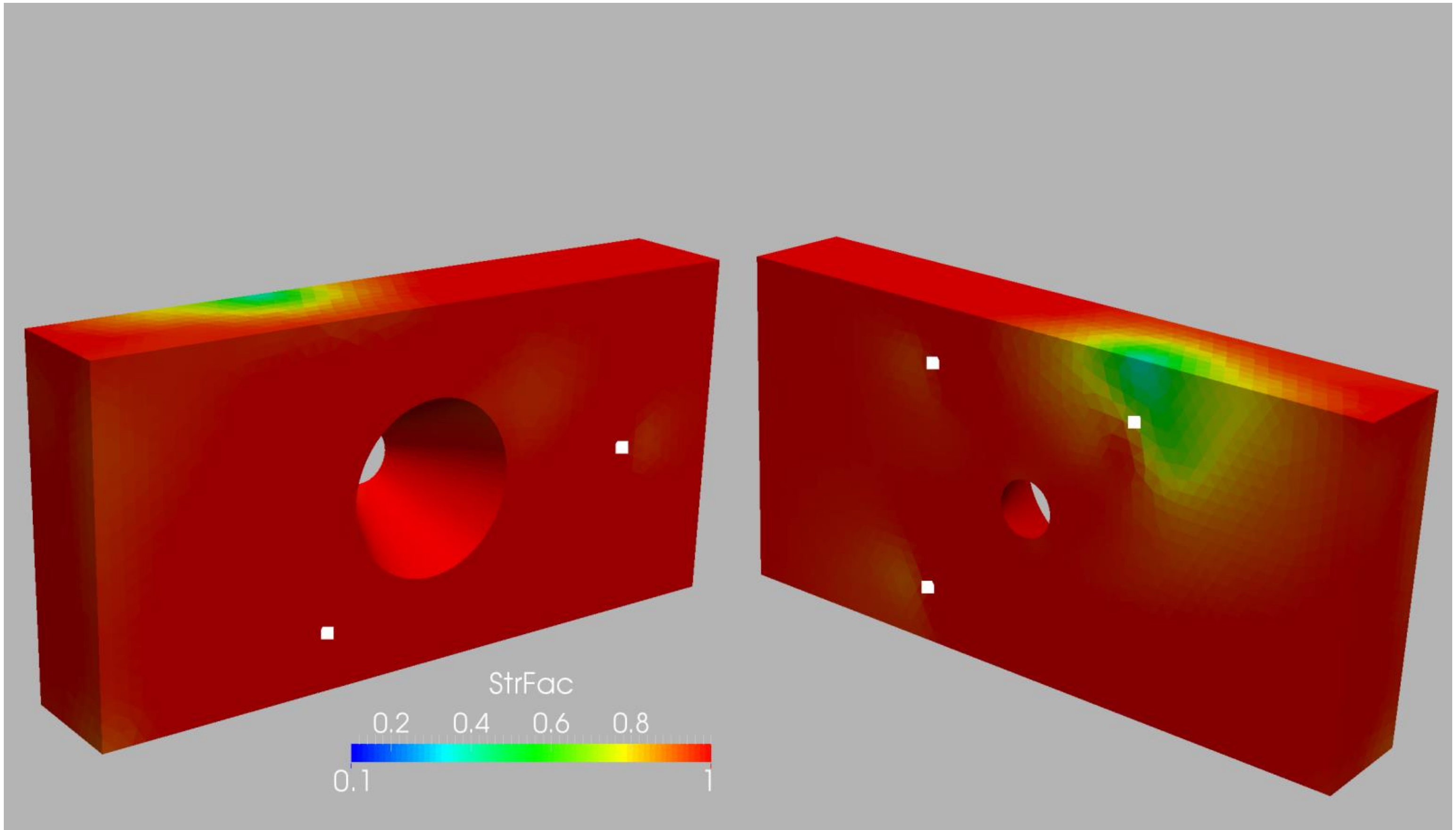
Number of Sensors Activated by Weakening an Element (Load 1)

Plate With Hole 3D



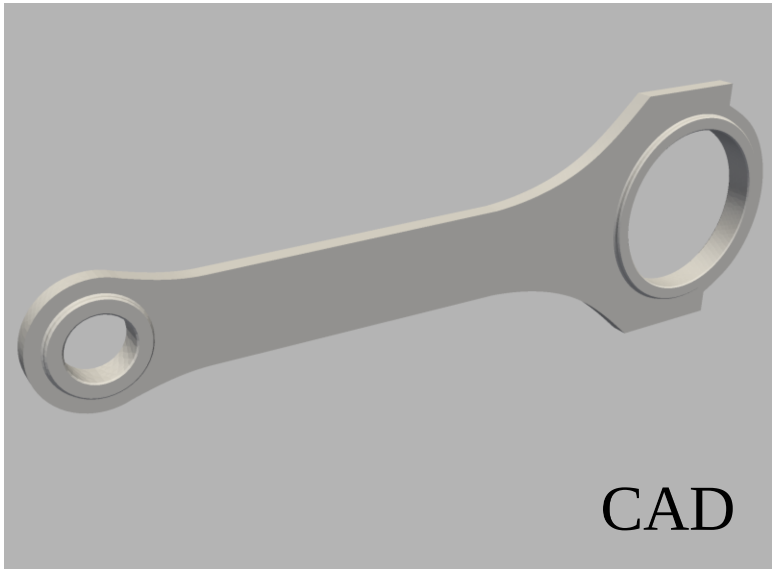
Optimal Sensor Locations and `Influence Zone'

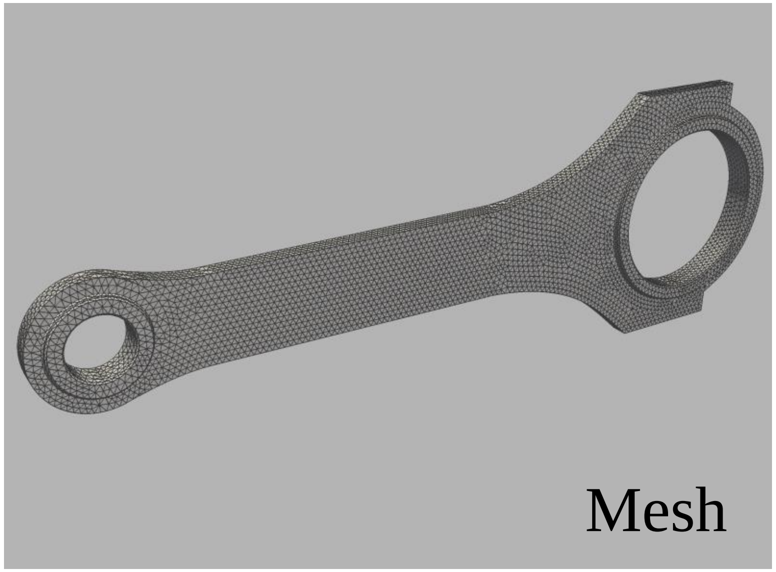
Plate With Hole 3D

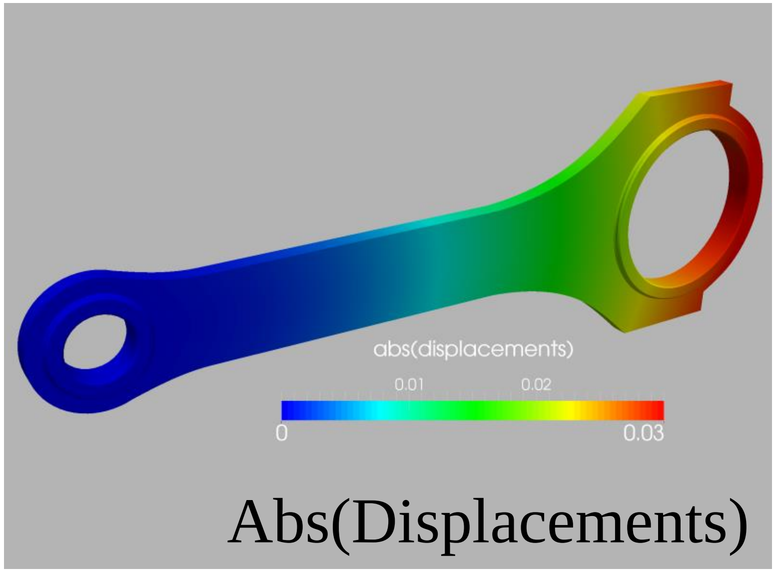


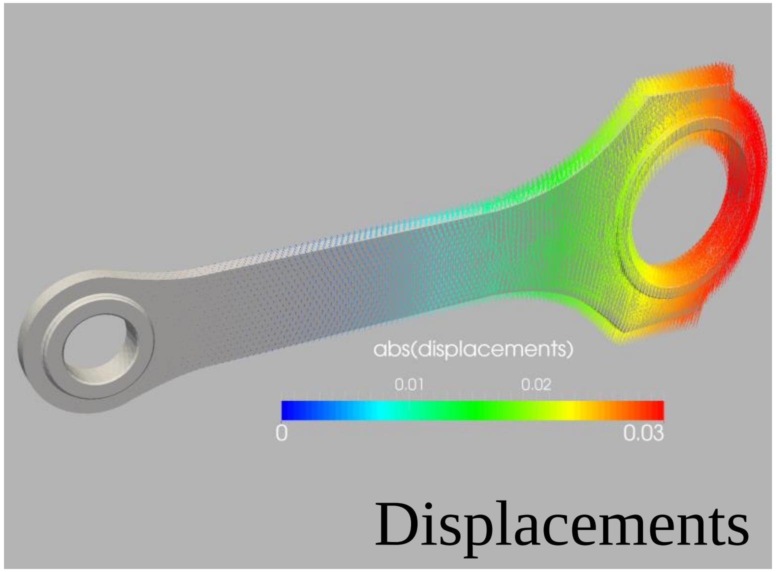
Computed Strength Factor (120Kels)

Connecting Rod

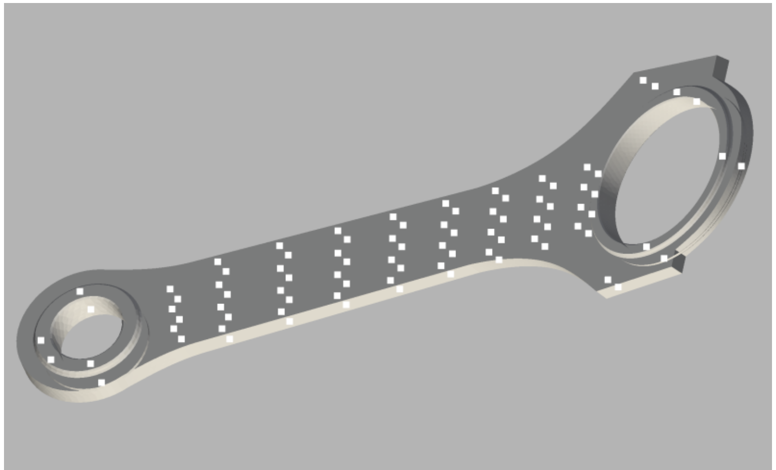




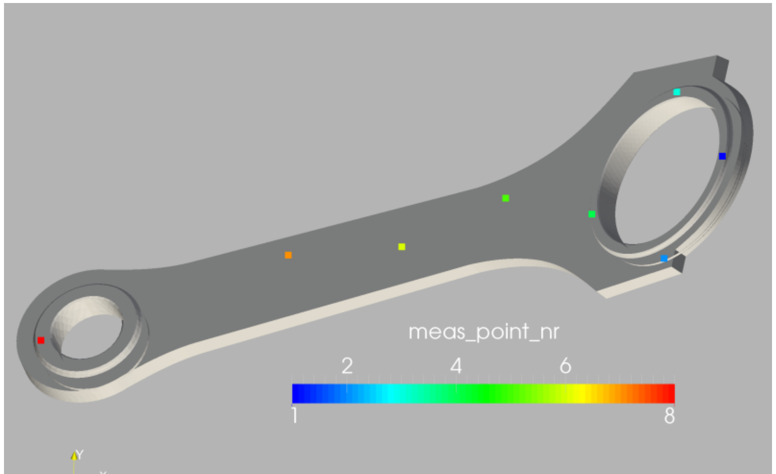




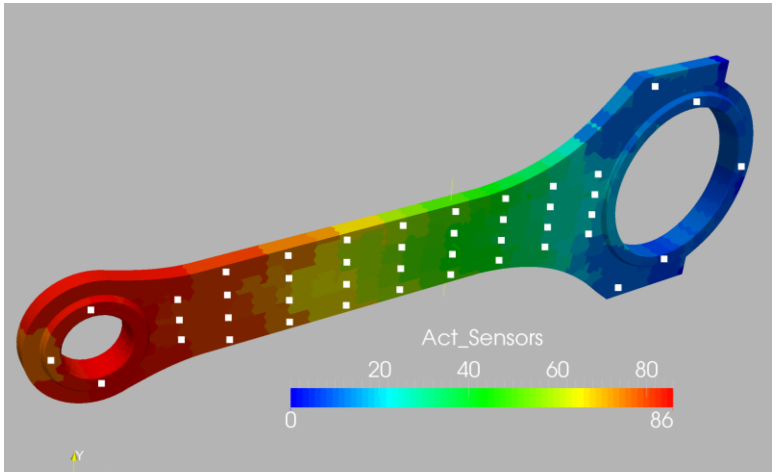
Connecting Rod



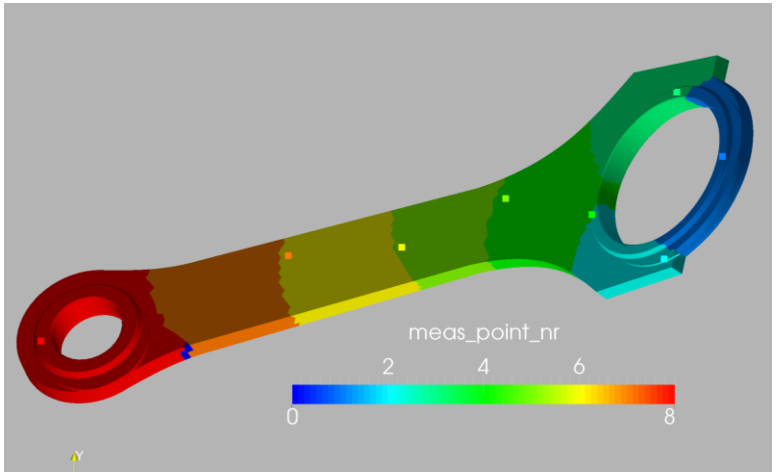
Possible Sensor Locations



Optimal Sensor Locations



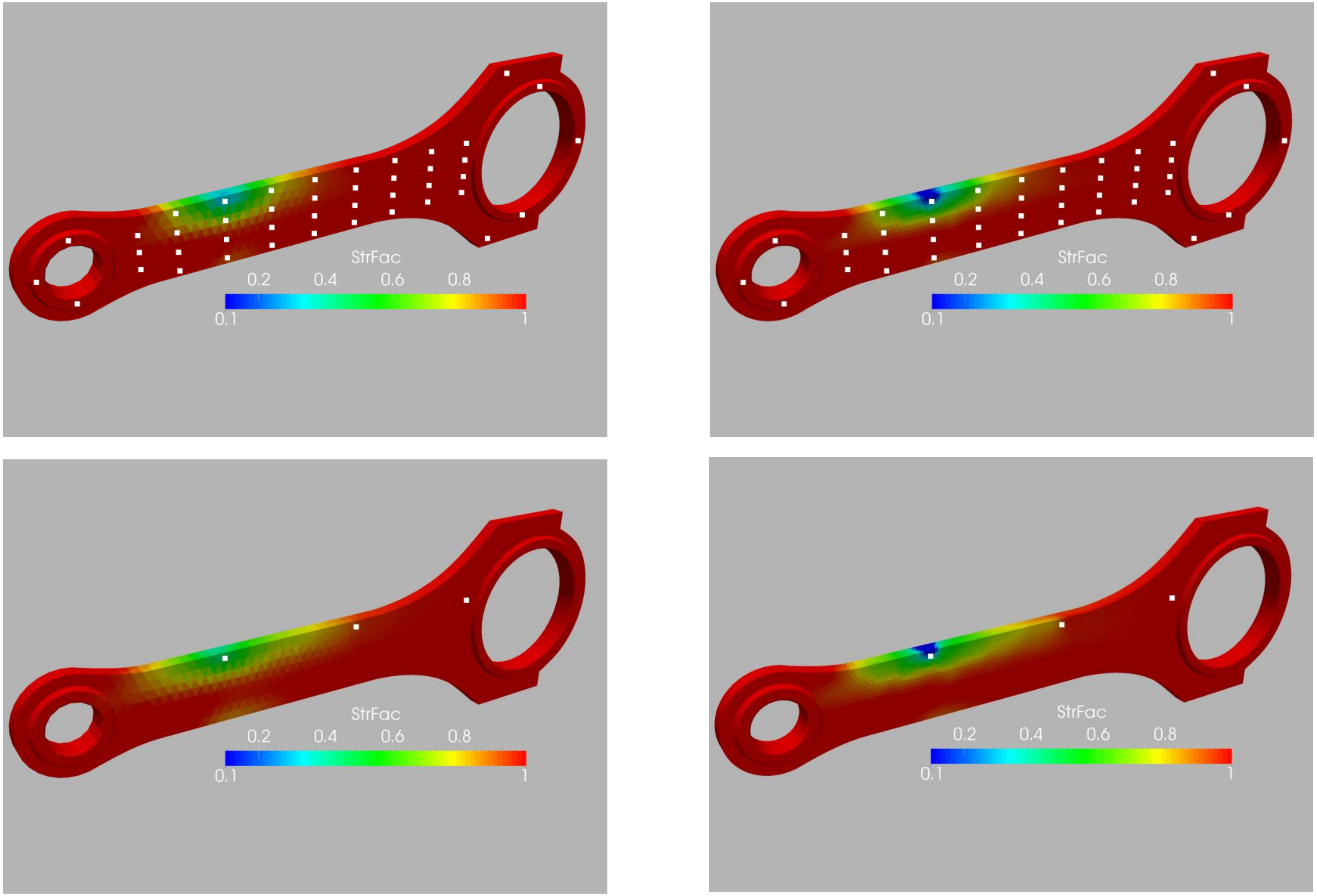
Number of Sensors Activated



`Zones of Influence'

Connecting Rod

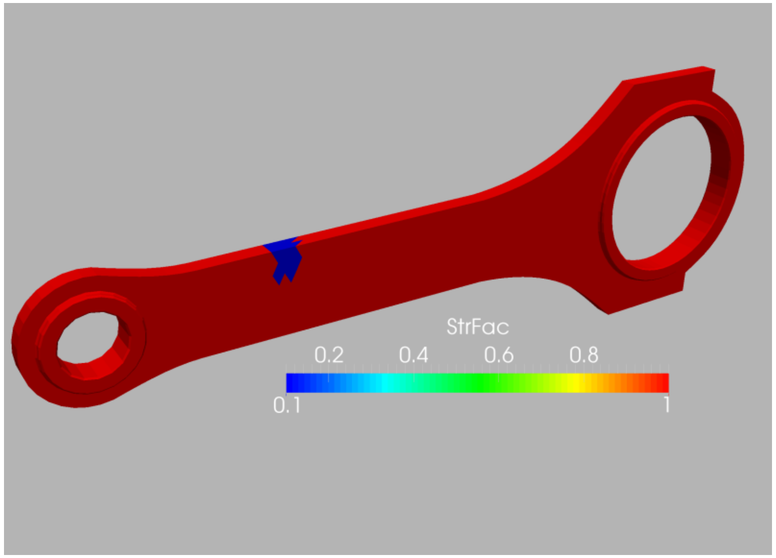
10Ktet 71Ktet

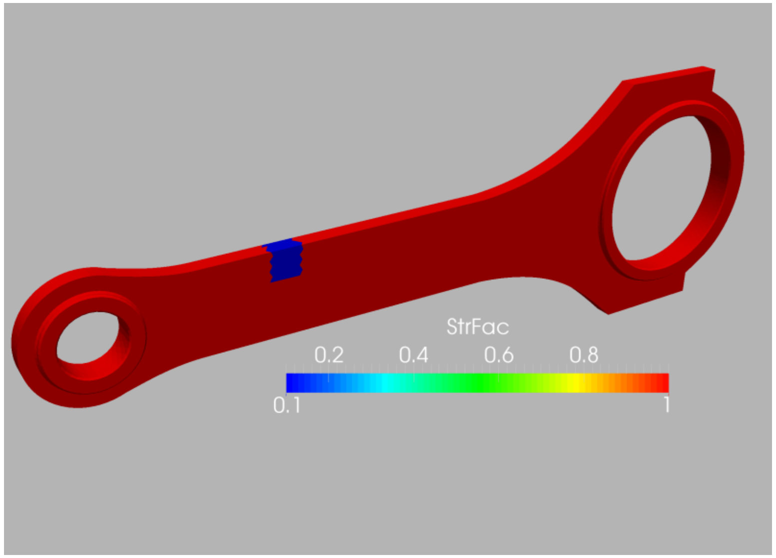


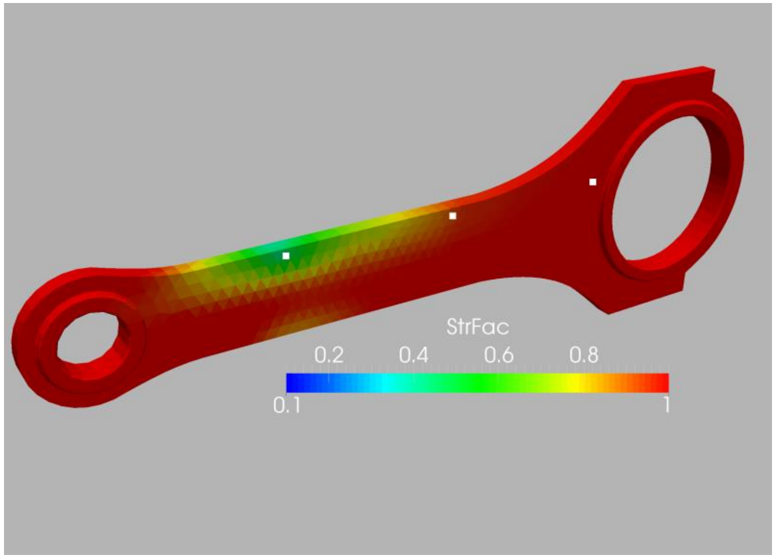
Connecting Rod

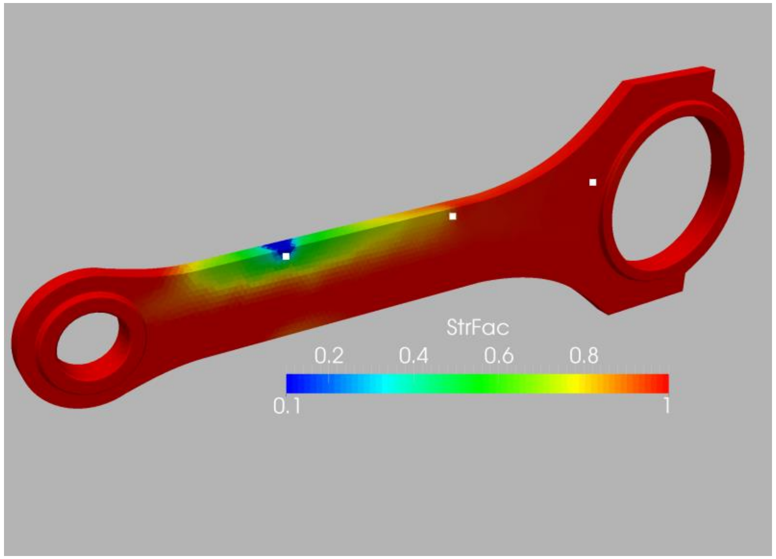
10Ktet

71Ktet

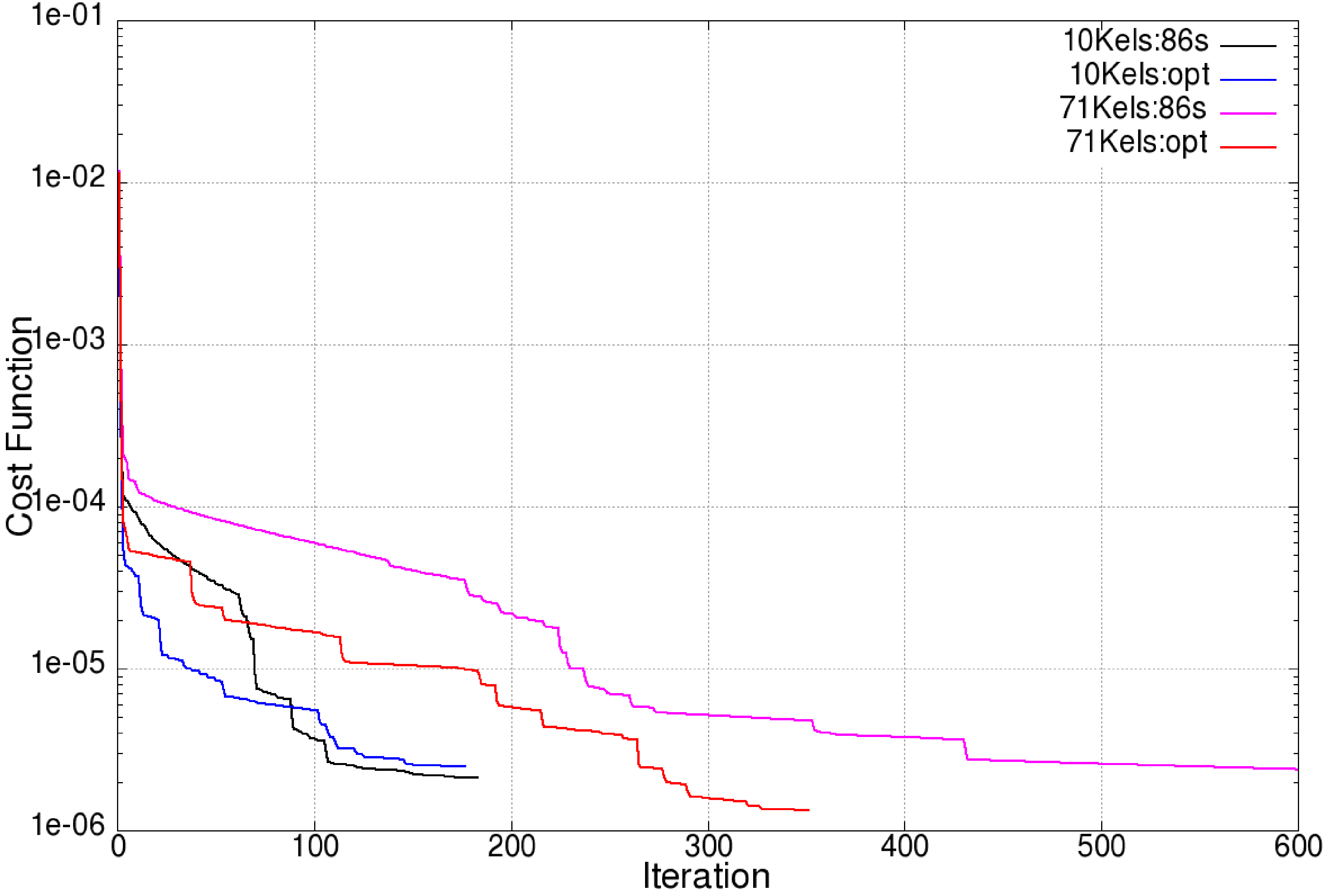




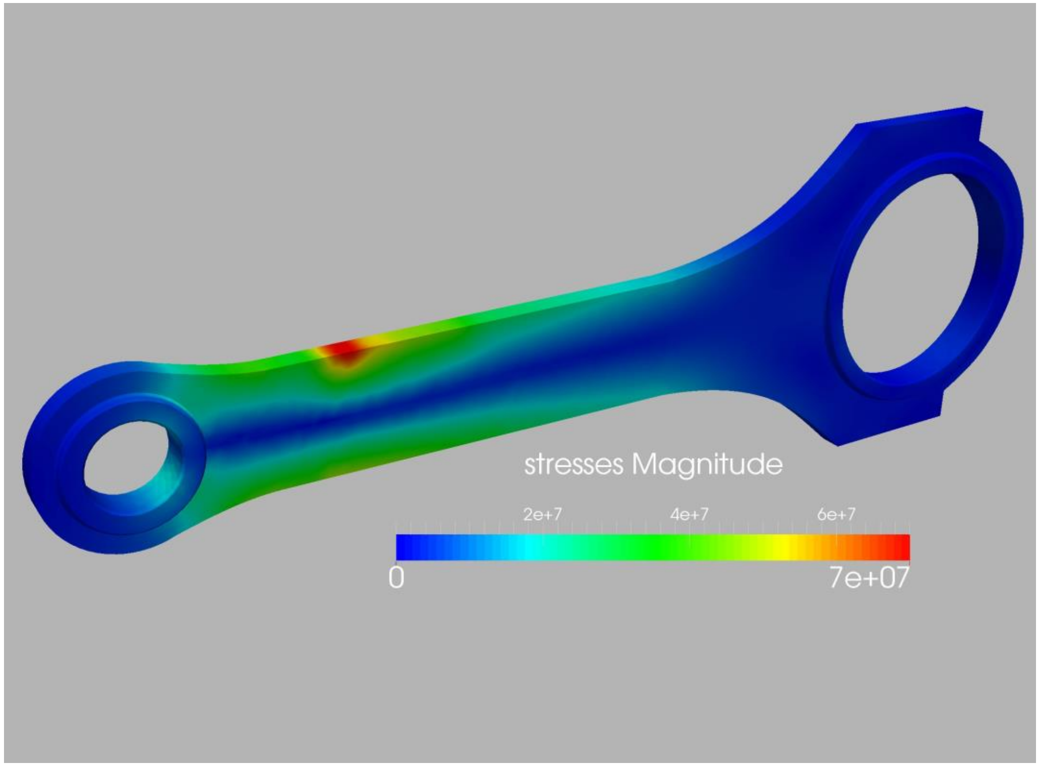




Connecting Rod



Connecting Rod



Open Questions

Seems of Work, But: Open Questions

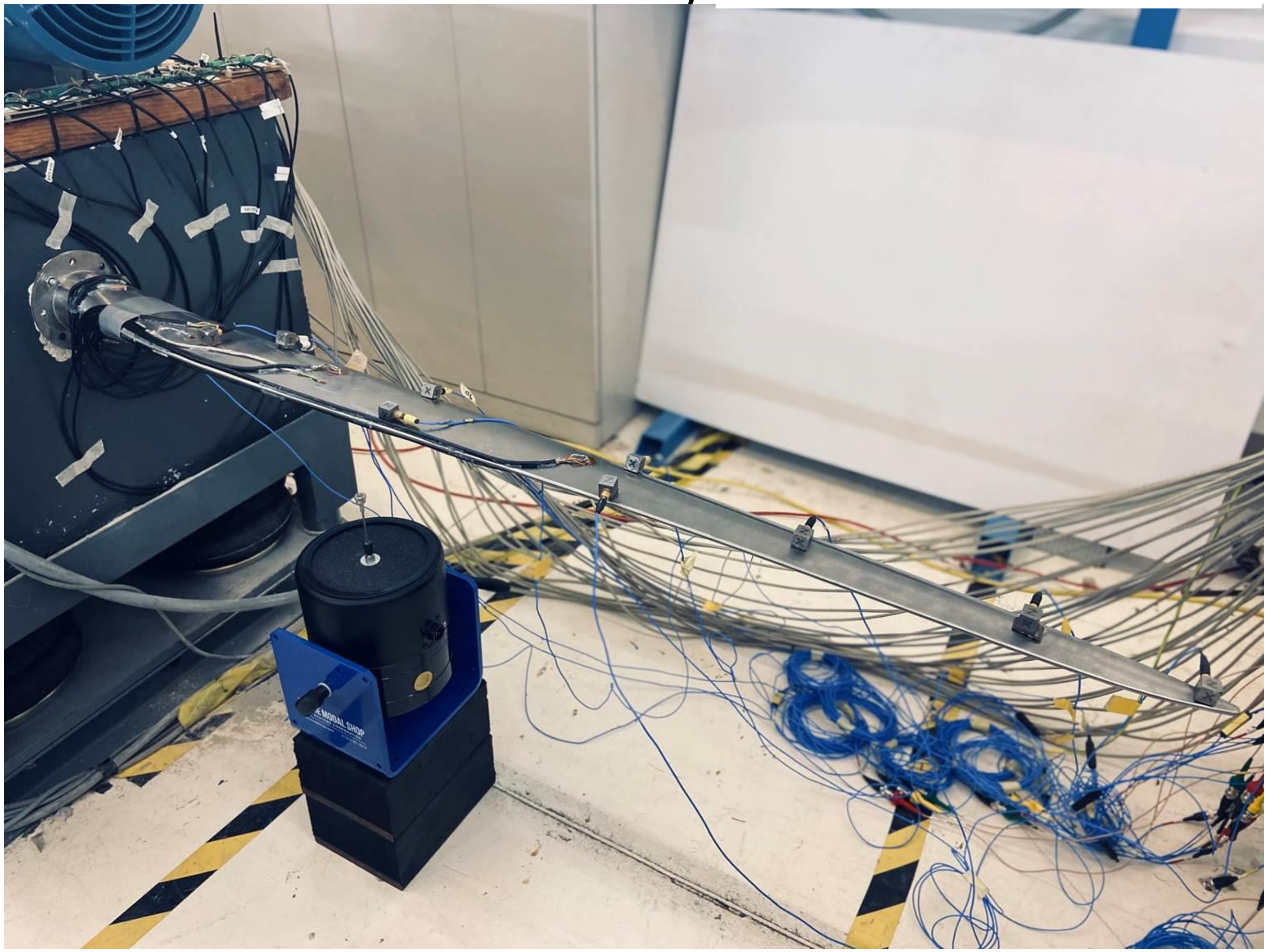
- Will It Work for Multiple Damage Regions ?
- Is the Solution Stable ?
- Is It Dependent on the Initial Solution ?
- Can It Be Extended to Eigenmodes ?
- Can We Zoom In ?
- Can It Be Extended to Plasticity ?
- What Happens If We Have Singularities ?
- Does It Work for Reinforced Concrete ?
- Stochastics/UQ ?
- Can We Recover Temperature Fields ?
- …

Will It Work for Multiple Damage Regions ?

- Try It

Blade

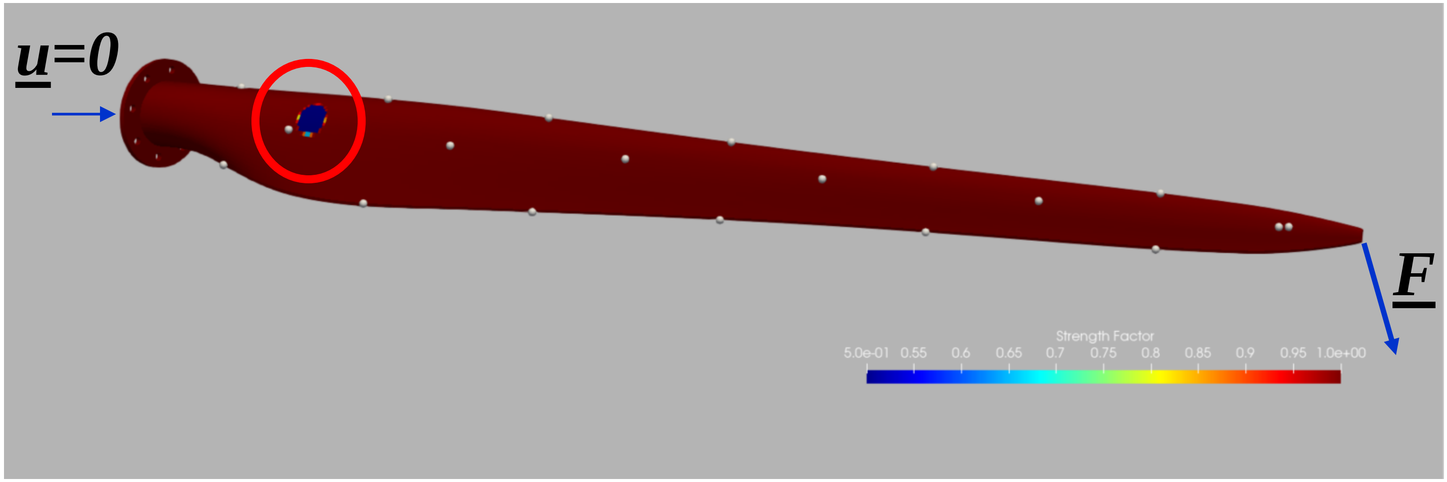
Siemens Project: xDT Titanium Blade
Courtesy S. Vettori and E. Di Lorenzo

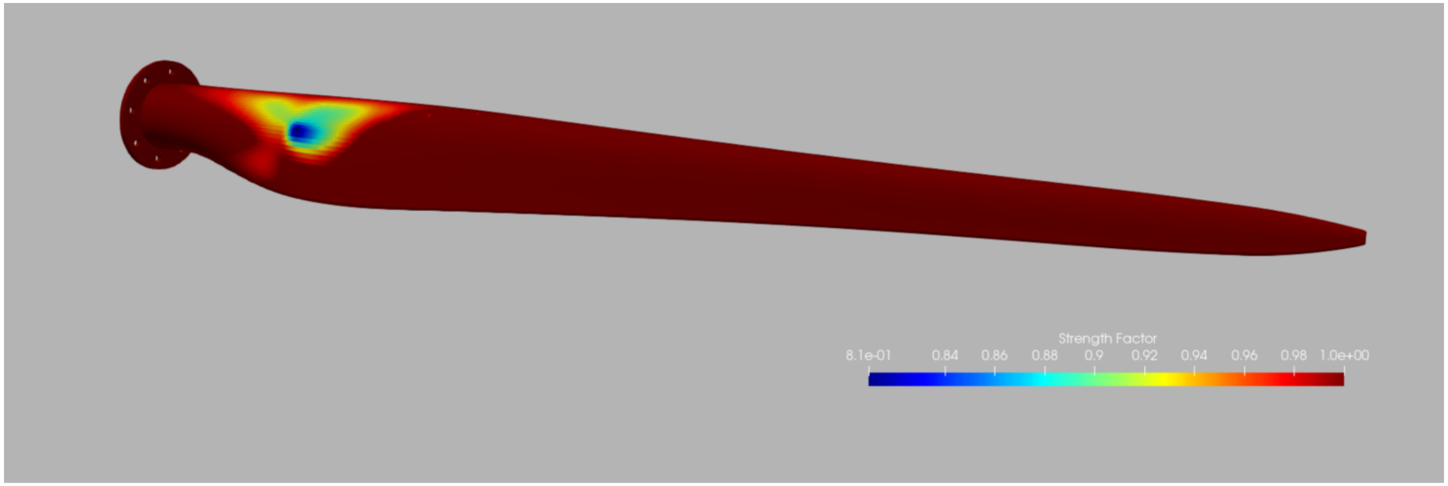


SIEMENS

Blade

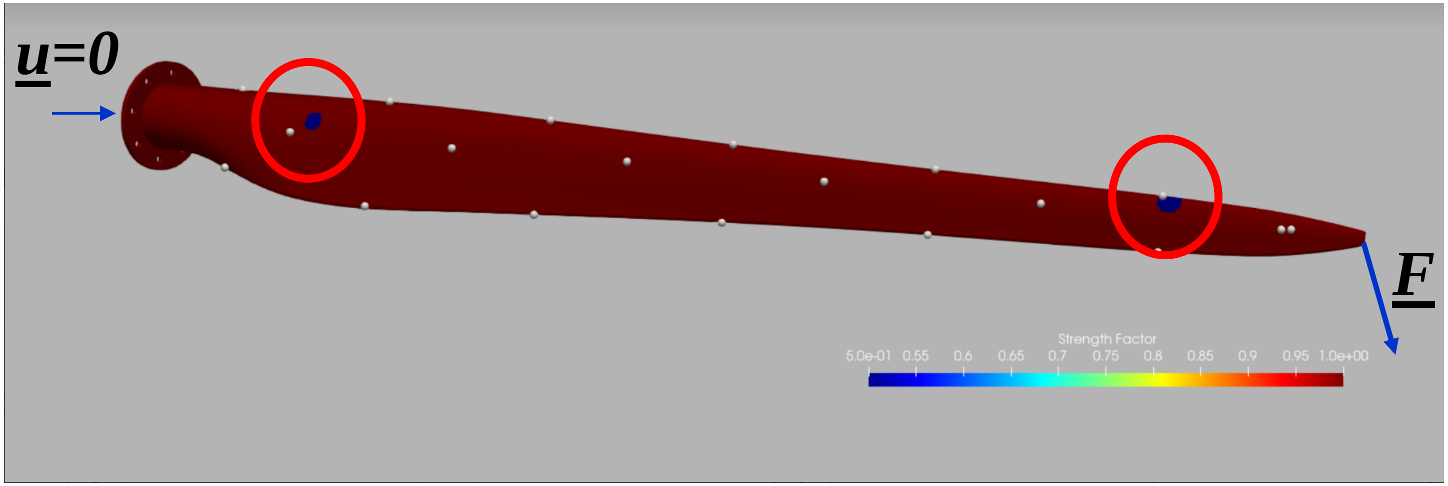
```
nelem=34K (Hex,Pr)
nsens=19
nload= 1
CALCULIX
```

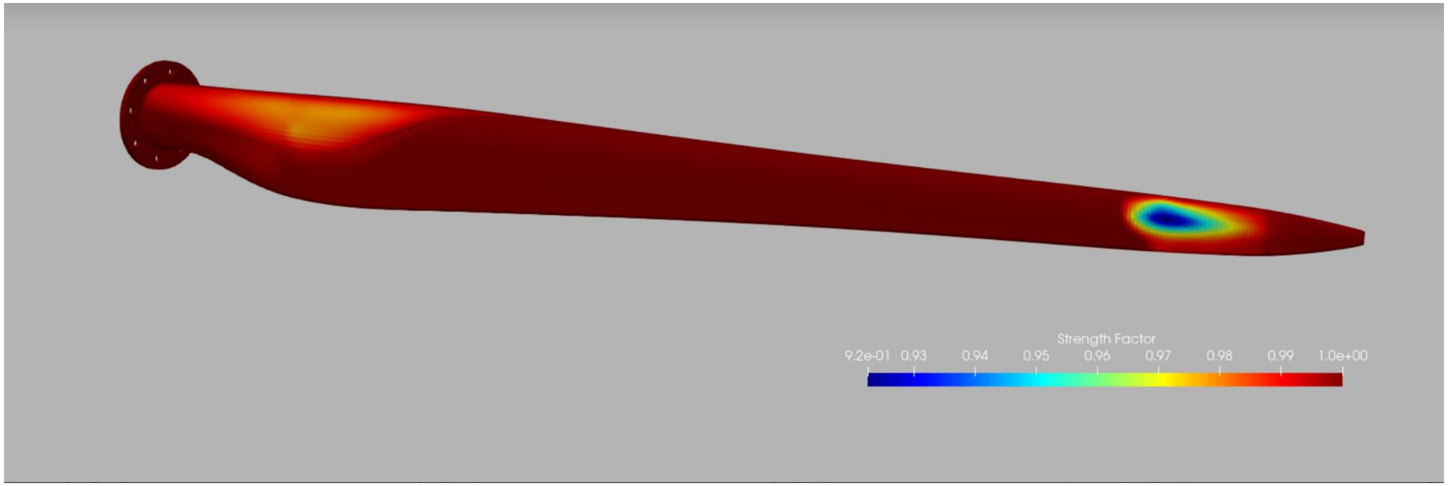




Blade

```
nelem=34K (Hex,Pr)
nsens=19
nload= 1
CALCULIX
```

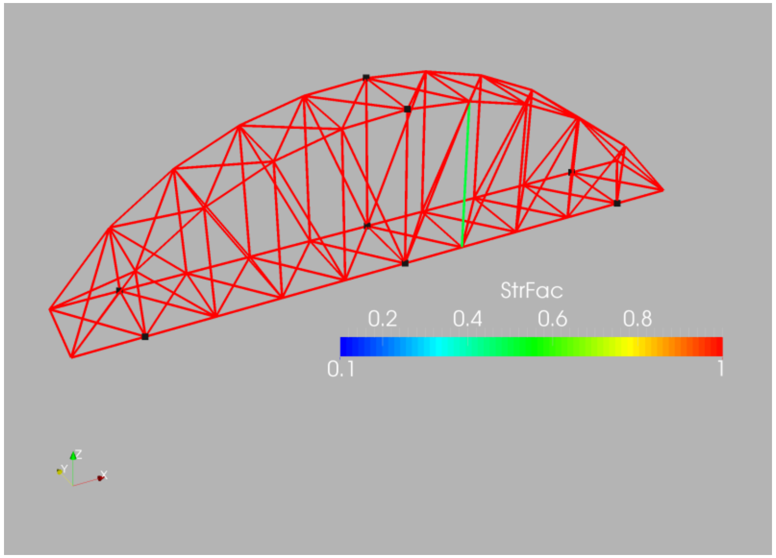


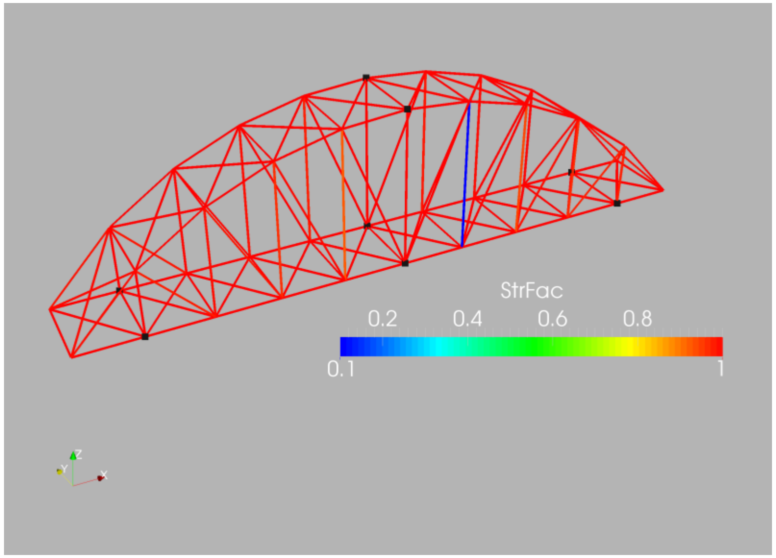


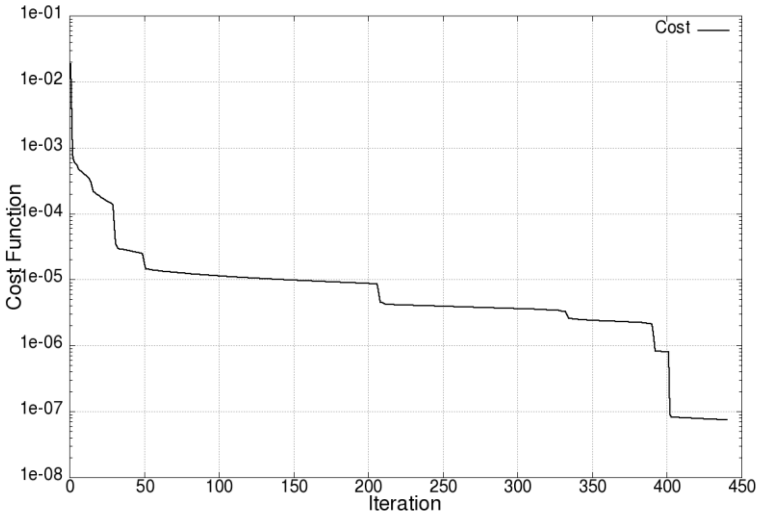
Is The Solution Stable ?

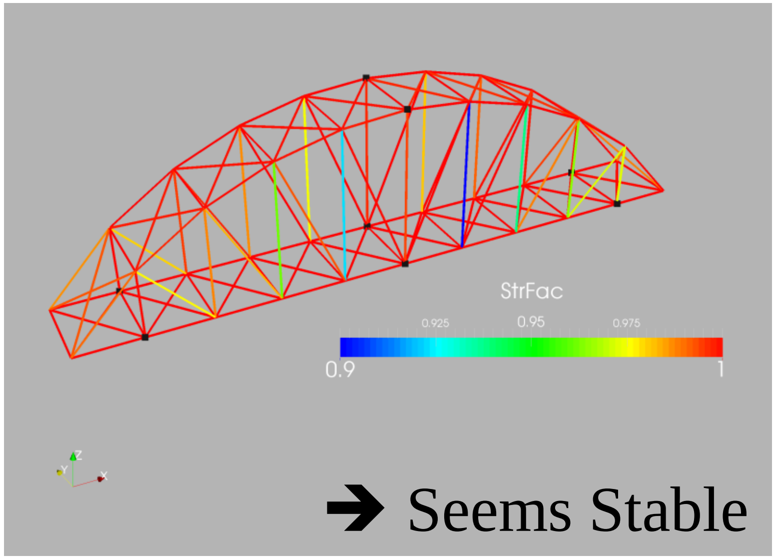
- Start From Close to the Exact Solution
- See If Exact Solution Is Obtained
- Here: Start With 0.2/0.5

Bridge: Start From 0.5









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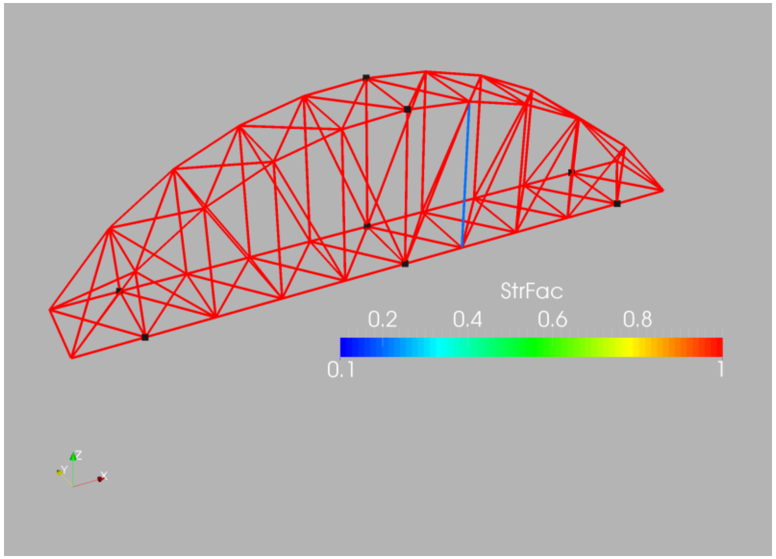
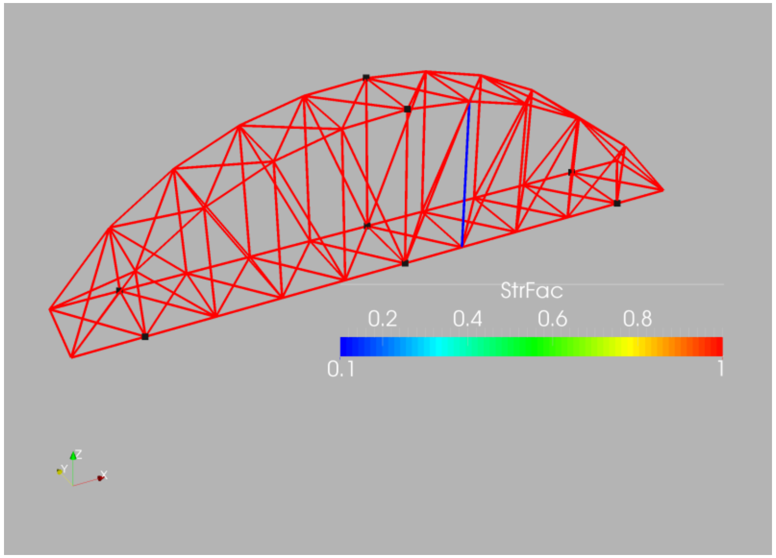
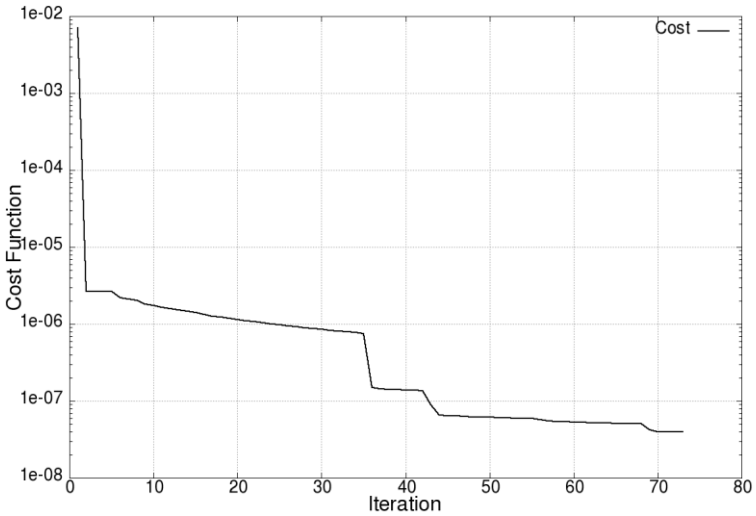
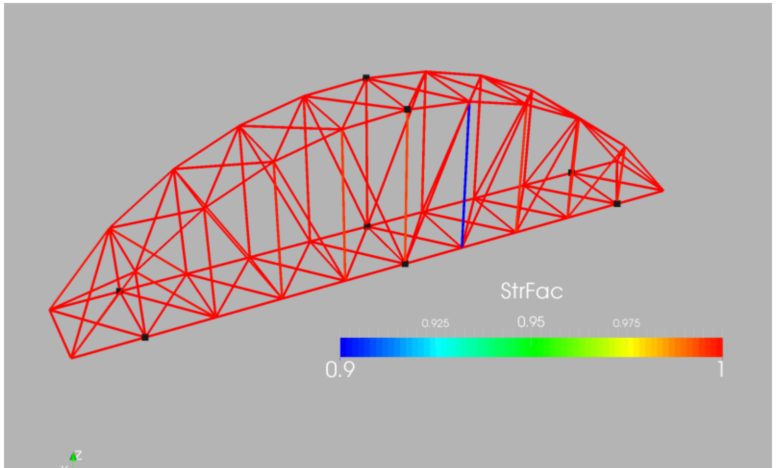
Bridge: Start From 0.2



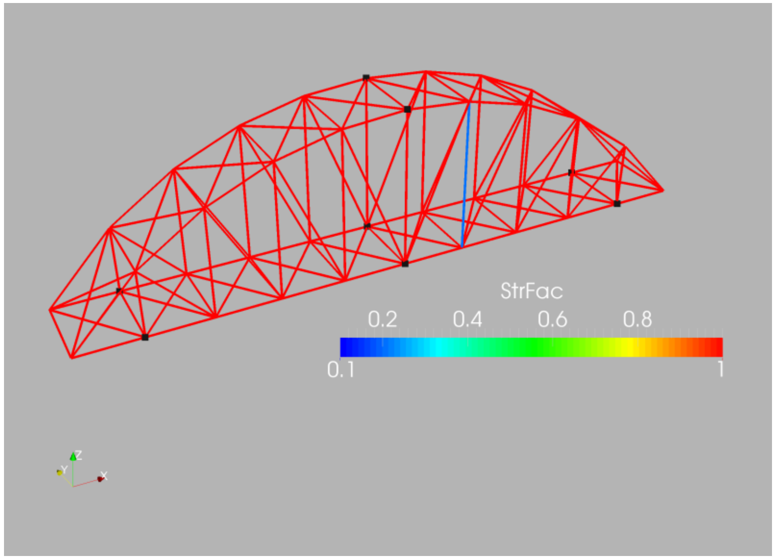


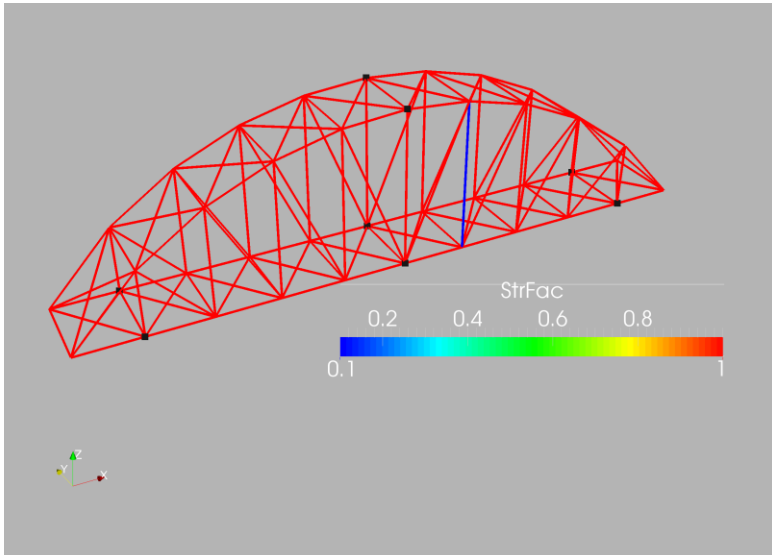


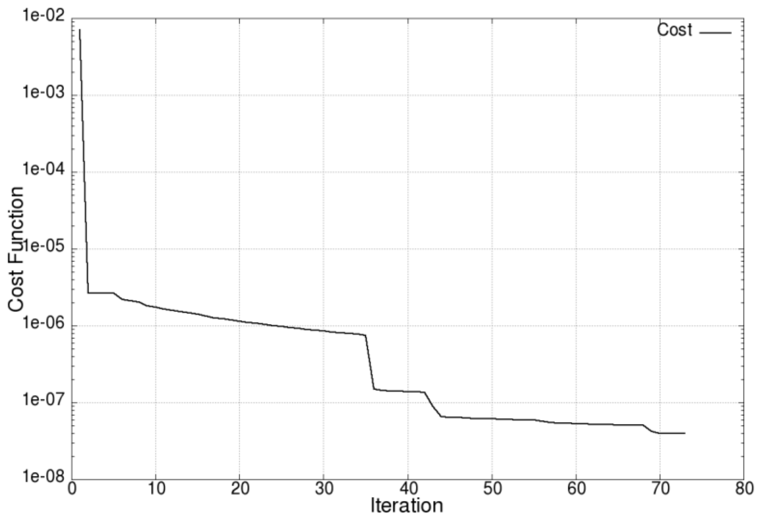


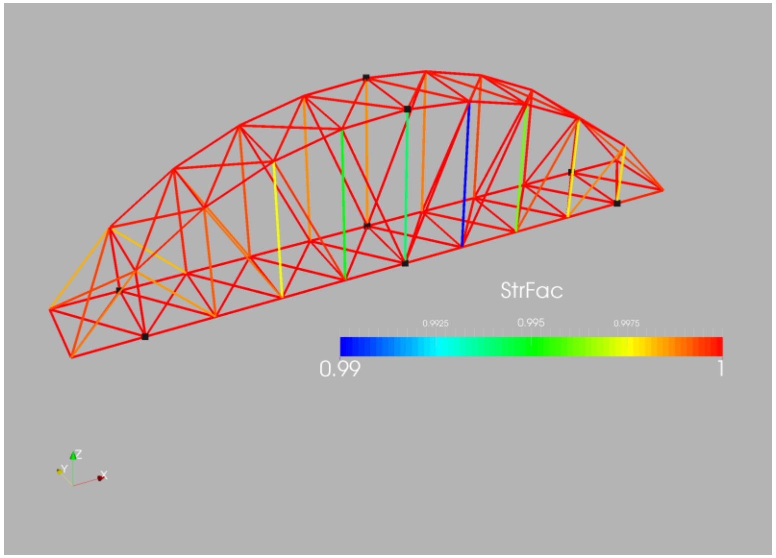
➔ Seems Stable

Bridge: Start From 0.2









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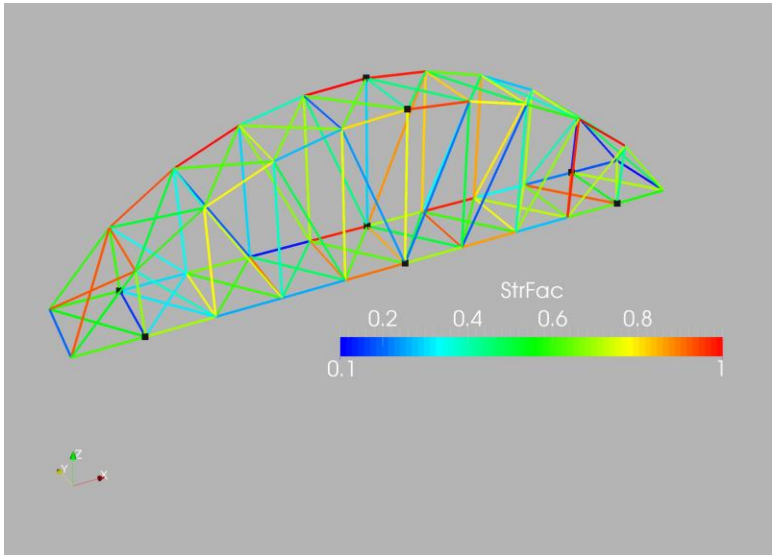
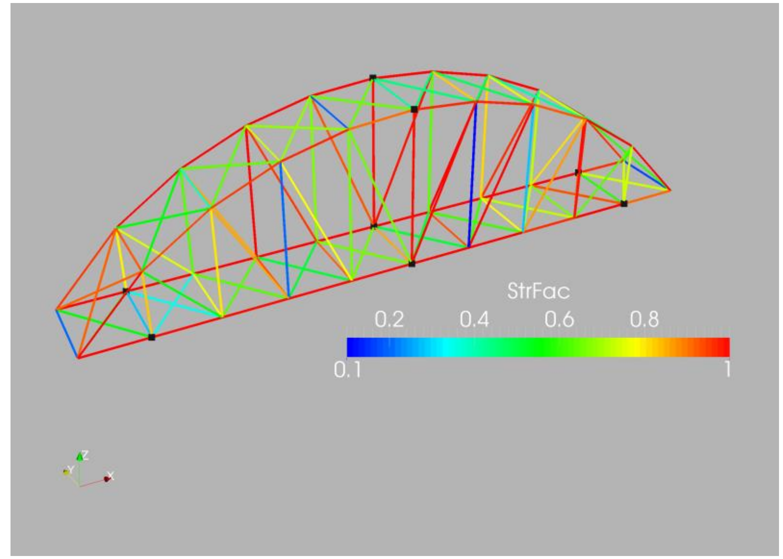
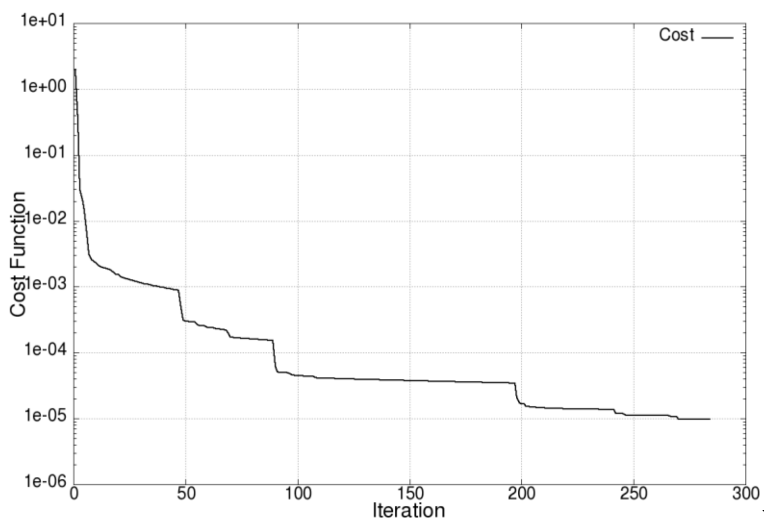
Is The Solution Dependent On Initial Conditions ?

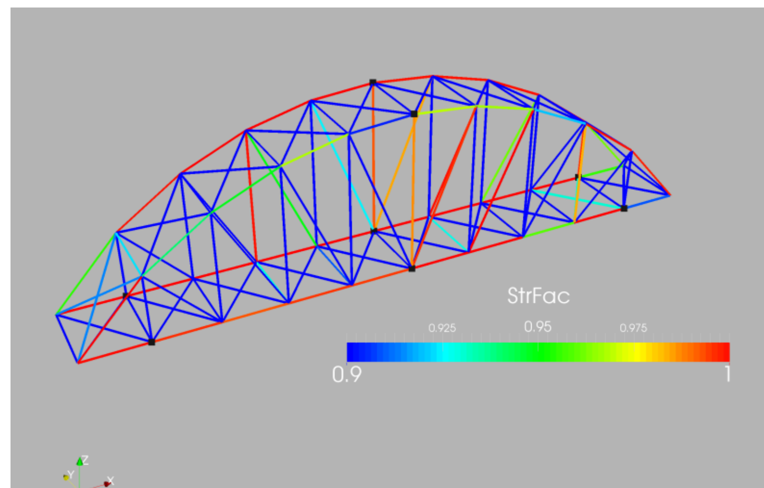
- Start From 1.0
- Start From Random

Bridge: Start From Random



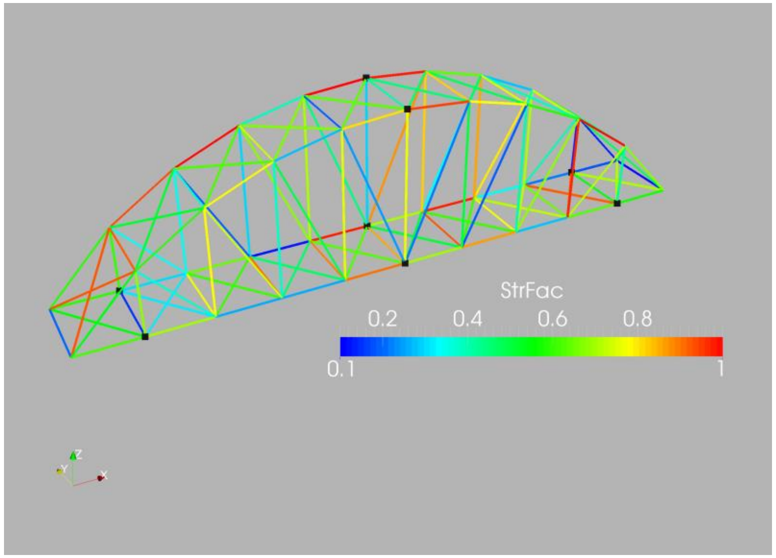


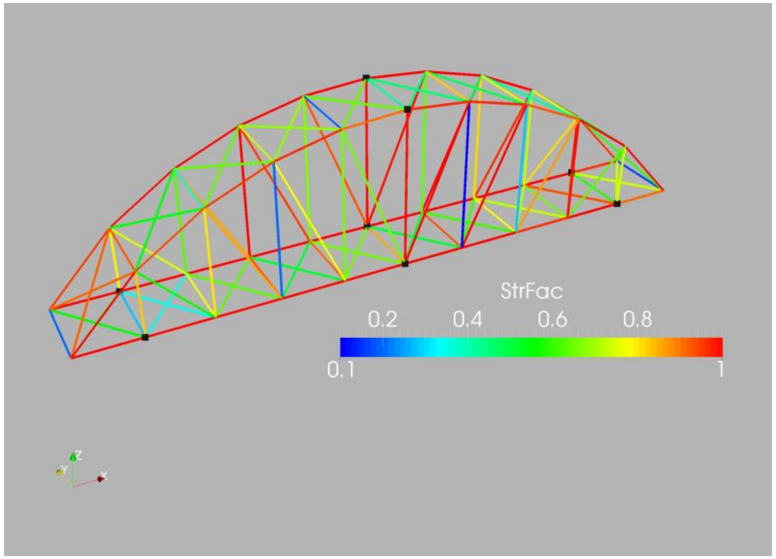


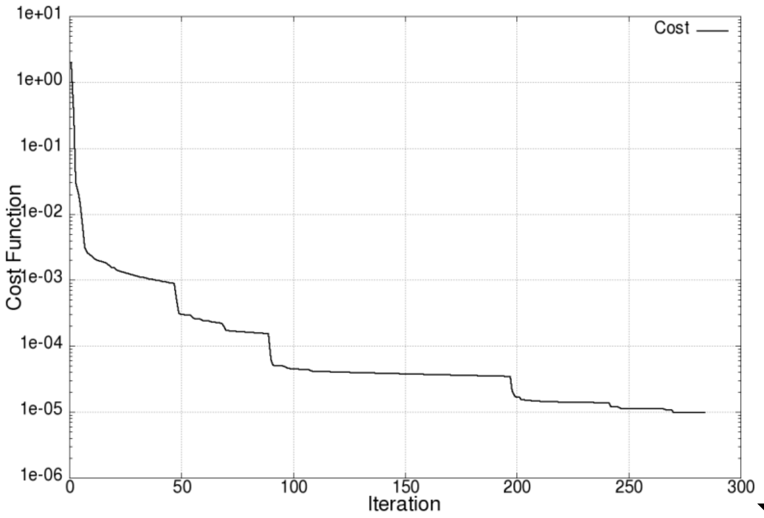


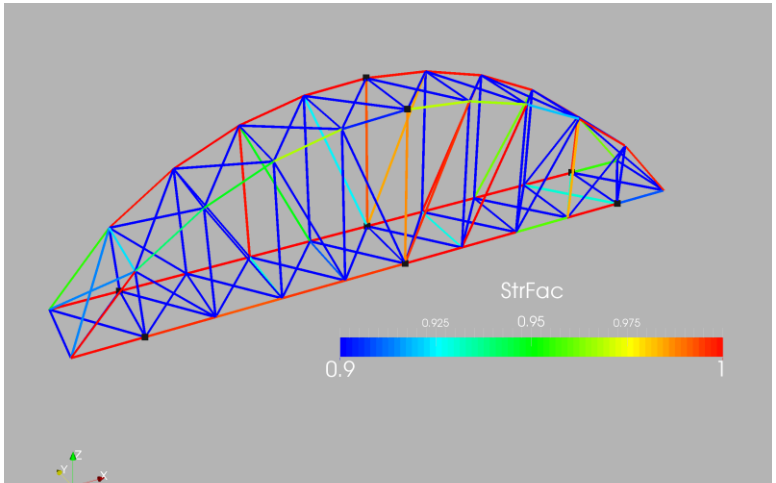
➔ Multiple Possible Solutions

Bridge: Start From Random







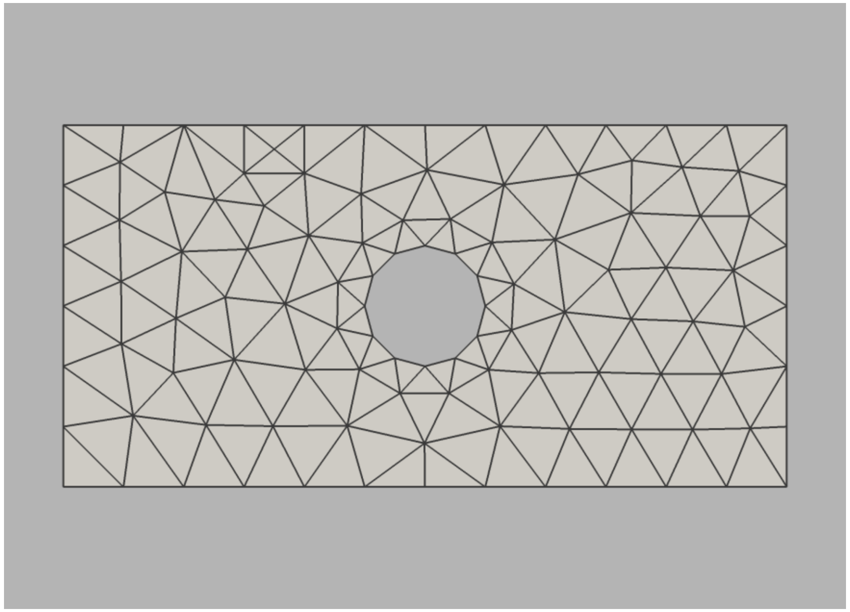


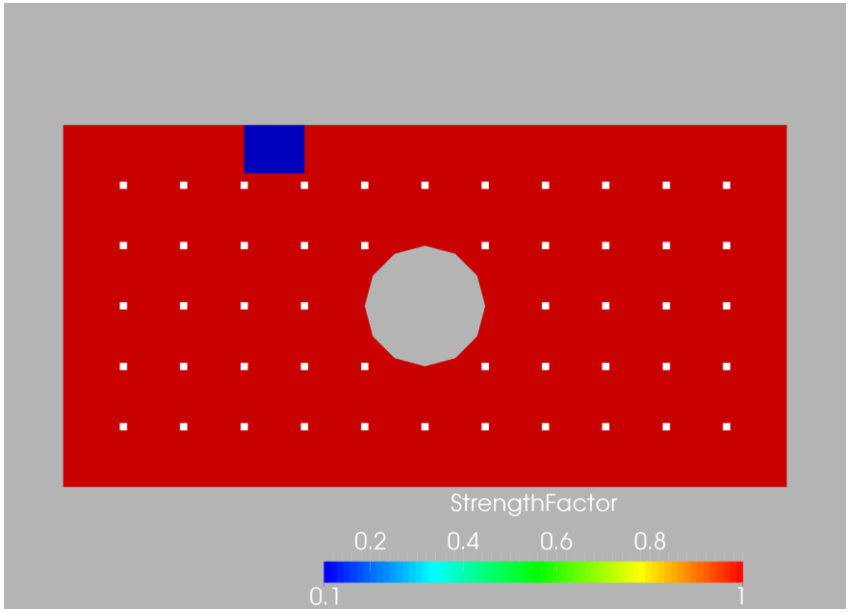
➔ Multiple Possible Solutions

➔ But: Know Starting Weakness

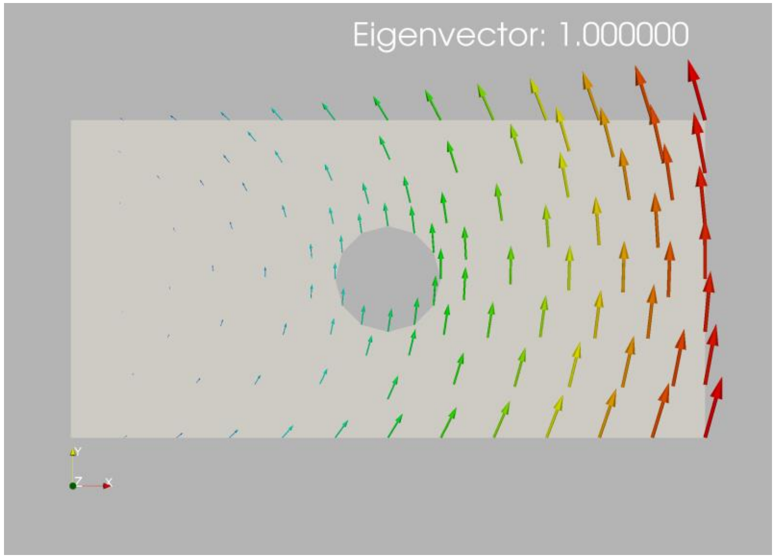
Can It Be Extended to Eigenmodes ?

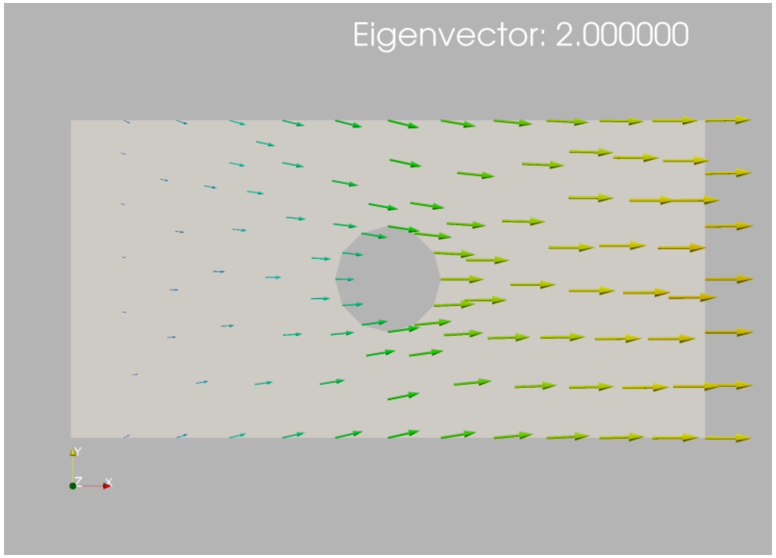
- Try It for Plate Case
- Cost Function: $\Sigma\, w_i\, (\lambda_i - \lambda_{i,meas})^2$

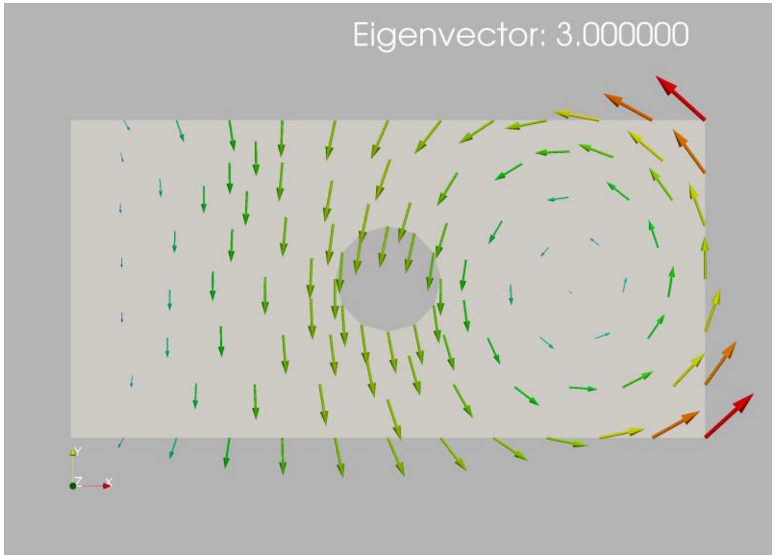


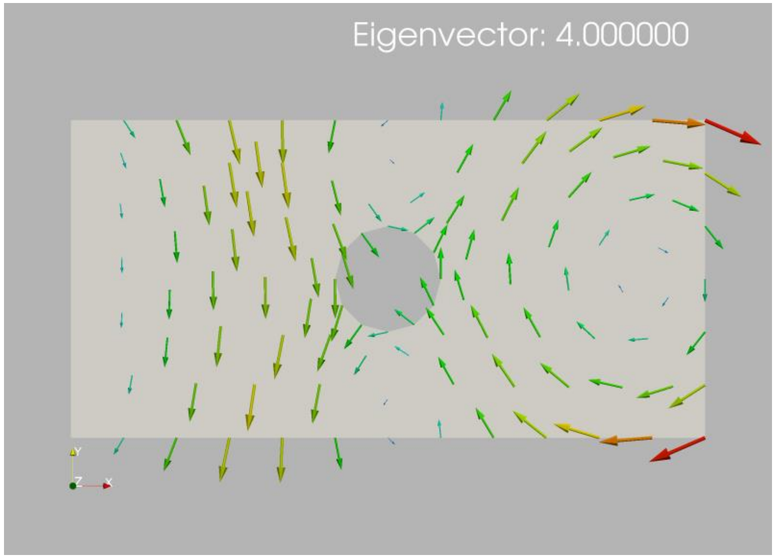


Eigenvectors

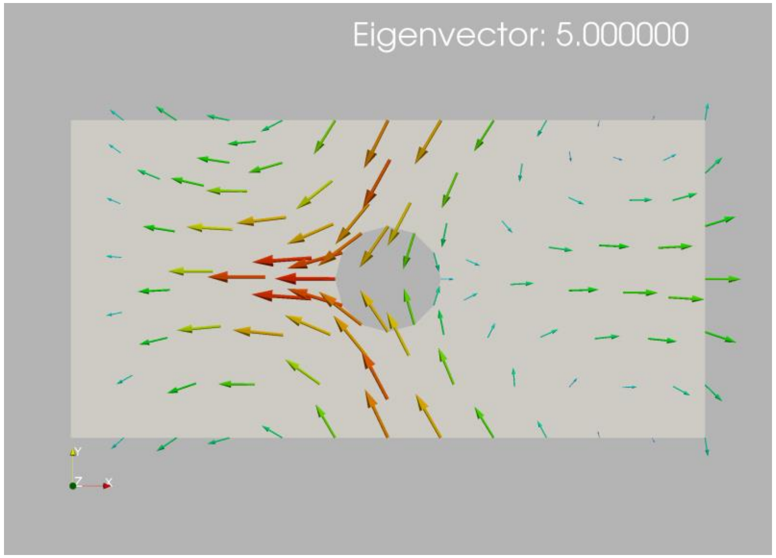


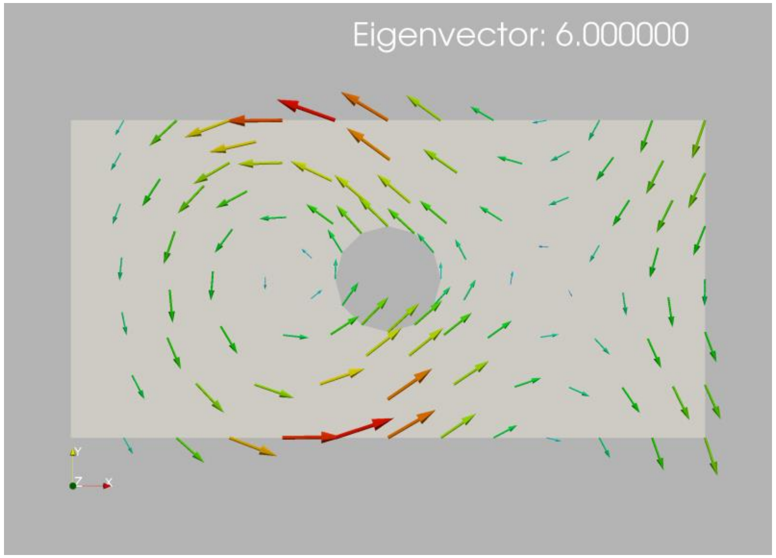


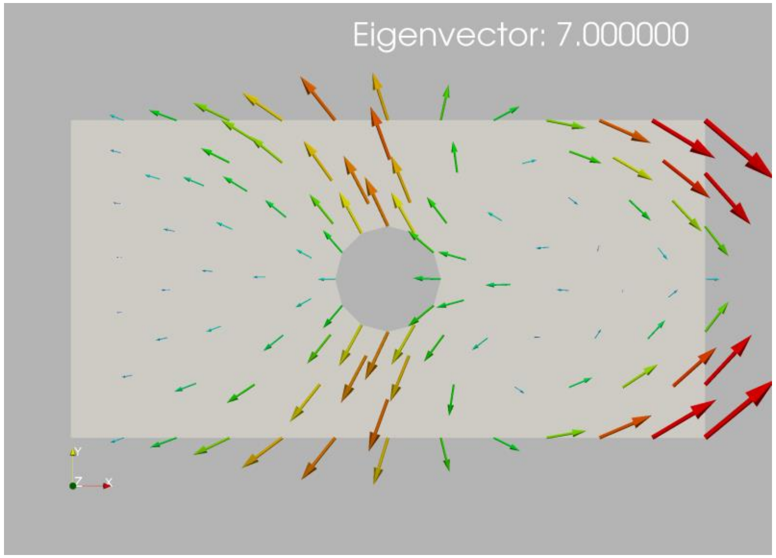


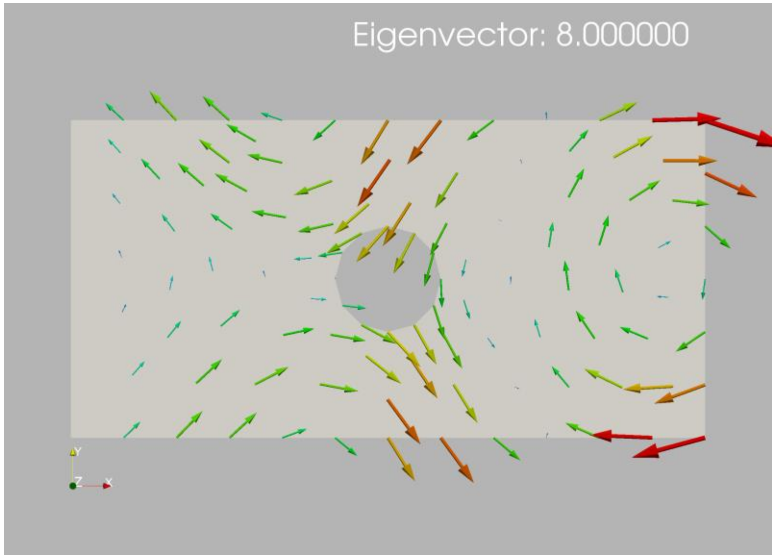


Eigenvectors

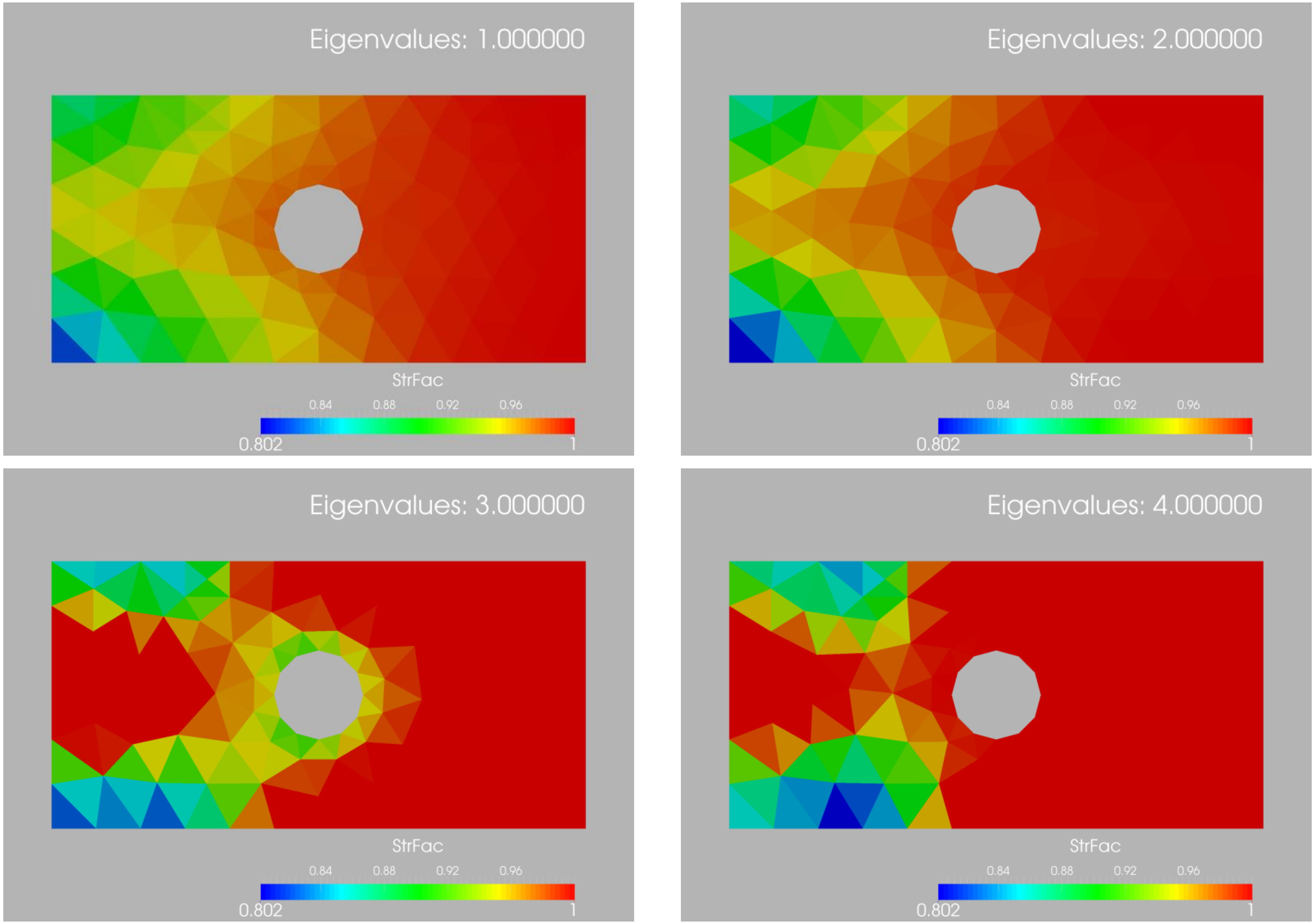


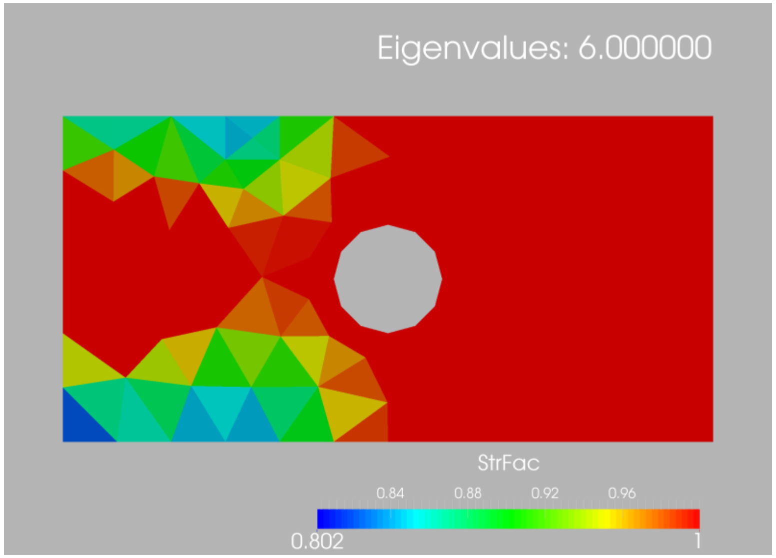


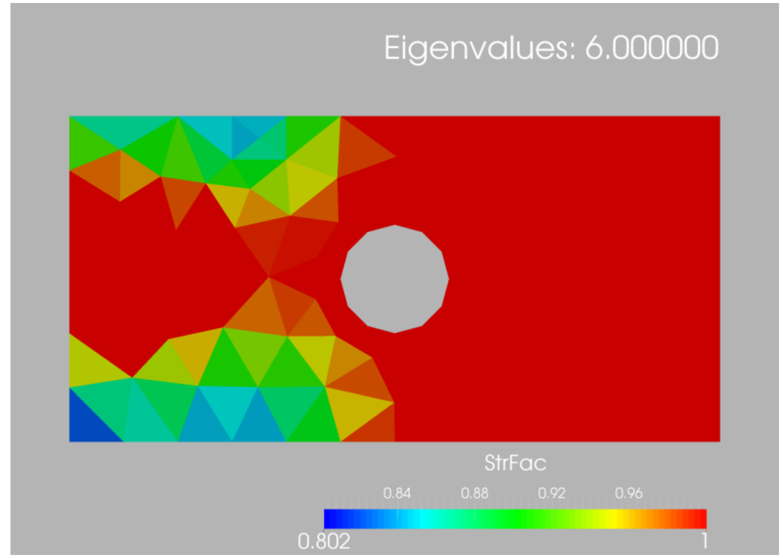


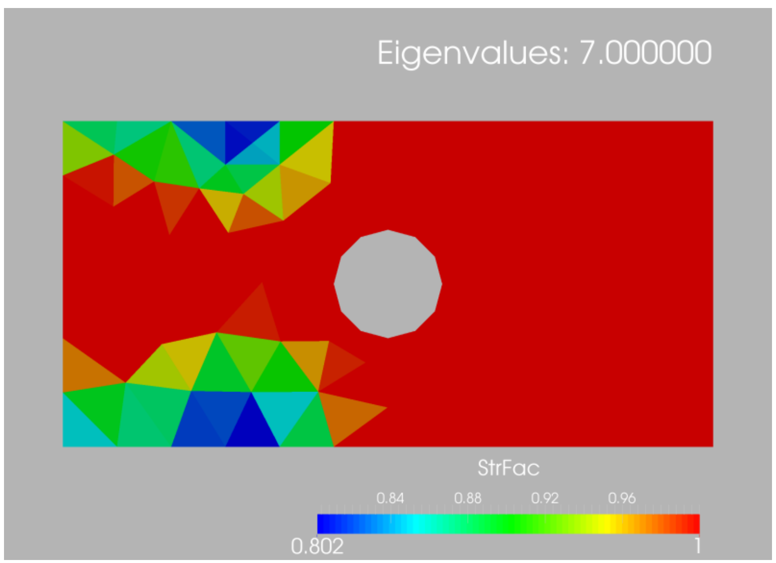


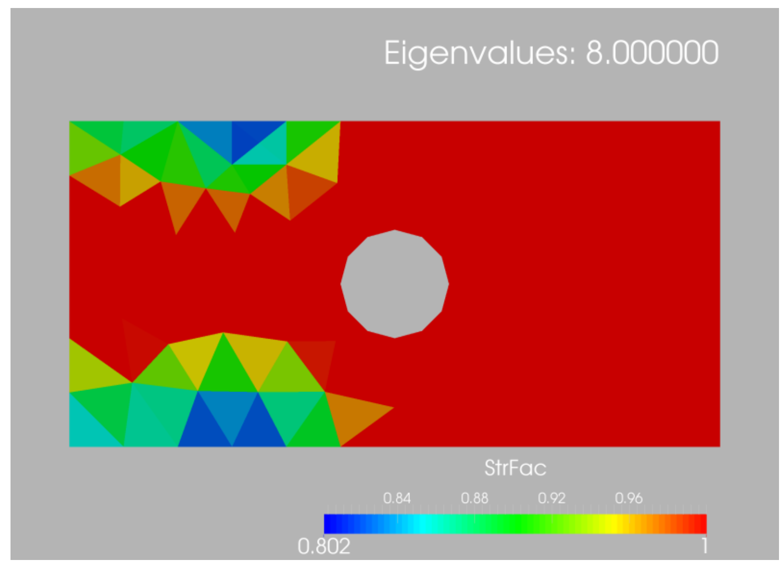
Weakening As A Function of Eigenmodes




Eigenvalues: 6.000000
StrFac
0.84
0.88
0.92
0.96
0.802
1


Eigenvalues: 6.000000
StrFac
0.84
0.88
0.92
0.96
0.802
1


Eigenvalues: 7.000000
StrFac
0.84
0.88
0.92
0.96
0.802
1


Eigenvalues: 8.000000
StrFac
0.84
0.88
0.92
0.96
0.802
1

Corollary for Eigenmodes

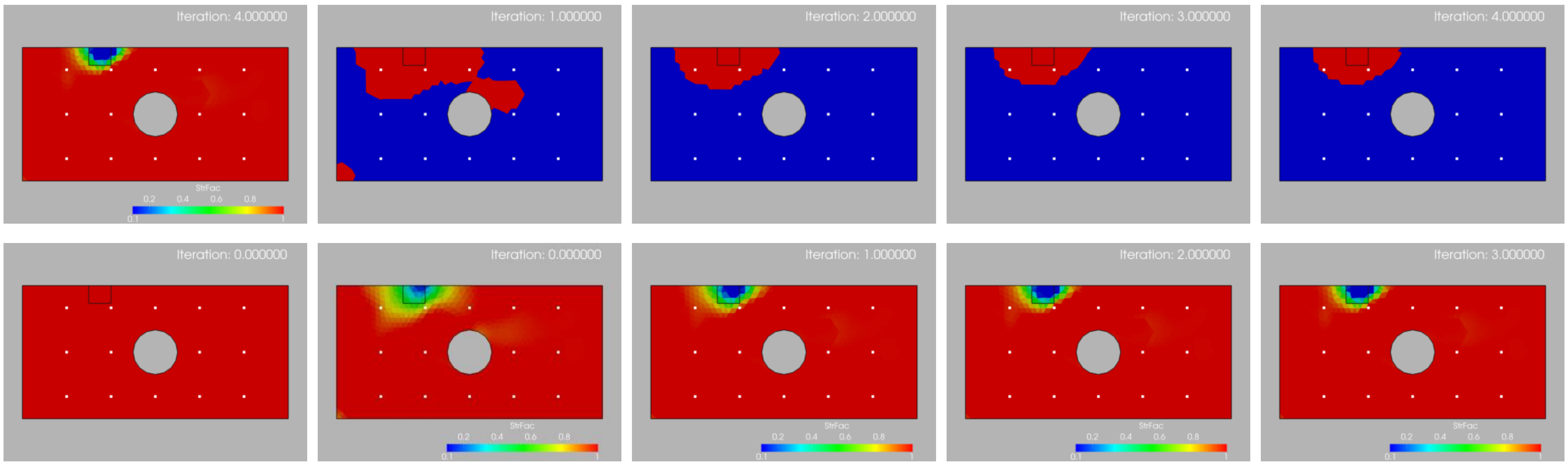
- Can Not Differentiate Symmetries
- ➔ Need `Extra Differentiator'
- Possible Options:
  - Eigenvalues + Eigenmodes [Difficult to Measure]
  - Eigenvalues + Displacements/Strains

Reduction of Search Space

- Observation 1: Iterations Correlate With NDOFs
  - The Larger the Mesh, the More Iterations
- Observation 2: After Few Iterations, `Interesting Regions' Already Apparent
- Idea: Remove From Consideration All Regions Where $\alpha=O(1)$

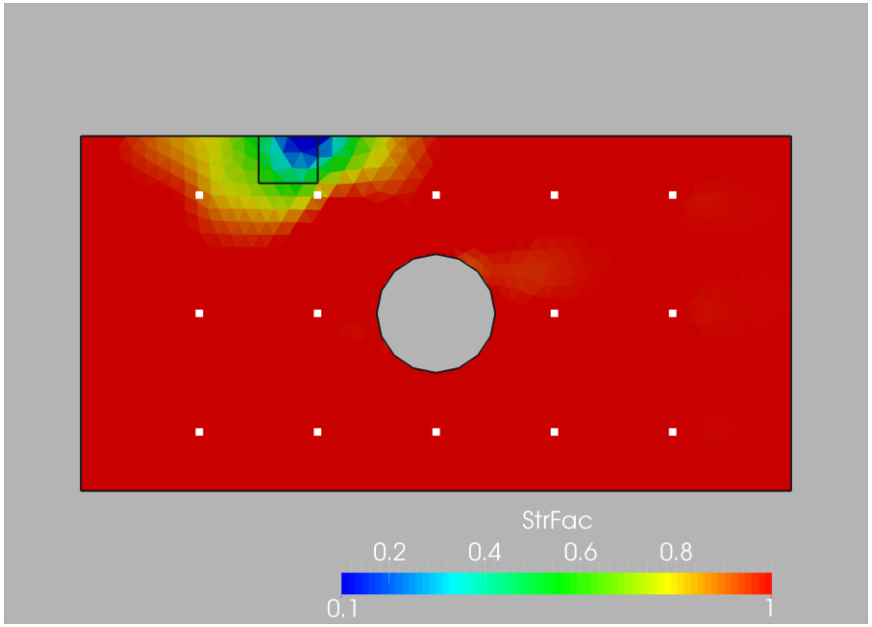
Plate With Hole: Fine Mesh

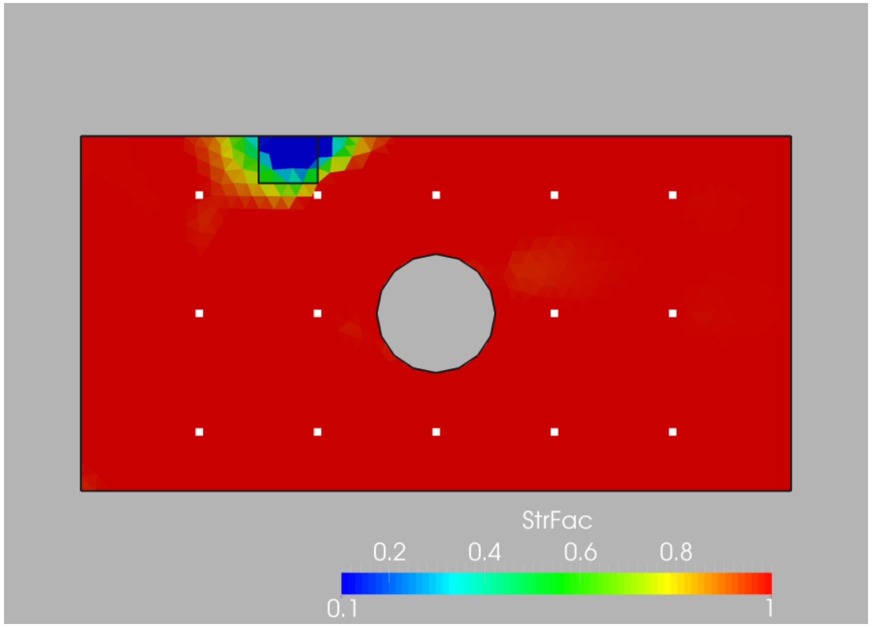
Active Zone: Red: Active, Blue: Inactive

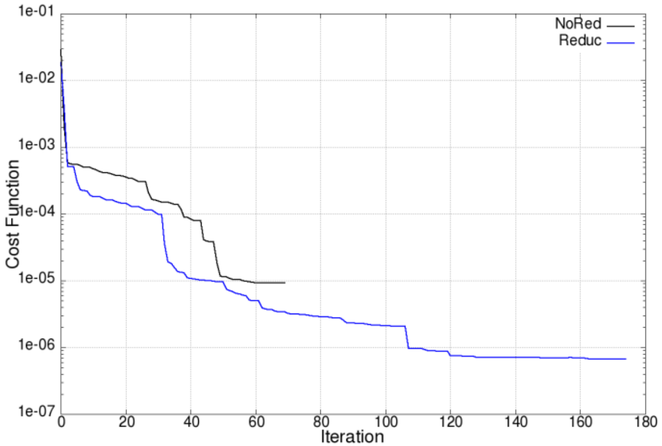


Strength Factor

Plate With Hole: Fine Mesh

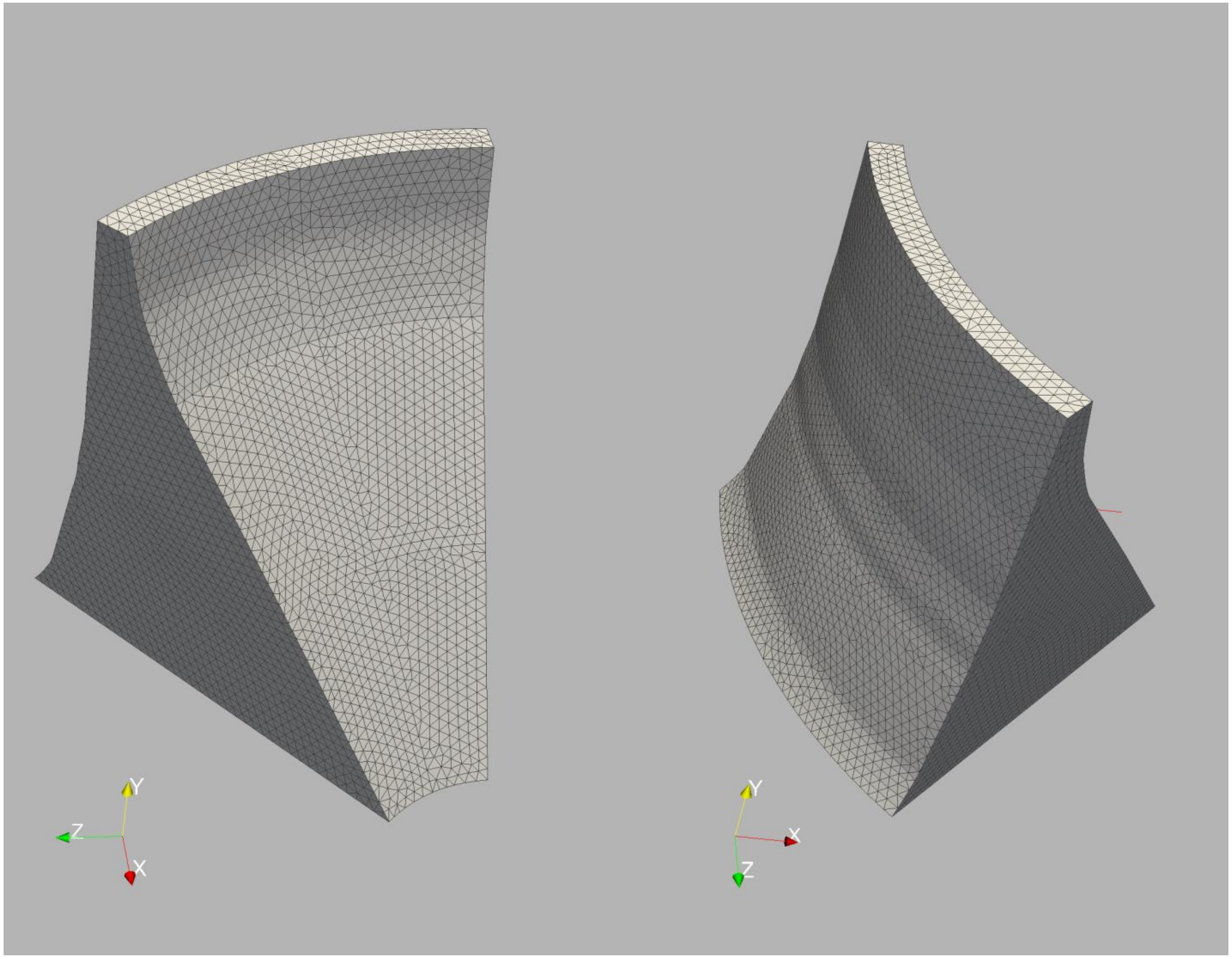




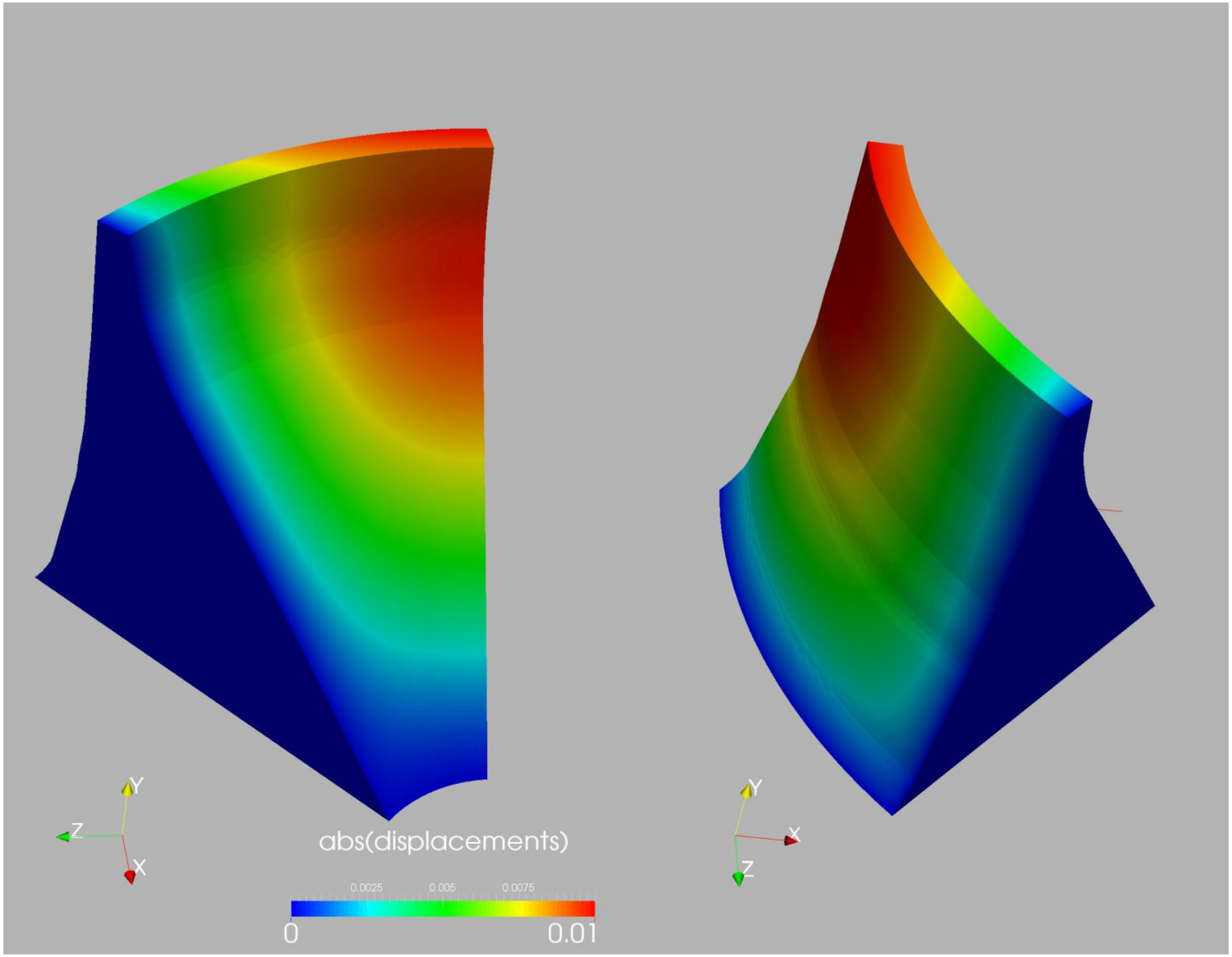


Hoover Dam

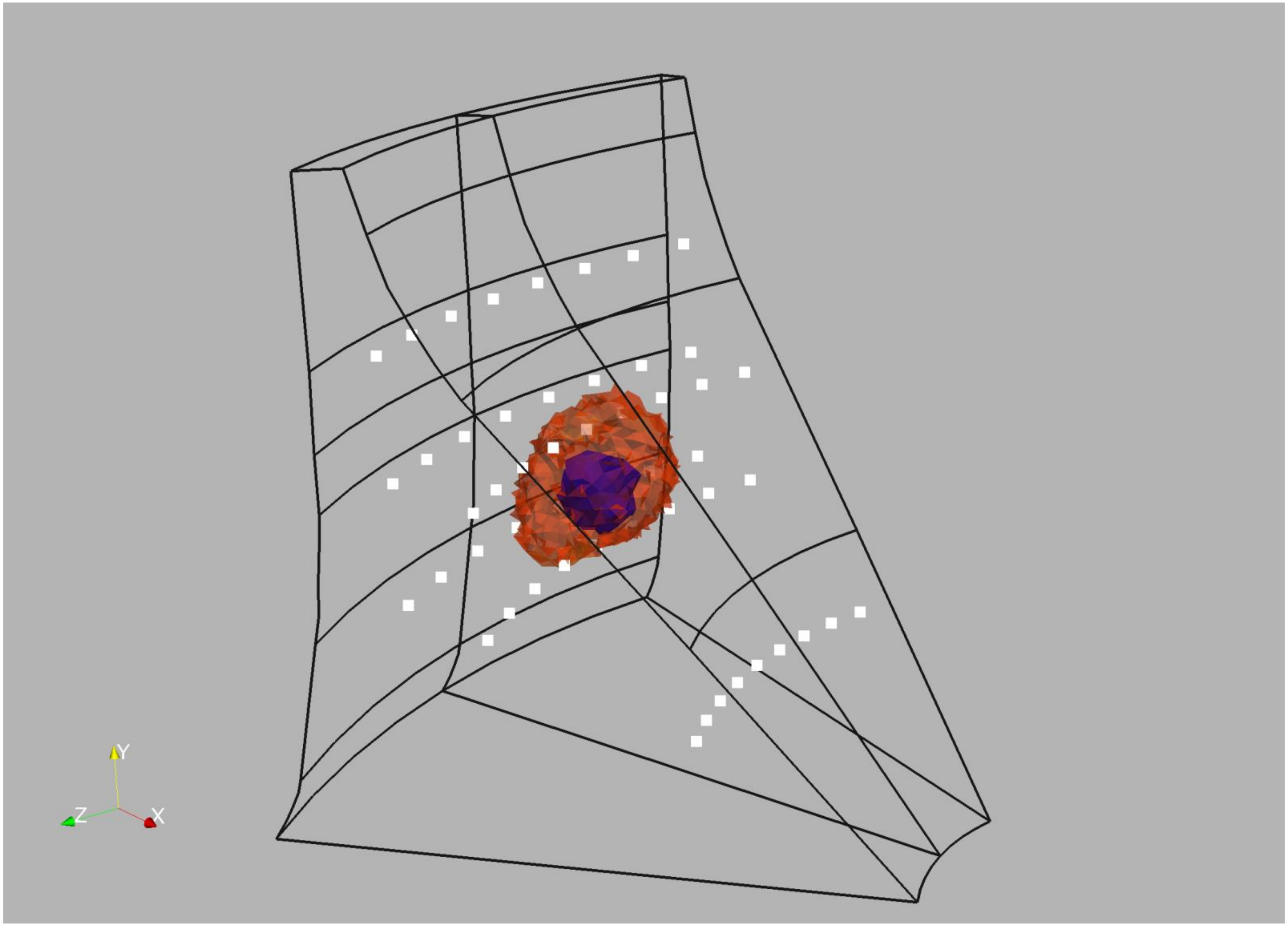
- FEELAST
- 200Kels



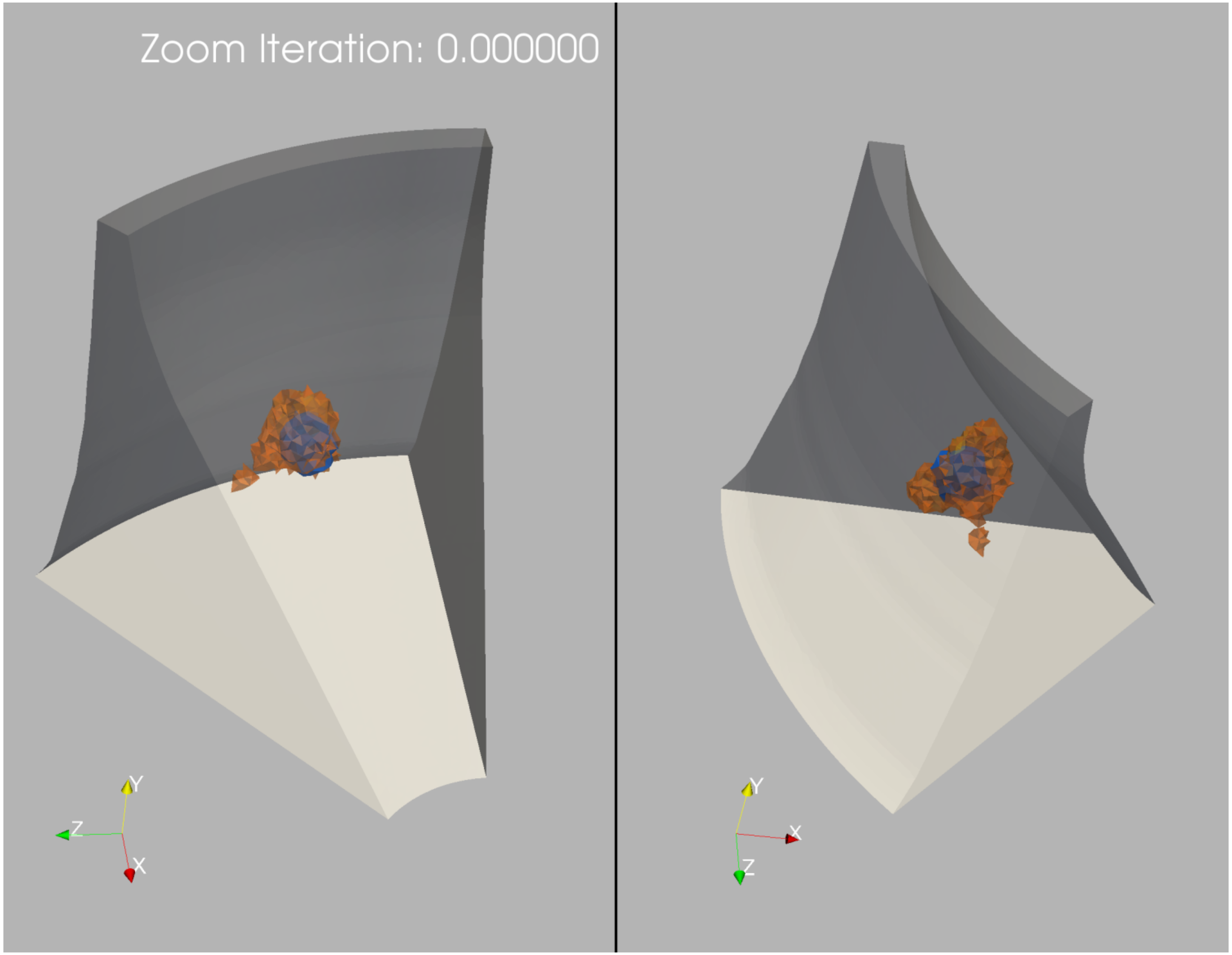
Hoover Dam: Displacements



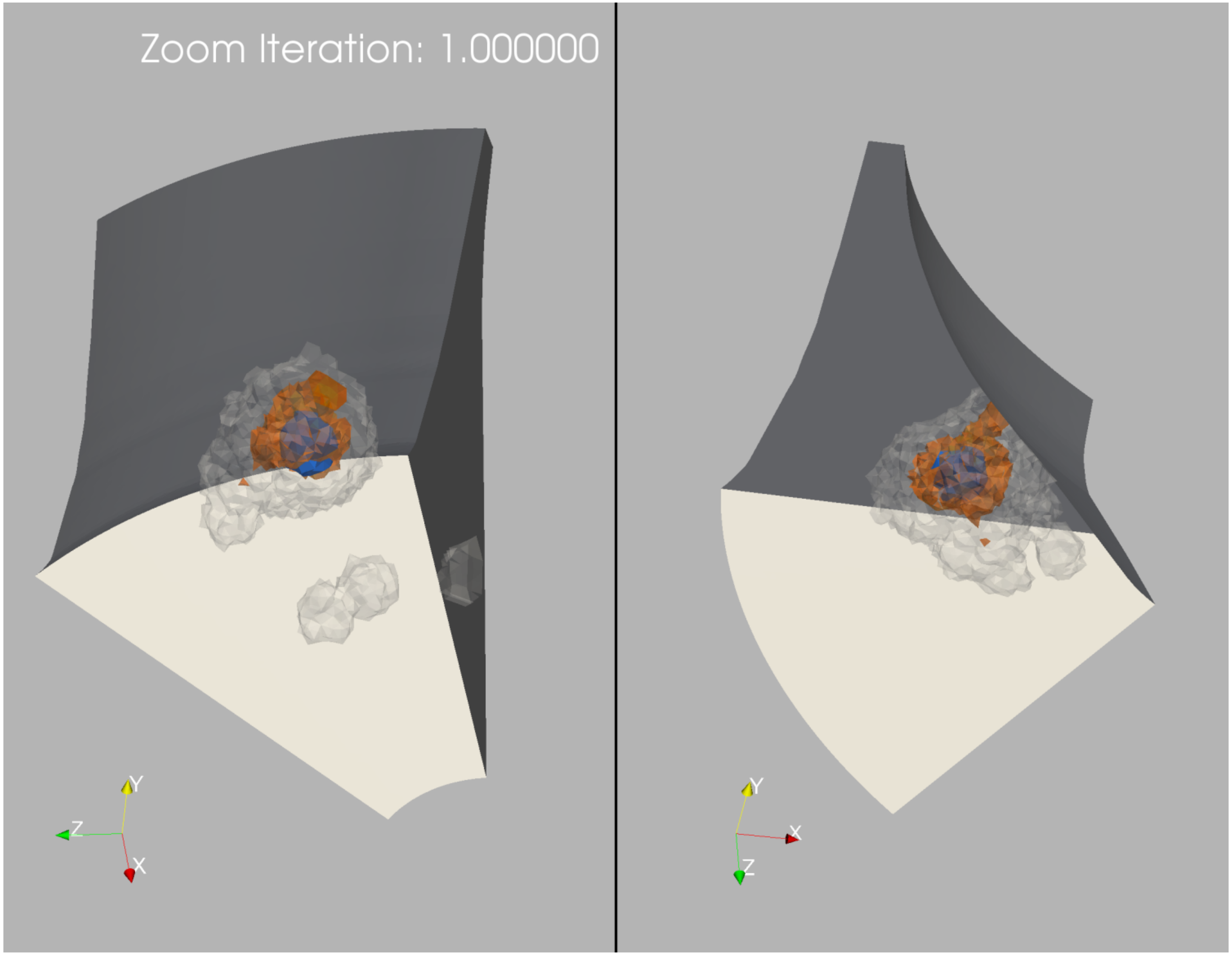
Hoover Dam: 51 Sensors



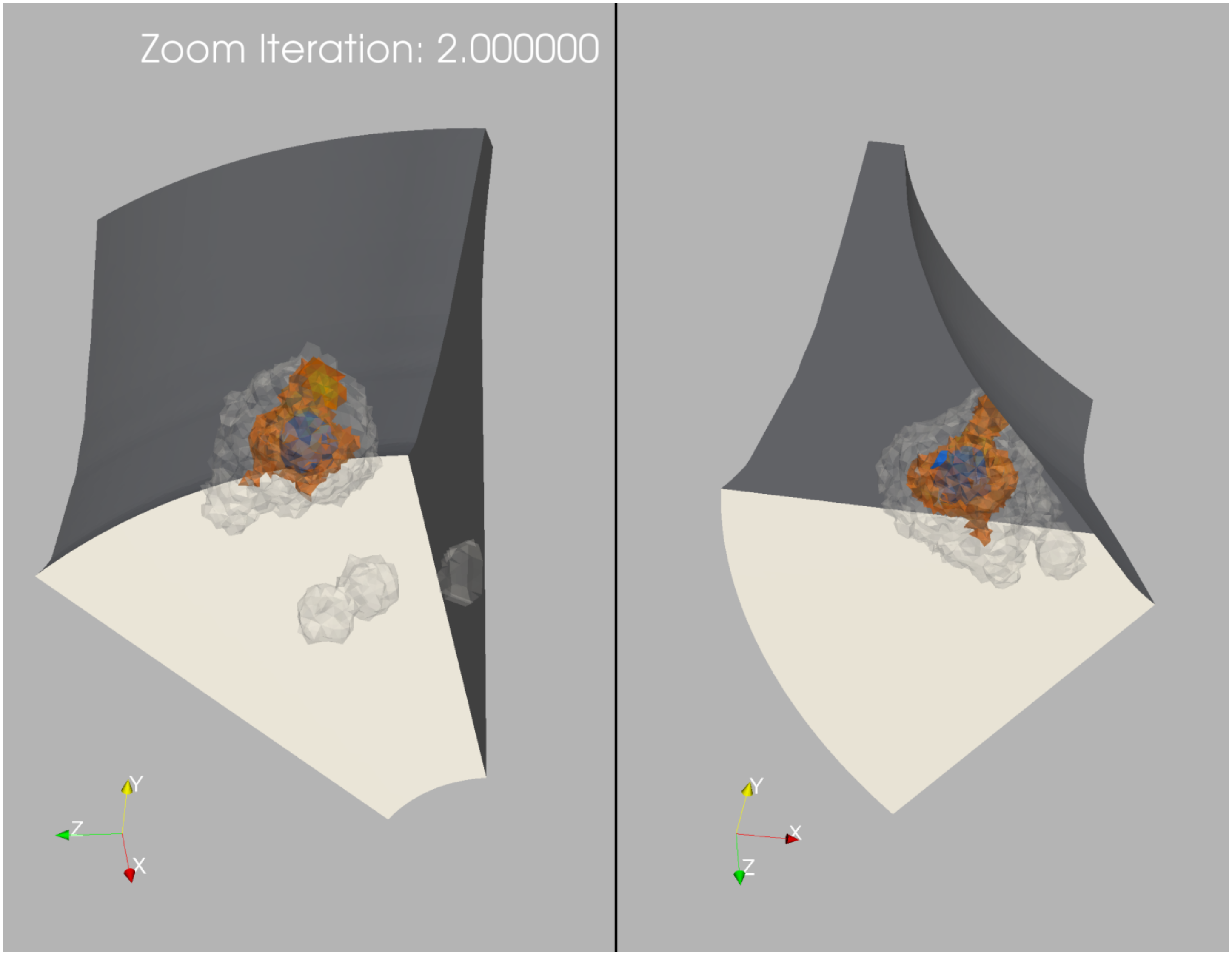
Hoover Dam



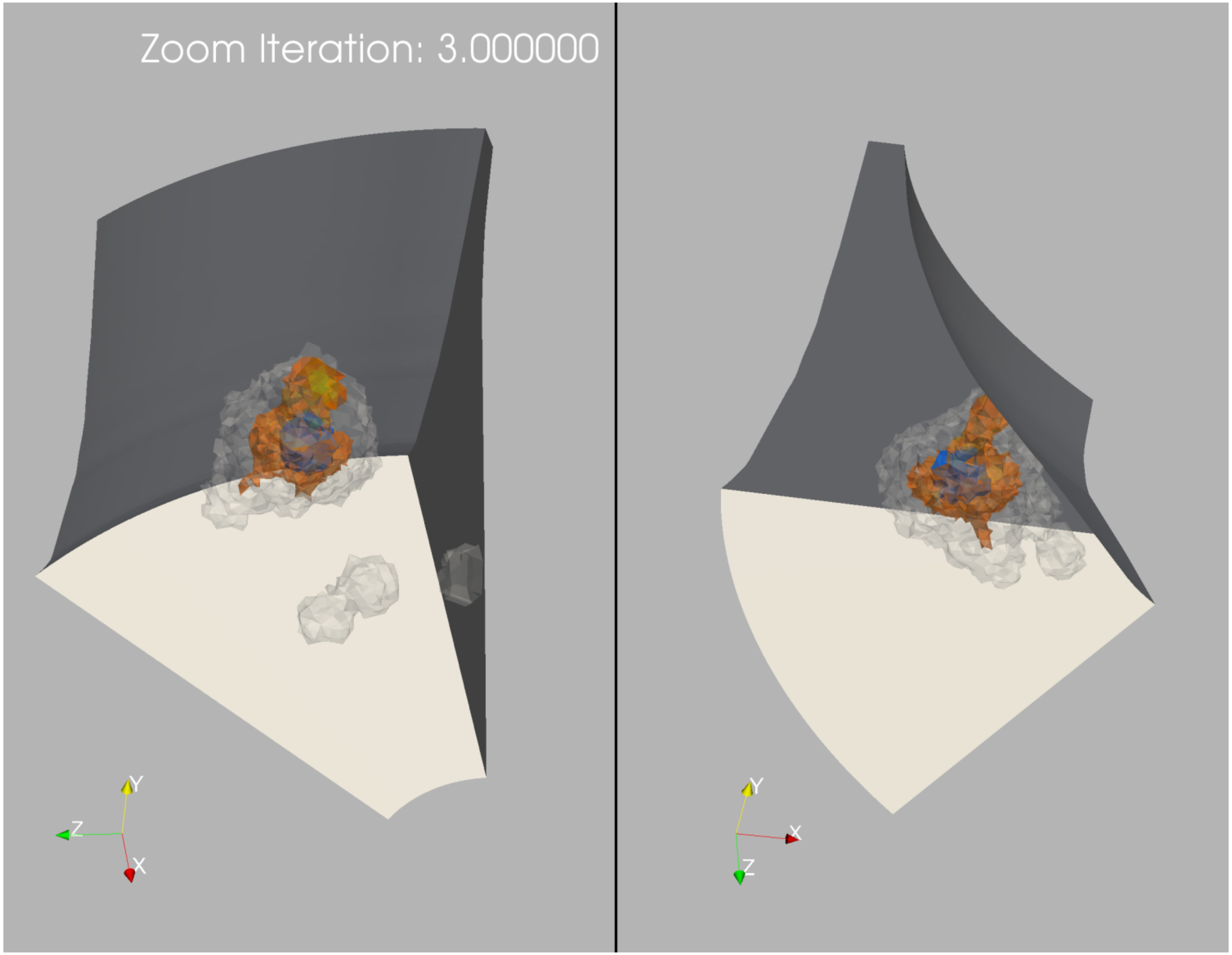
Hoover Dam



Hoover Dam



Hoover Dam



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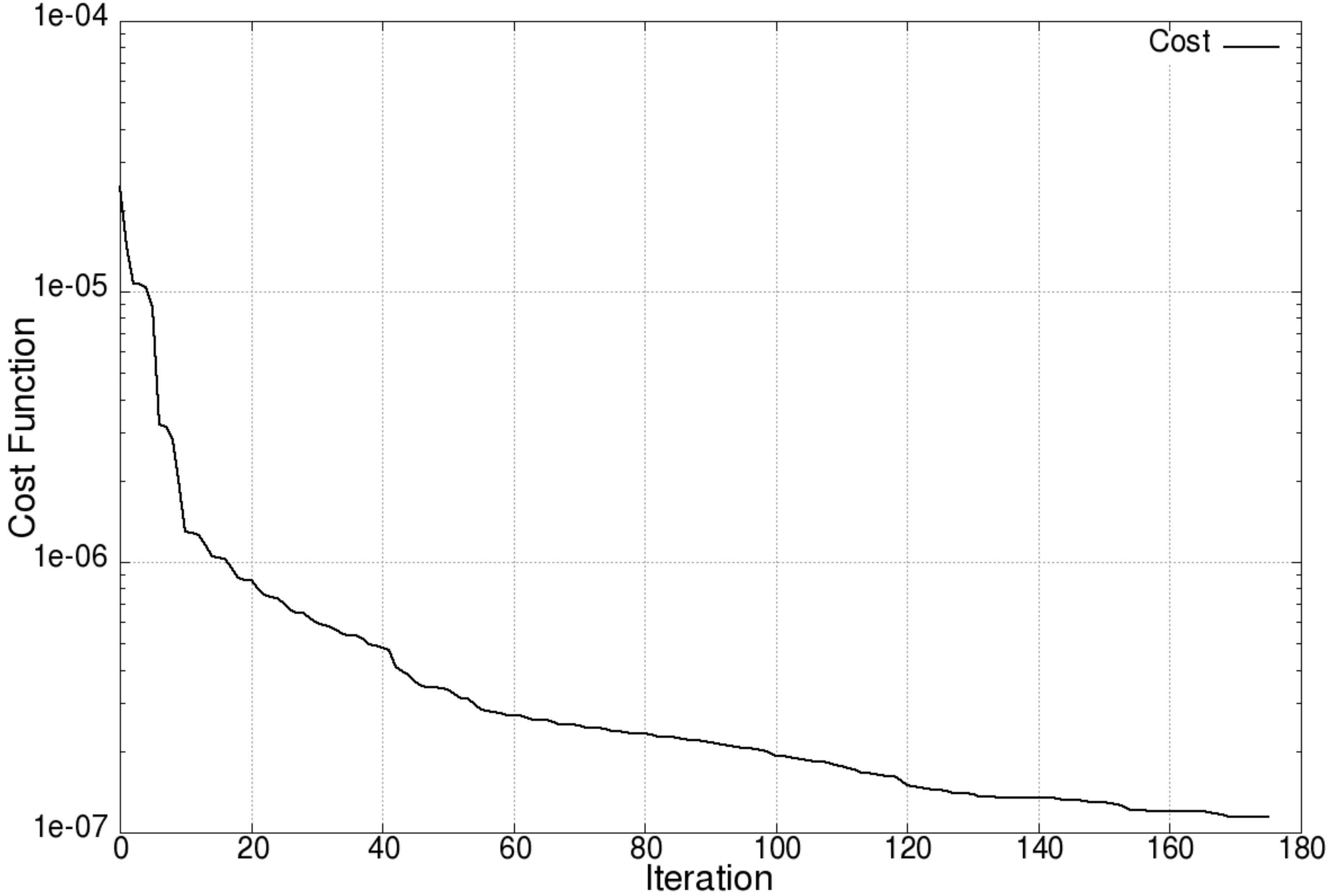
Hoover Dam



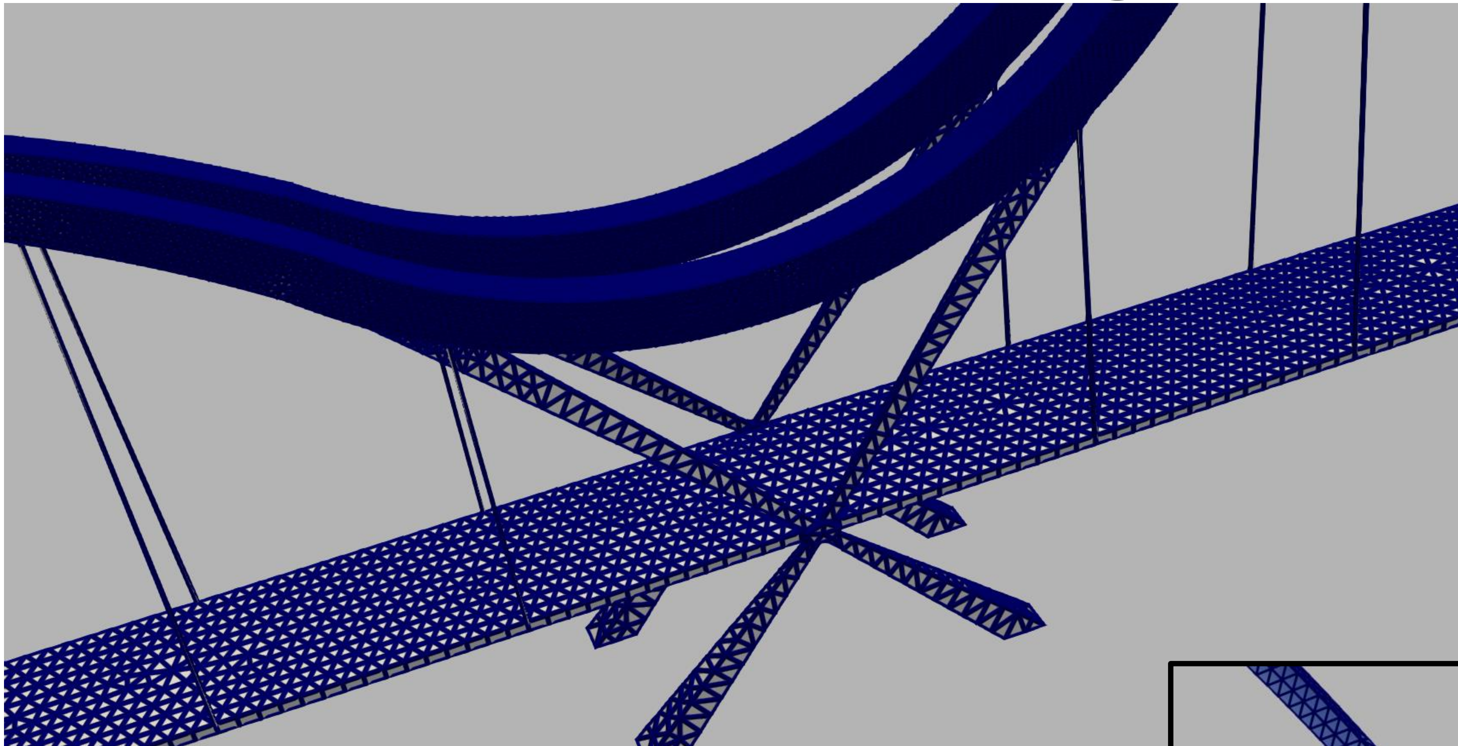
Cost
1e-04
1e-05
1e-06
1e-07
Cost Function
0
20
40
60
80
100
120
140
160
180
Iteration

Singularities

- Present in Many Hi-Fi Models
- Usually Ignored by Designers/Engineers
  - `These Are The Devils We Know'
  - St. Venant's Principle (Effects Are Local)
- Could Pose Problem for Weakness Detection
  - `Singularity Pollutes Signal:Noise Ratio'
- Option 1: Change Model
  - Labour Intensive
  - Probably Not Viable in Practice
- Option 2: Filter/Damp Out Regions
  - Detection ?
  - Automation ?
  - Use ?

Infinity Bridge (1)

Infinity Bridge (2)

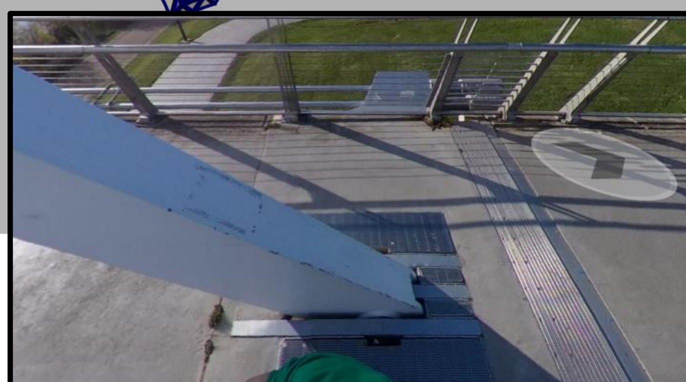


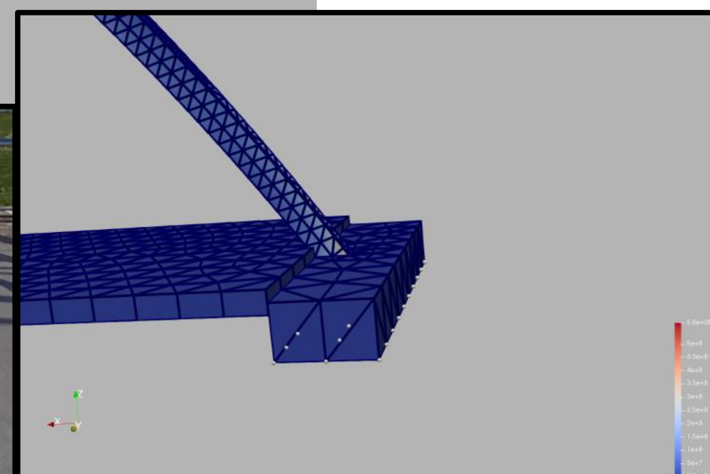
Calculix

Element Types:

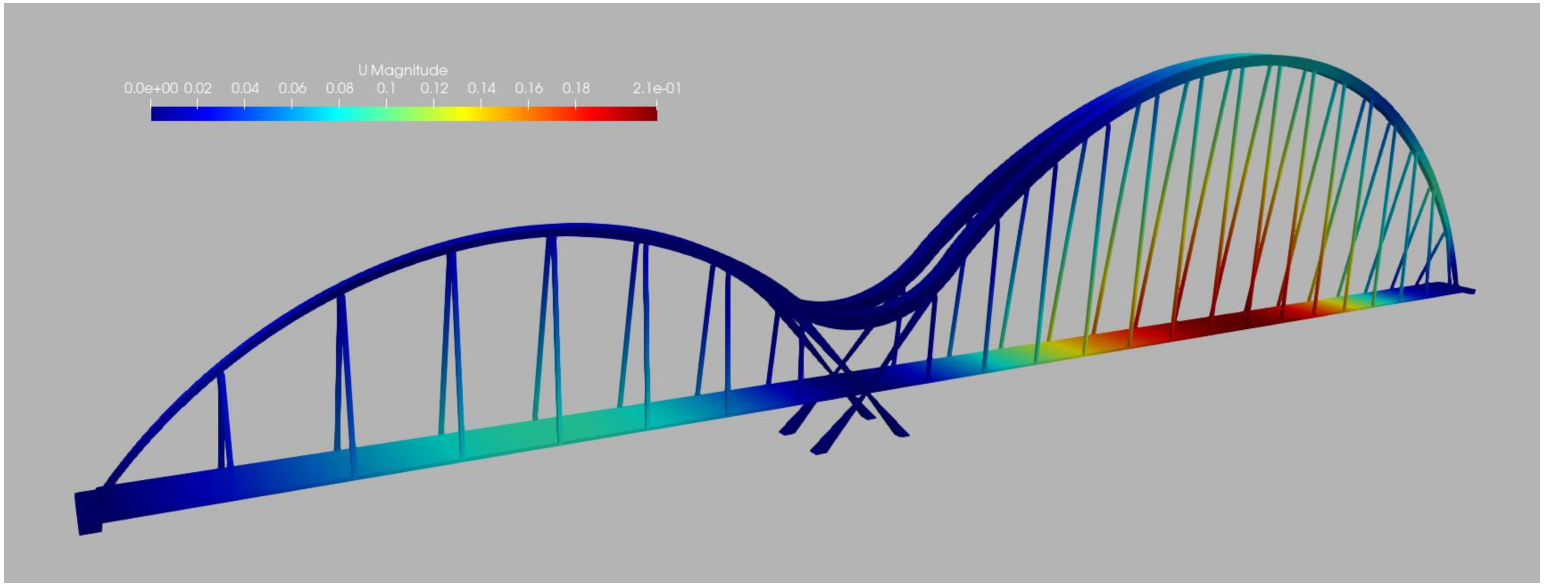
Trusses, Beams,

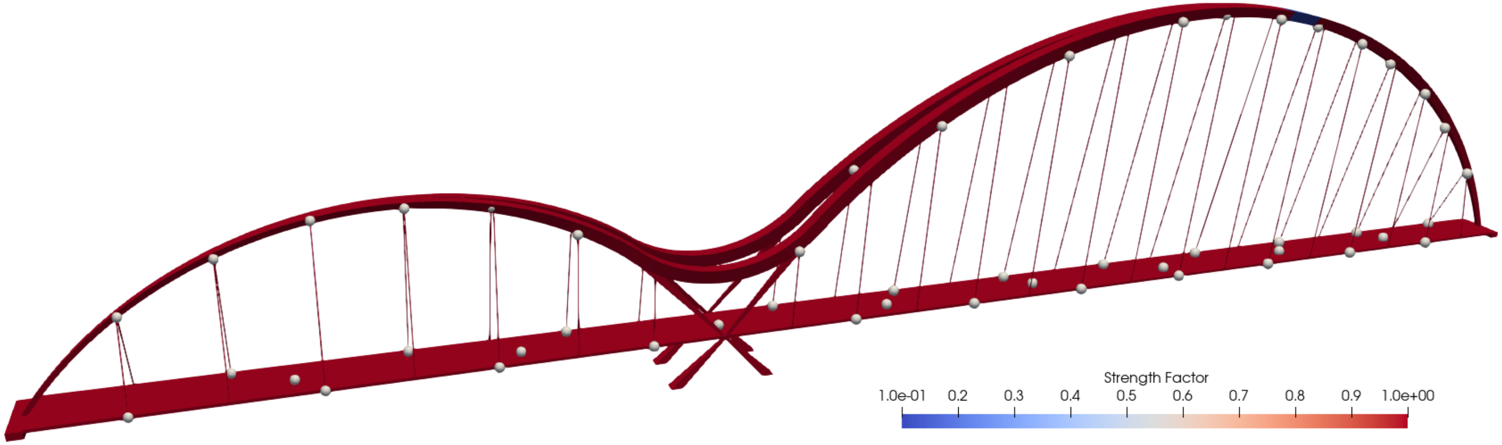
Plates, Solids



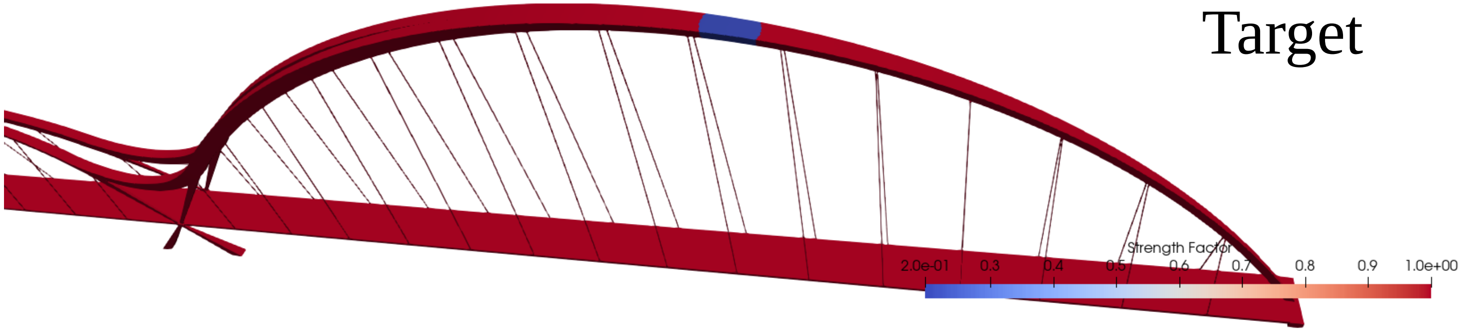


Infinity Bridge (3)

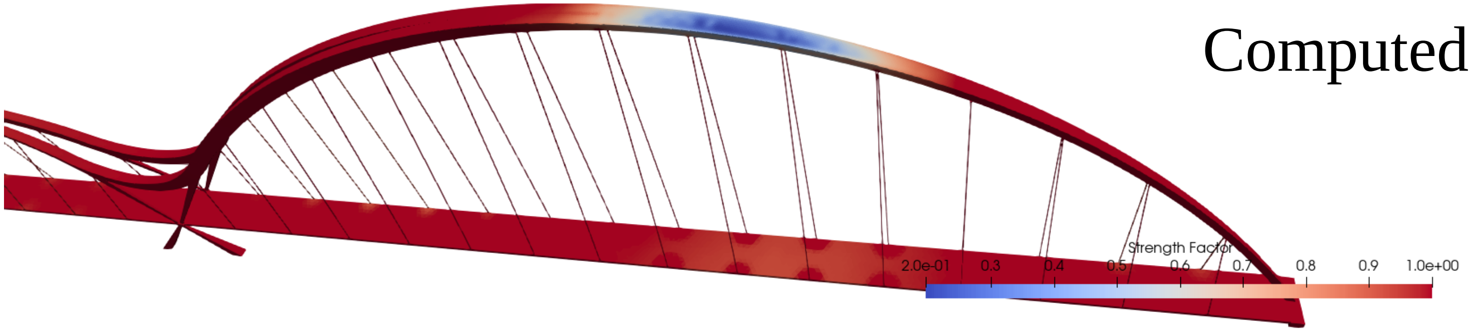




Infinity Bridge (4)



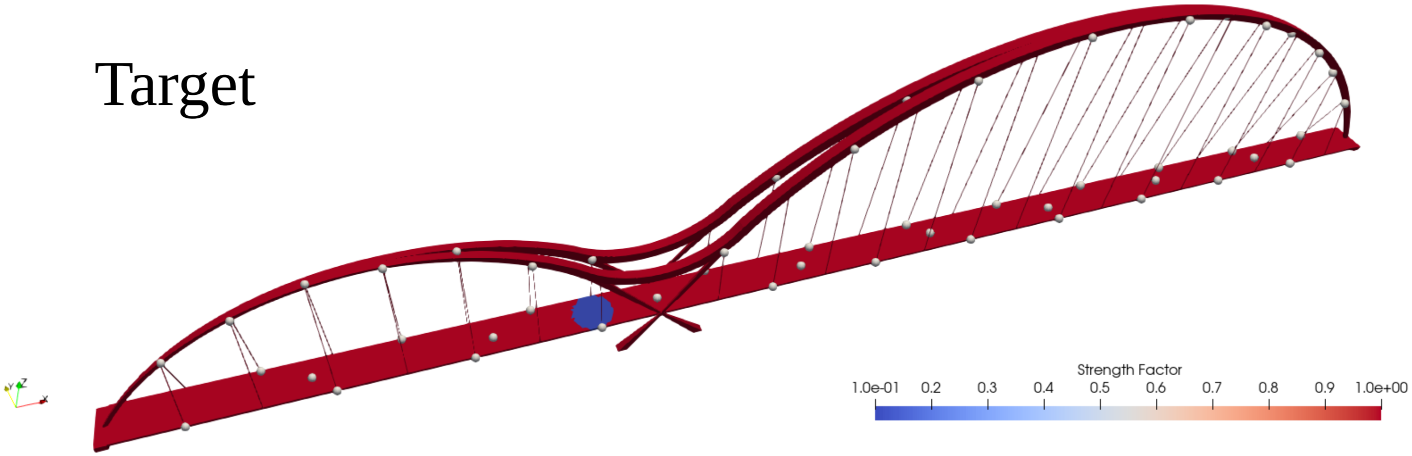
Target



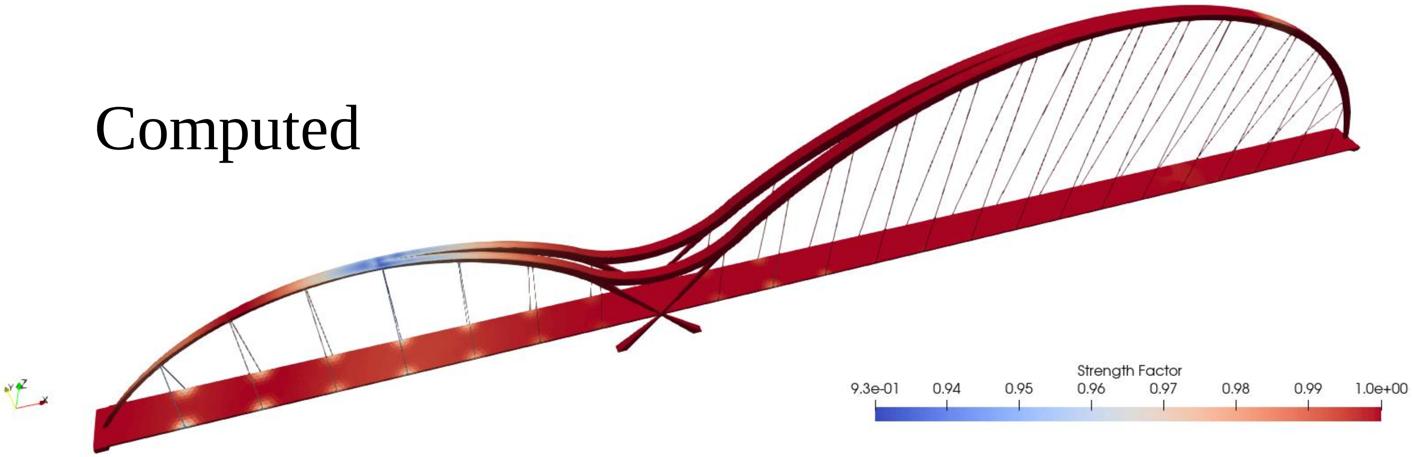
Computed

Infinity Bridge (5)

Target



Computed



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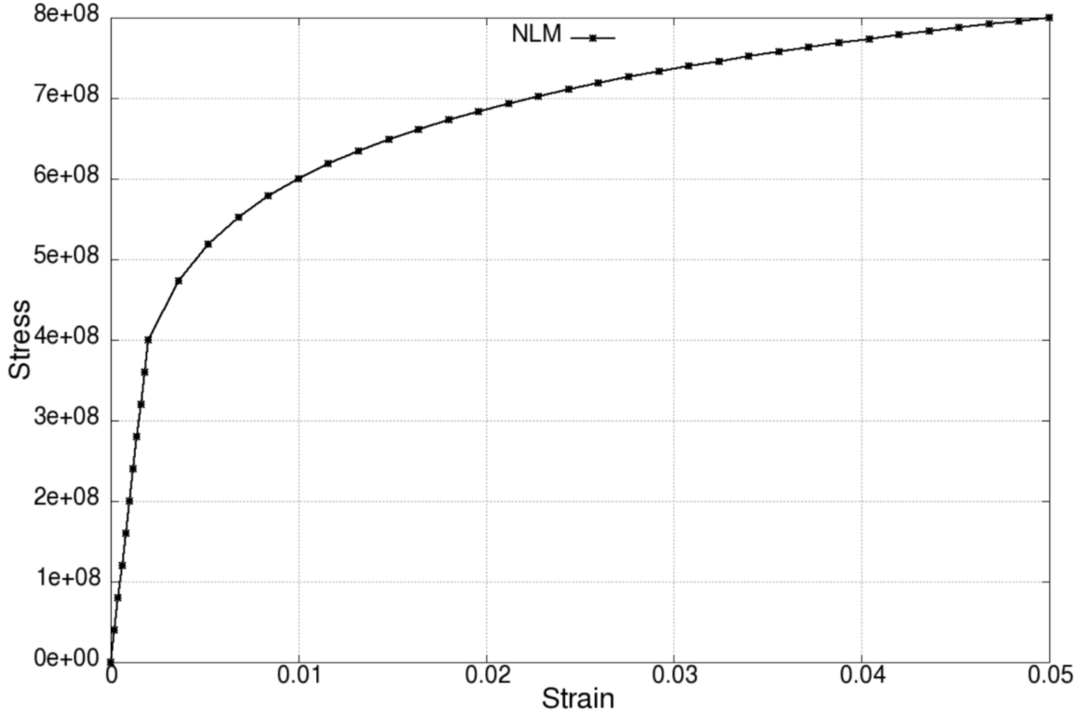
Elasto-Plastic

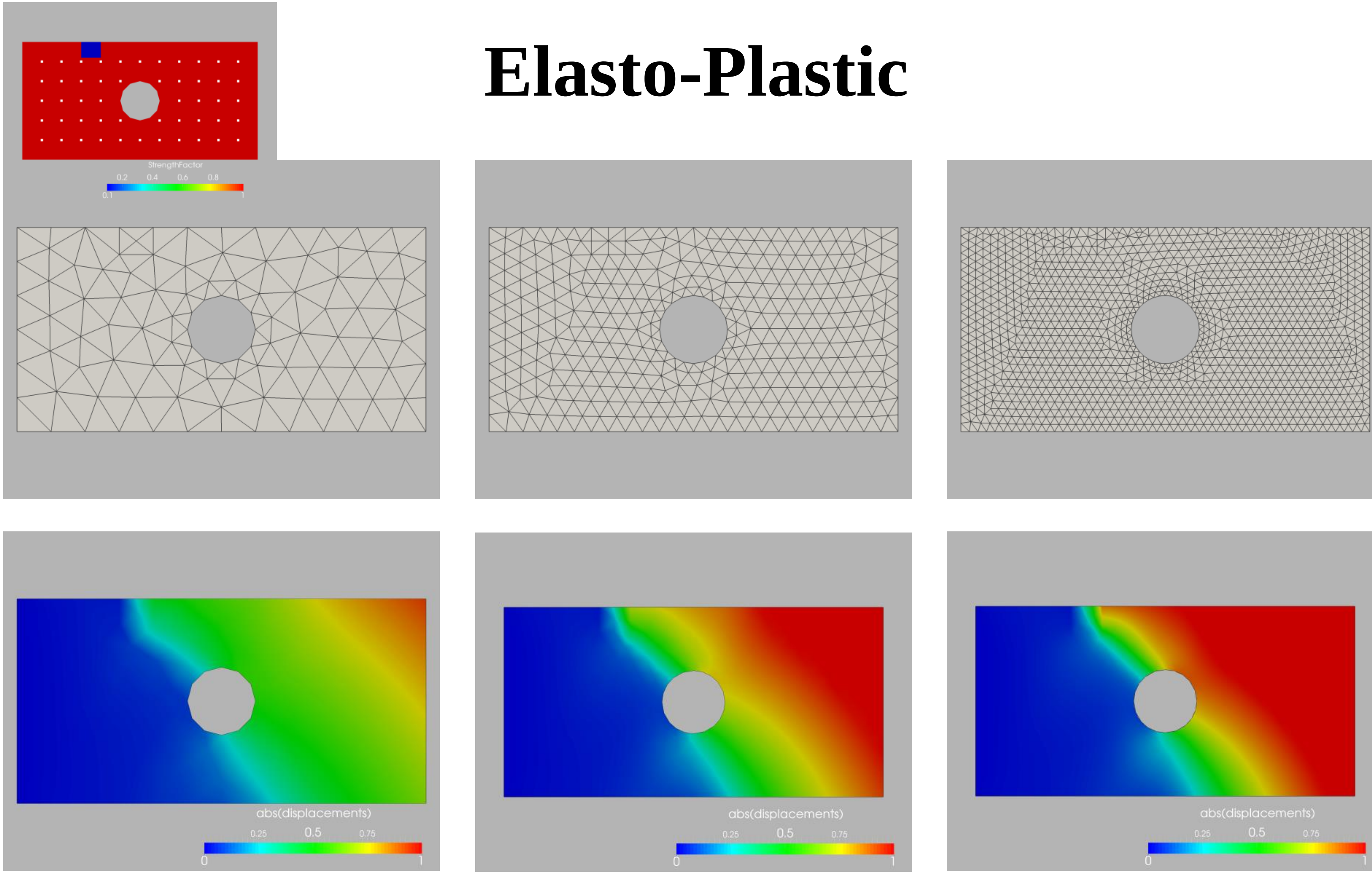
- Nonlinear Material

$$\epsilon \leq \epsilon_0 : E = E_0$$

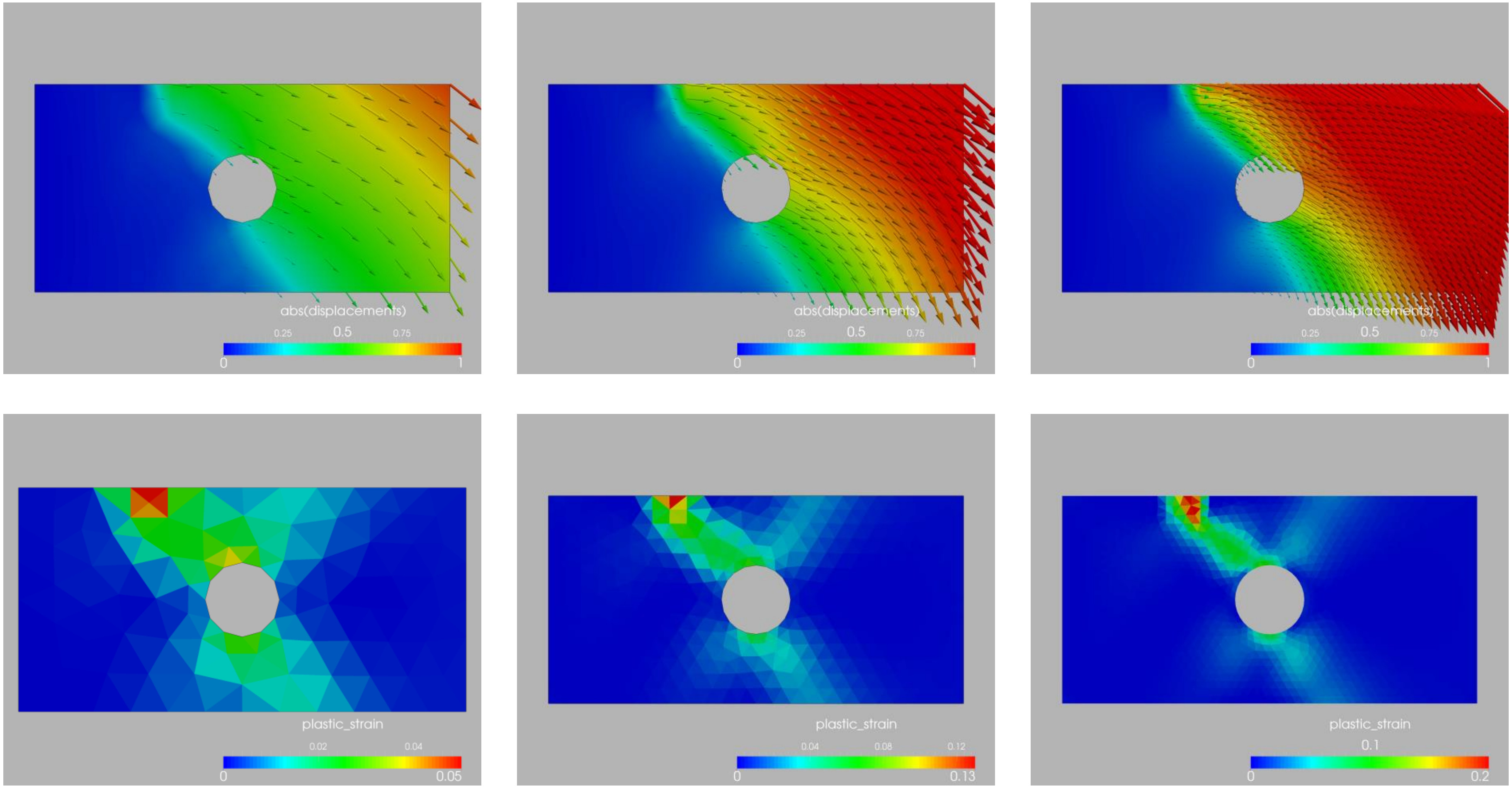
$$\epsilon \geq \epsilon_0 : E_0\epsilon_0 + (E_1\epsilon_1 - E_0\epsilon_0)(\log(\epsilon/\epsilon_0))/(\log(\epsilon_1/\epsilon_0))$$



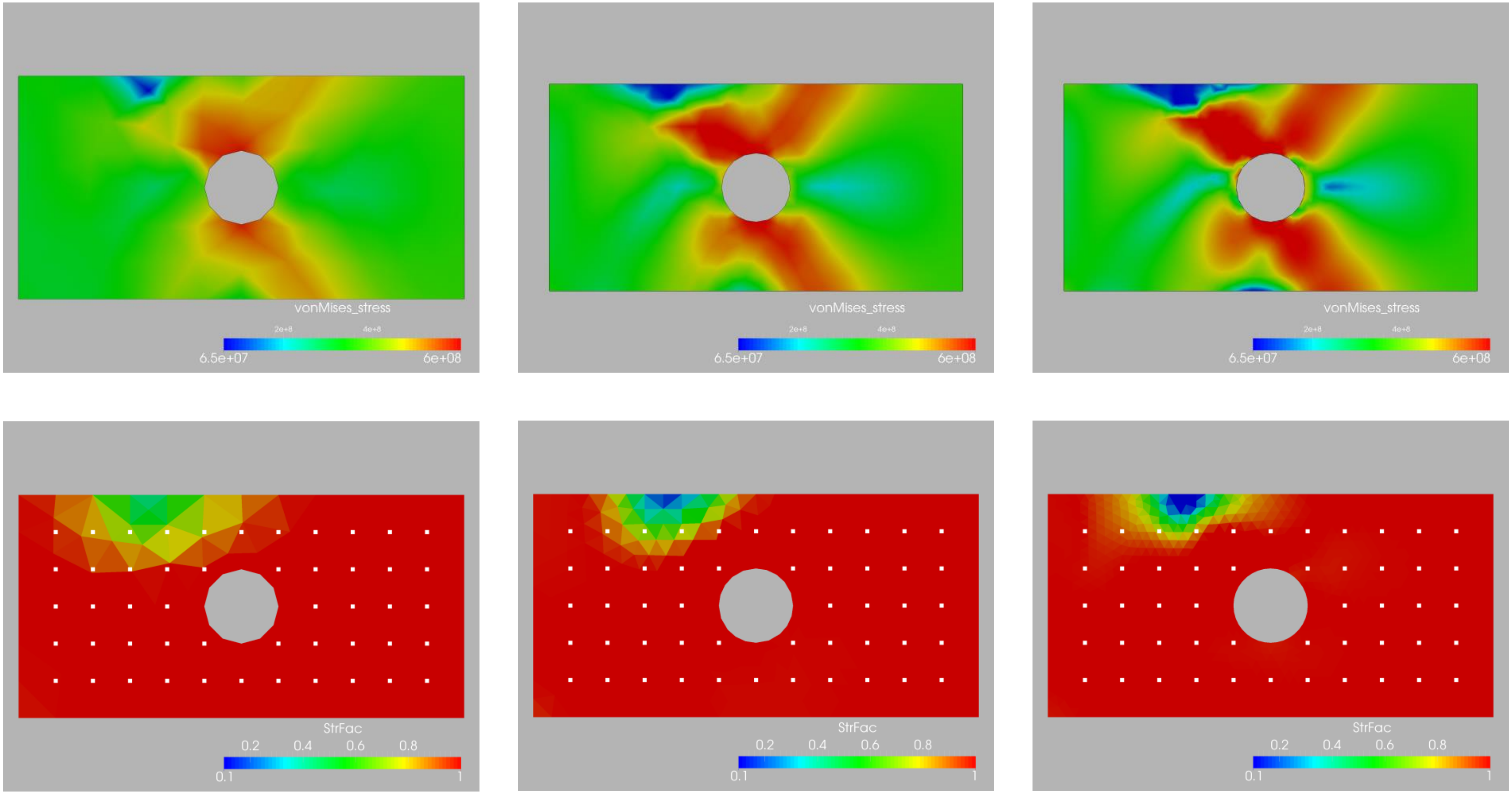
Elasto-Plastic



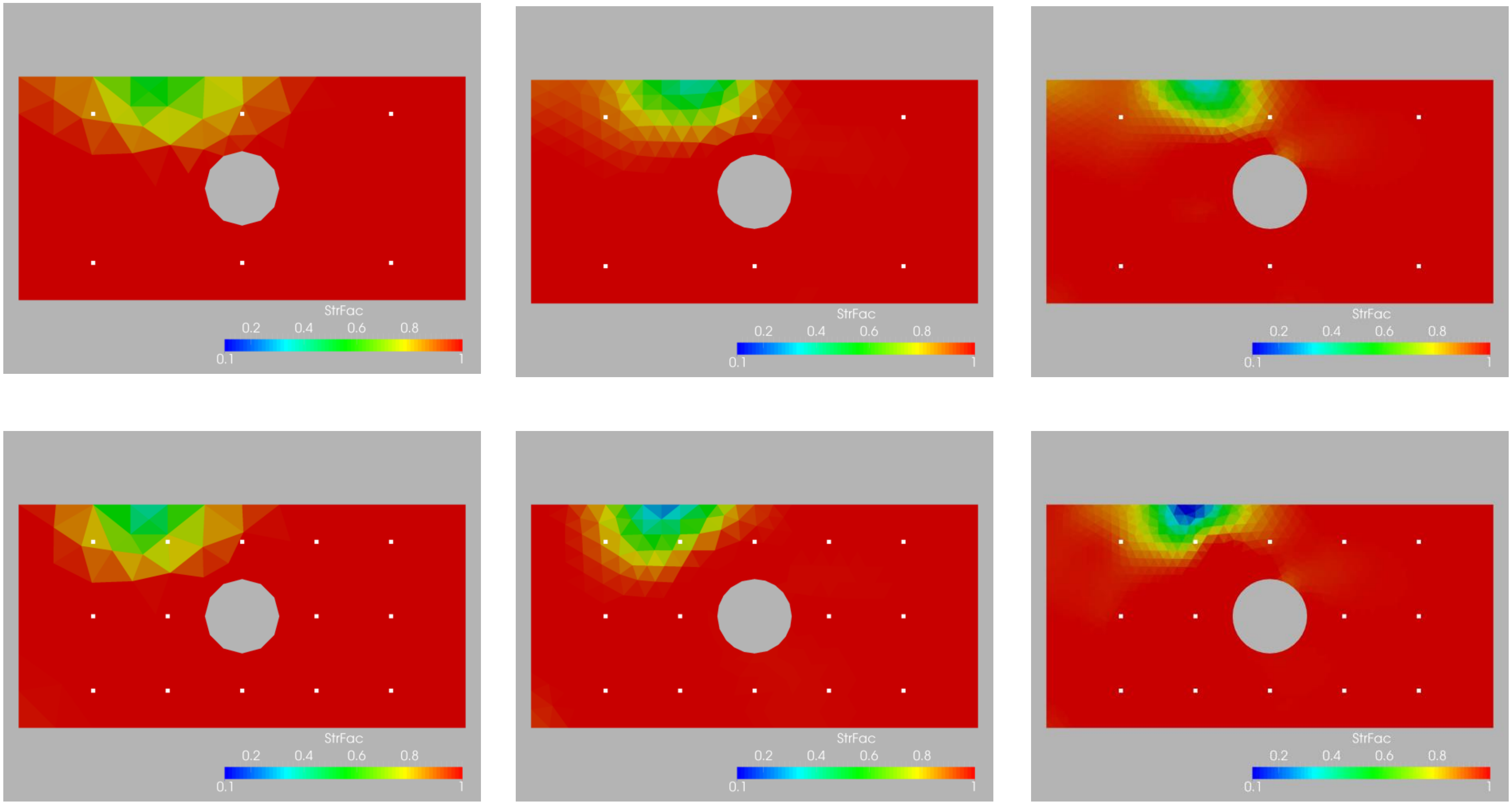
Elasto-Plastic



Elasto-Plastic



Elasto-Plastic



Recovery of Temperature Fields

Recovery of Temperature Fields (1)

- Real Object (Bridge, Building, …) May Have Deformations Due to Thermal Stresses
- ➔ Need to `Remove' These To Assess Effect of [Real] Damage
- ➔ Try to Recover Temperature Field From Displacements/Strains

Recovery of Temperature Fields (2)

- Same Notation as Before
- Optimization Problem: Find Temperature Variation

$$I(\mathbf{u}, \Delta T) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}^{md}(\mathbf{u}_{ij}^{md} - \mathbf{I}_{ij}^{d}\cdot\mathbf{u}_i)^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m} w_{ij}^{ms}(\mathbf{s}_{ij}^{ms} - \mathbf{I}_{ij}^{s}\cdot\mathbf{s}_i)^2$$

- Subject To:

$$\mathbf{K}\cdot\mathbf{u} = \mathbf{f}_{ext} + \mathbf{f}_{\Delta T}$$

Recovery of Temperature Fields (3)

- Extended Lagrangian

$$L(\mathbf{u}, \Delta T, \tilde{\mathbf{u}}) = I(\mathbf{u}, \Delta T) + \tilde{\mathbf{u}}^t \cdot (\mathbf{K} \cdot \mathbf{u} - \mathbf{f}_{ext} - \mathbf{f}_{\Delta T})$$

- Gradients/Derivatives

$$\frac{dL}{d\tilde{\mathbf{u}}} = \mathbf{K} \cdot \mathbf{u} - \mathbf{f}_{ext} - \mathbf{f}_{\Delta T} = 0$$

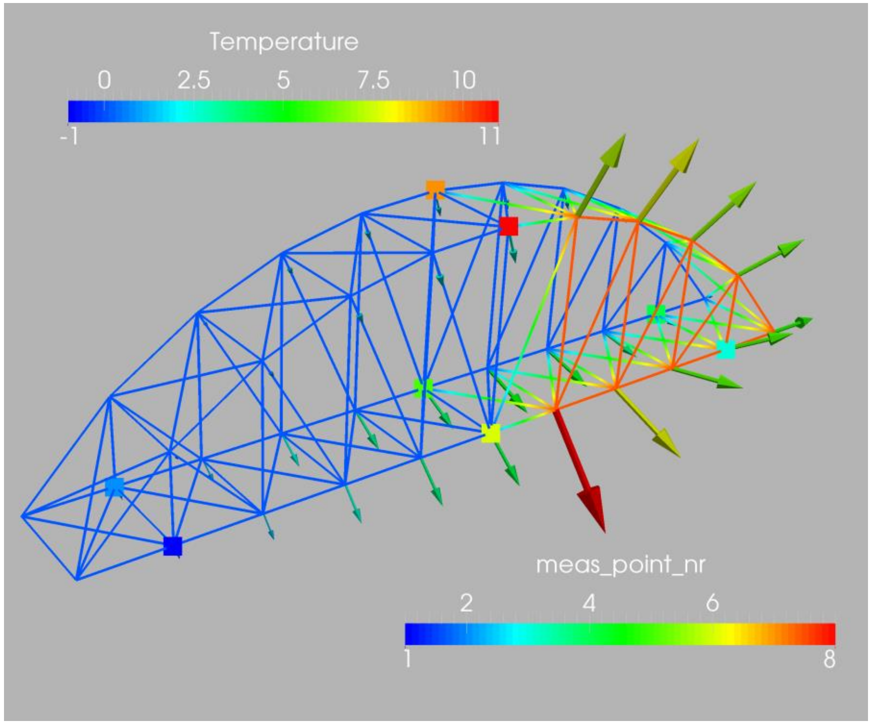
$$\frac{dL}{d\mathbf{u}} = \sum_{j=1}^{m} w_j^{md} (\mathbf{u}_j^{md} - \mathbf{I}_j^d \cdot \mathbf{u}) + \sum_{j=1}^{m} w_j^{ms} (\mathbf{s}_j^{ms} - \mathbf{J}_j^s \cdot \mathbf{s}) + \mathbf{K}^t \cdot \tilde{\mathbf{u}} = 0$$

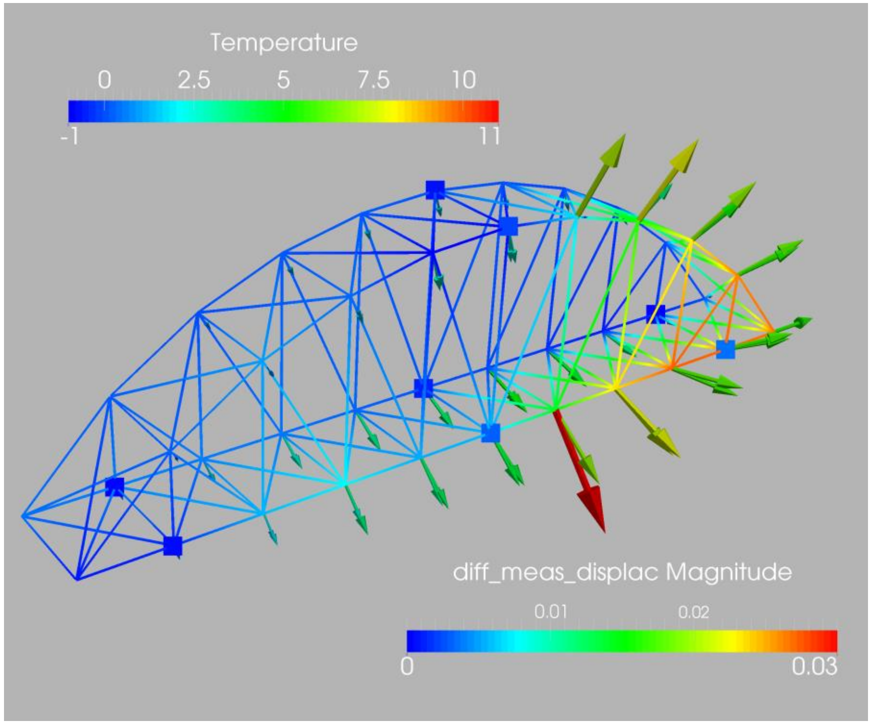
$$\frac{dL}{d\Delta T} = \tilde{\mathbf{u}}^t \cdot \frac{d\mathbf{f}_{\Delta T}}{d\Delta T} \quad .$$

Recovery of Temperature Fields (4)

- Forward Problem: Additional Thermal Stress Terms
- Adjoint Equation: Same as Before (!)
- Gradient: Different
  - Several Options [Element-, Point-Based, …]

Bridge: 8 Sensors

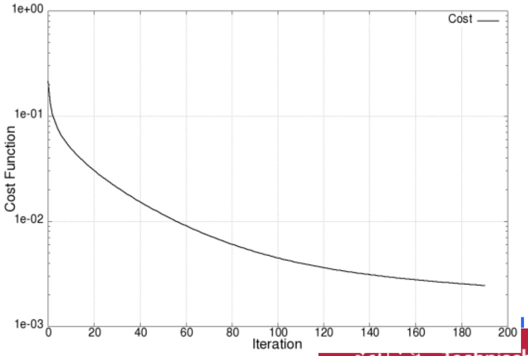




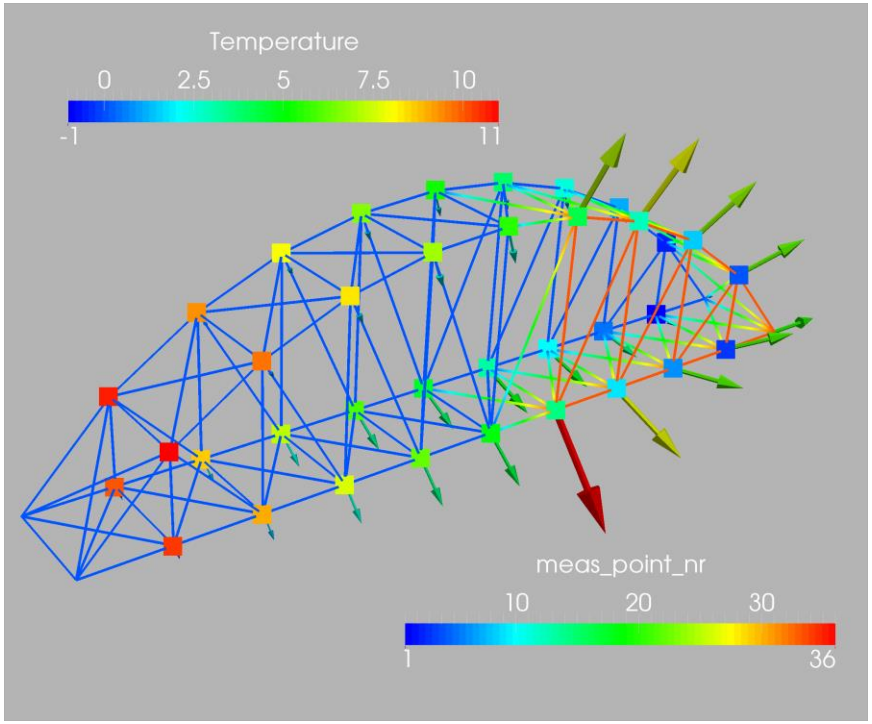
Material: Steel

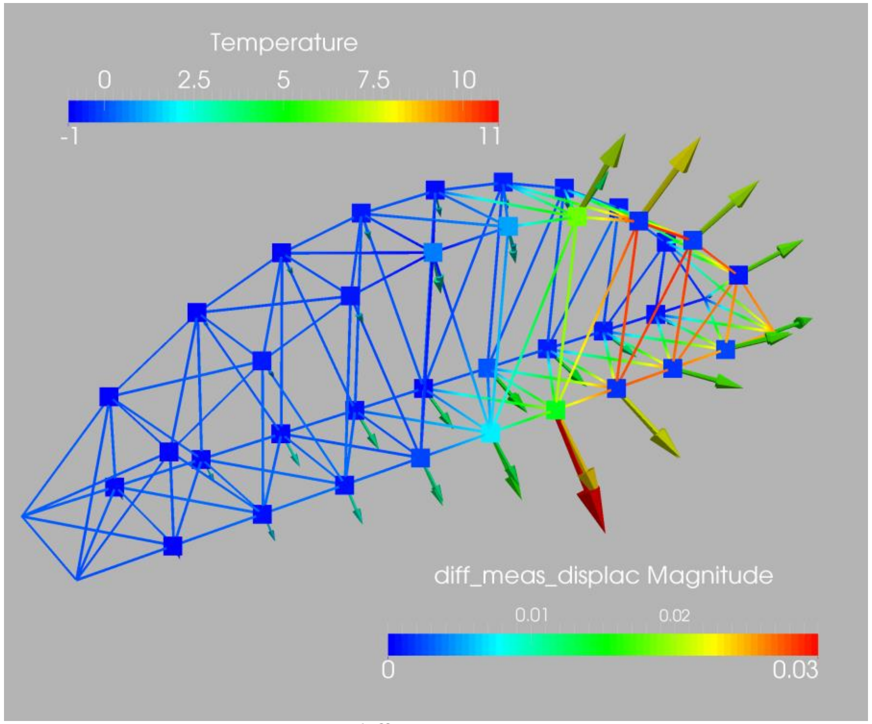
Trusses: A=1-100cm^2

FEELAST



Bridge: 36 Sensors

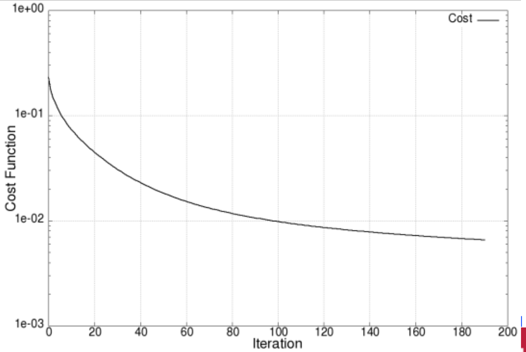




Material: Steel

Trusses: A=1-100cm^2

FEELAST



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Recovery of Material Parameters

Recovery of Material Parameters (1)

- Real Object (Bridge, Building, …) May Have Material Parameters Different from Design/Plan
- ➔ `Design/Analysis/Building Code’ FEM Model Incorrect
- ➔ Need to `Adjust’ Material Parameters To Assess Real Structure

- ➔ Try to Recover Material Parameters From Displacements/Strains

Recovery of Material Parameters (2)

- Same Notation as Before
- Assume: $\sigma = \sigma(\epsilon, \boldsymbol{\mu})$
- Optimization Problem: Find Material Parameters

$$I(\mathbf{u}_{1,..,n}, \boldsymbol{\mu}) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij}^{md} (\mathbf{u}_{ij}^{md} - \mathbf{I}_{ij}^{d} \mathbf{u}_i)^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij}^{ms} (\mathbf{s}_{ij}^{ms} - \mathbf{I}_{ij}^{s} \mathbf{s}_i)^2$$

- Subject To:

$$\mathbf{K}(\boldsymbol{\mu}) \cdot \mathbf{u}_i = \mathbf{f}_i \quad , \quad i = 1, n$$

Recovery of Material Parameters (3)

- Extended Lagrangian

$$L(\mathbf{u}_{1,..,n}, \boldsymbol{\mu}, \tilde{\mathbf{u}}_{1,..,n}) = I(\mathbf{u}_{1,..,n}, \boldsymbol{\mu}) + \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^t \cdot (\mathbf{K} \cdot \mathbf{u}_i - \mathbf{f}_i)$$

- Gradients/Derivatives

$$\frac{dL}{d\tilde{\mathbf{u}}_i} = \mathbf{K} \cdot \mathbf{u}_i - \mathbf{f}_i = 0$$

$$\frac{dL}{d\mathbf{u}} = \sum_{j=1}^{m} w_{ij}^{md} \mathbf{I}_{ij}^{d} (\mathbf{u}_{ij}^{md} - \mathbf{I}_{ij}^{d} \cdot \mathbf{u}_i) + \sum_{j=1}^{m} w_{ij}^{ms} \mathbf{J}_{ij}^{s} (\mathbf{s}_{ij}^{ms} - \mathbf{I}_{ij}^{s} \cdot \mathbf{s}_i) + \mathbf{K}^t \cdot \tilde{\mathbf{u}}_i = 0$$

$$\frac{dL}{d\mu_k} = \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^t \cdot \frac{d\mathbf{K}}{d\mu_k} \cdot \mathbf{u}_i$$

Recovery of Material Parameters (4)

- Forward and Adjoint Problem: Same as Before (!)
  - Same Matrices, Same RHSides
- Forward Problem: Nonlinear
  - Keep Last Stiffness Matrix for the Adjoint
- Derivative wrt Material Parameters: Done **Numerically**
  - Finite Differences, Done at Element Level
  - Ensures Generality

$$\frac{d\mathbf{K}}{d\mu_k} = \sum_{e=1}^{N_e} \frac{d\mathbf{K}_e}{d\mu_k}$$

Plate With Hole (MP1)

- Linear Elastic
- Material Parameters: E, ν
- 4, 6 and 14 Sensors

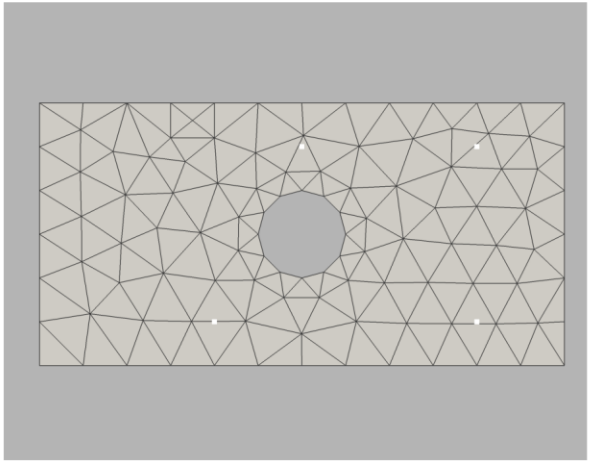
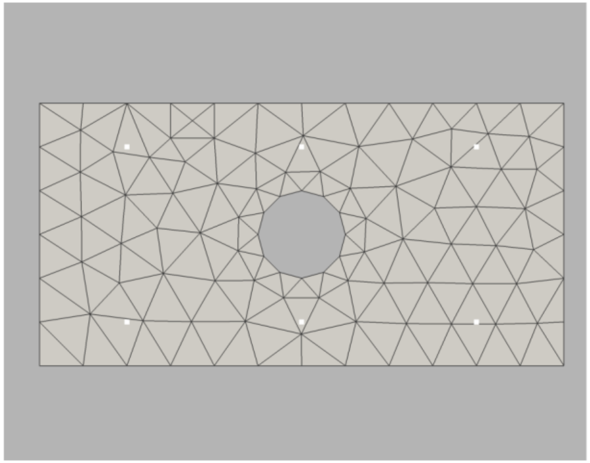
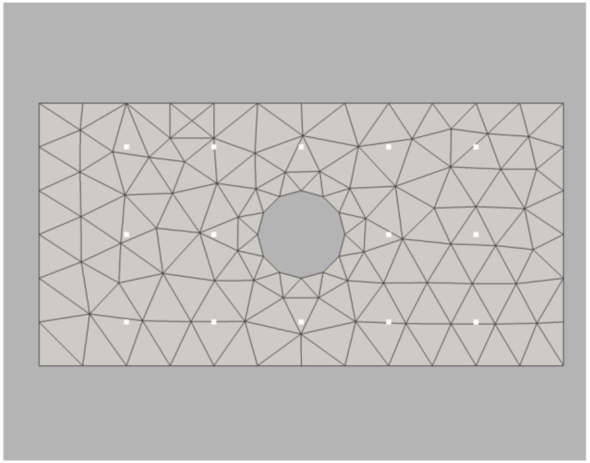




Plate With Hole (MP2)

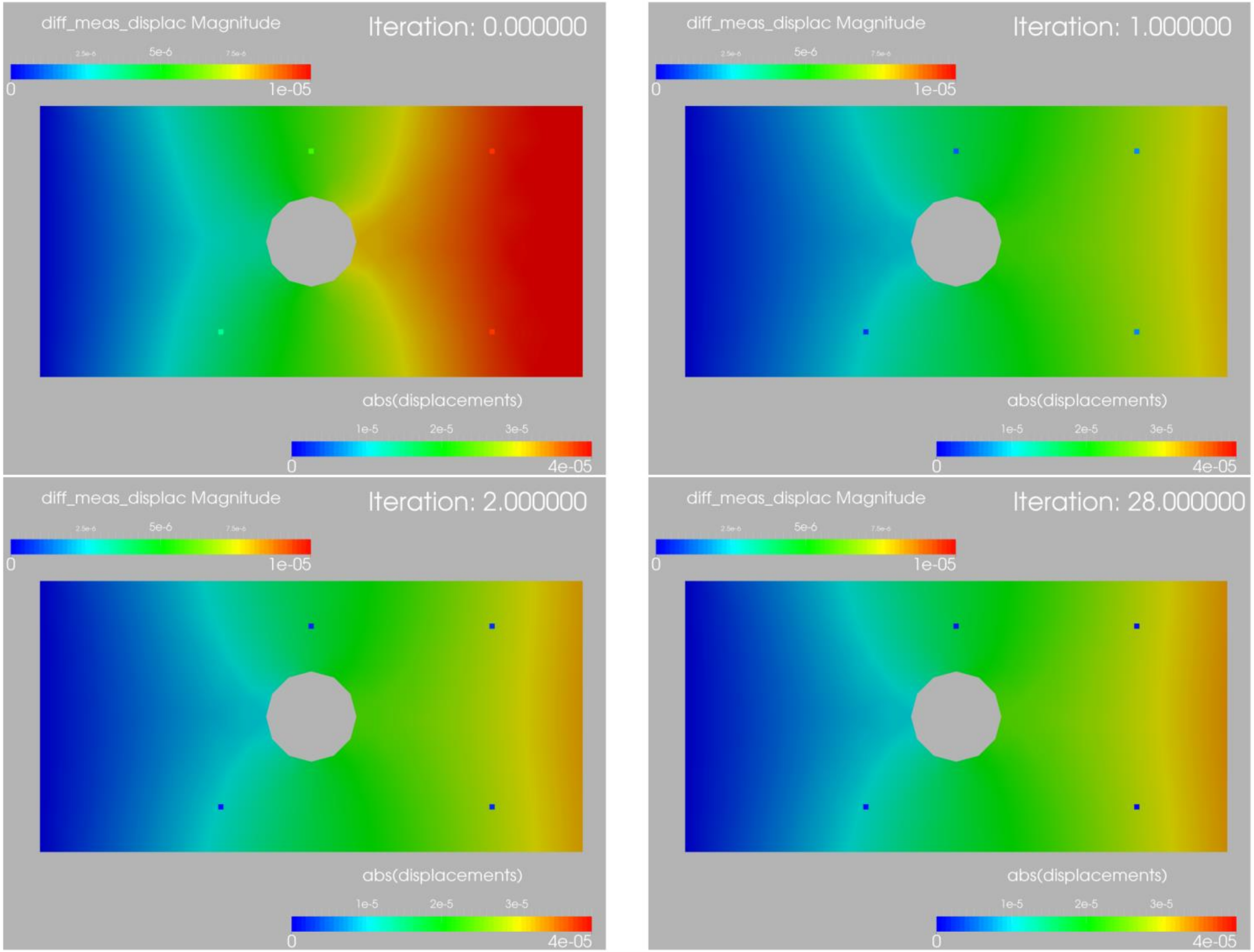
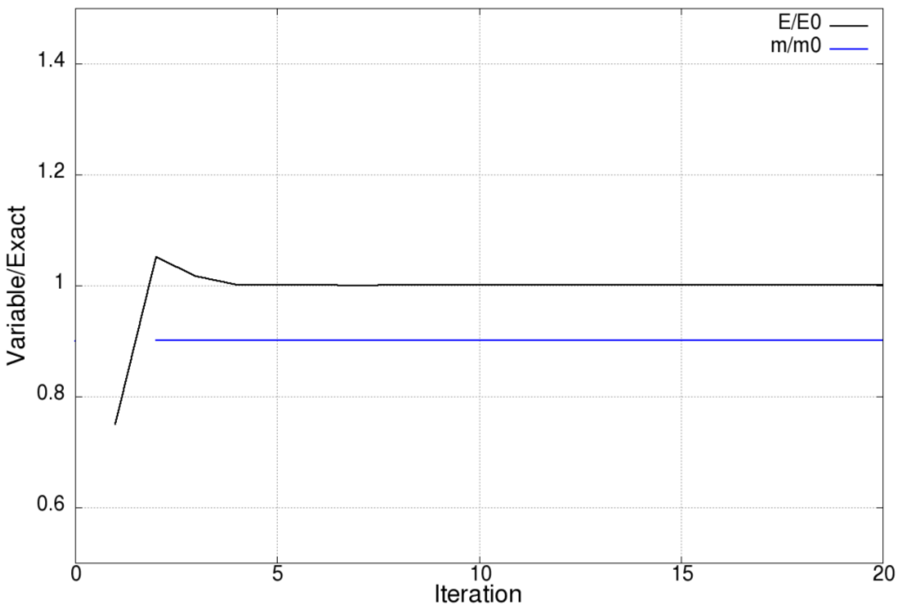


Plate With Hole (MP3a)

4 Sensors



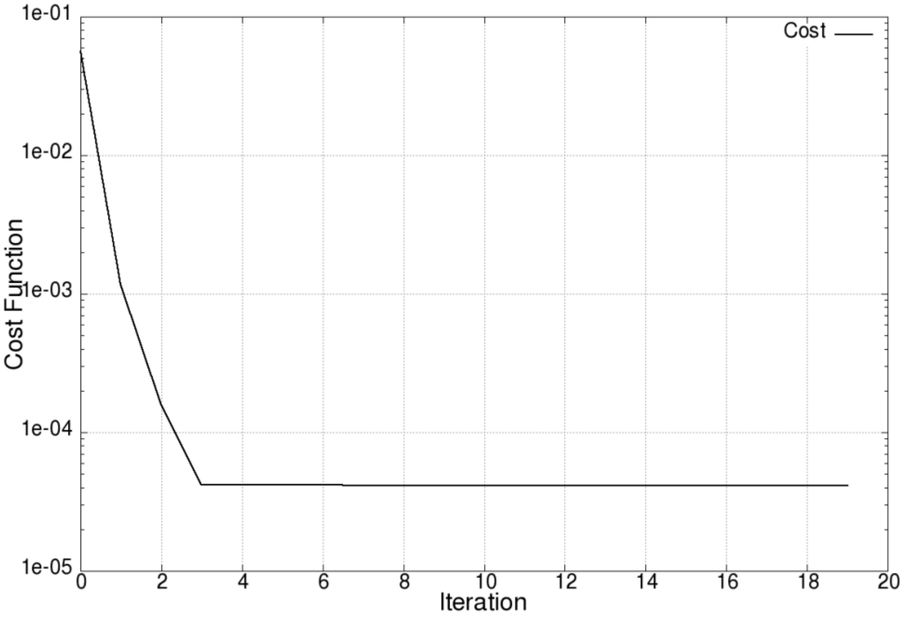
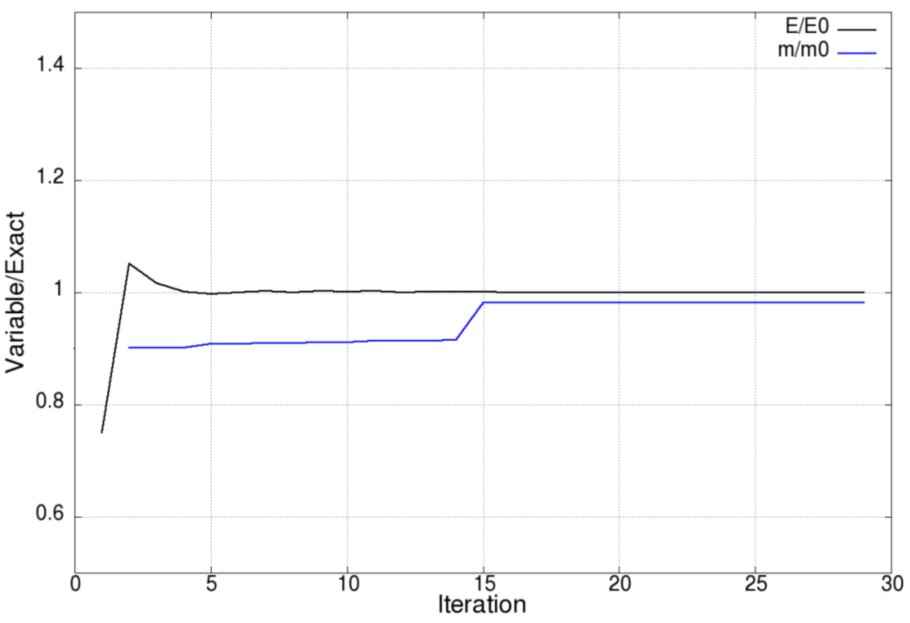


Plate With Hole (MP3b)

6 Sensors



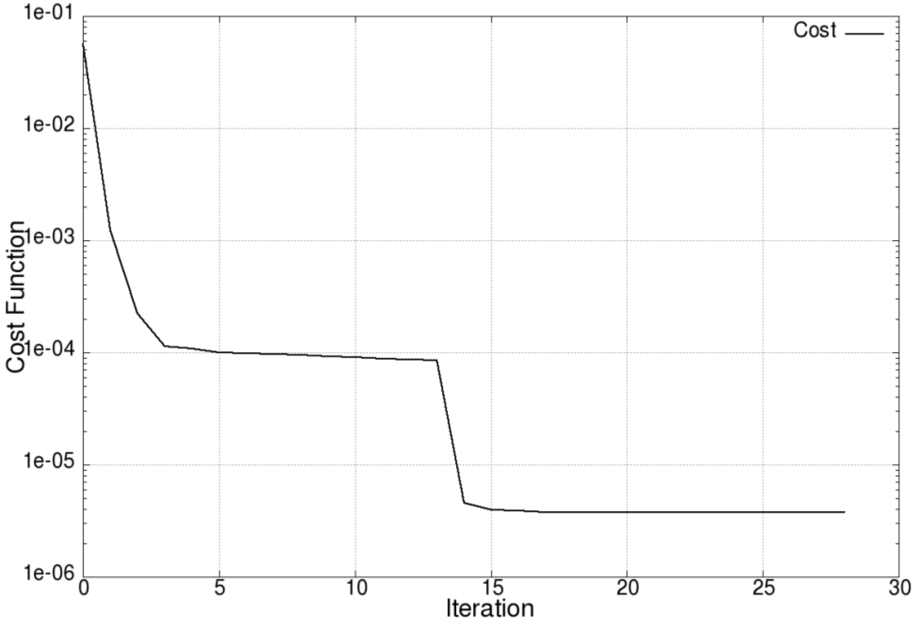
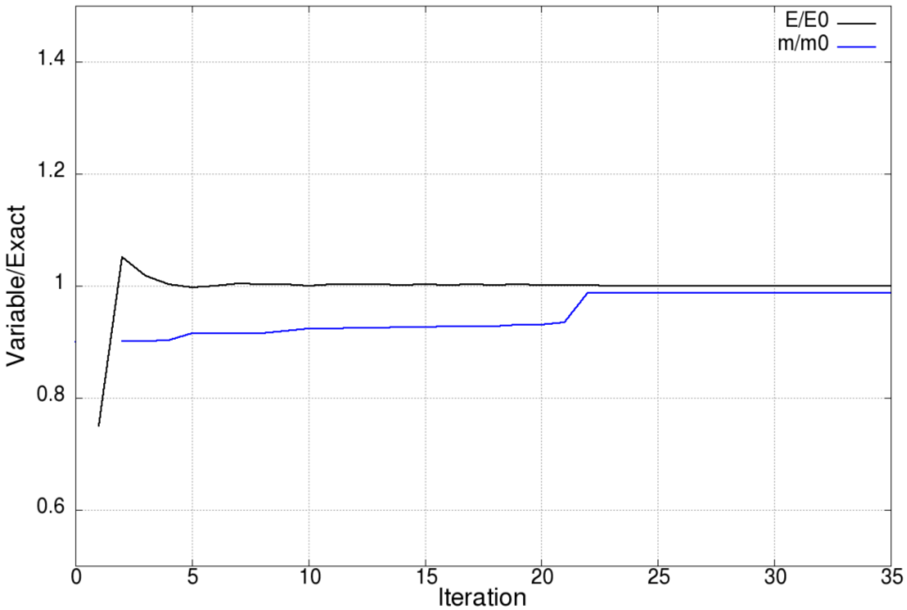
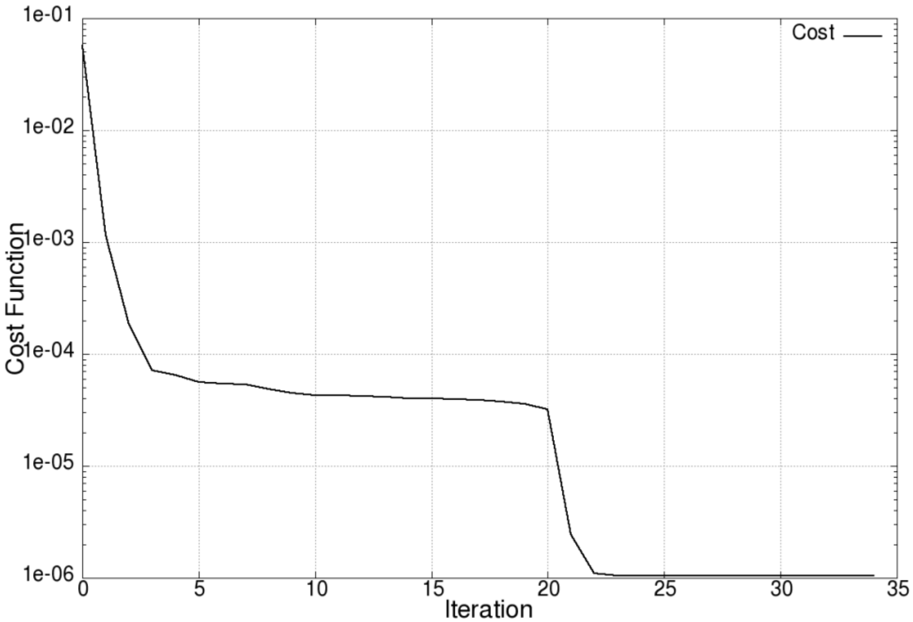


Plate With Hole (MP3c)

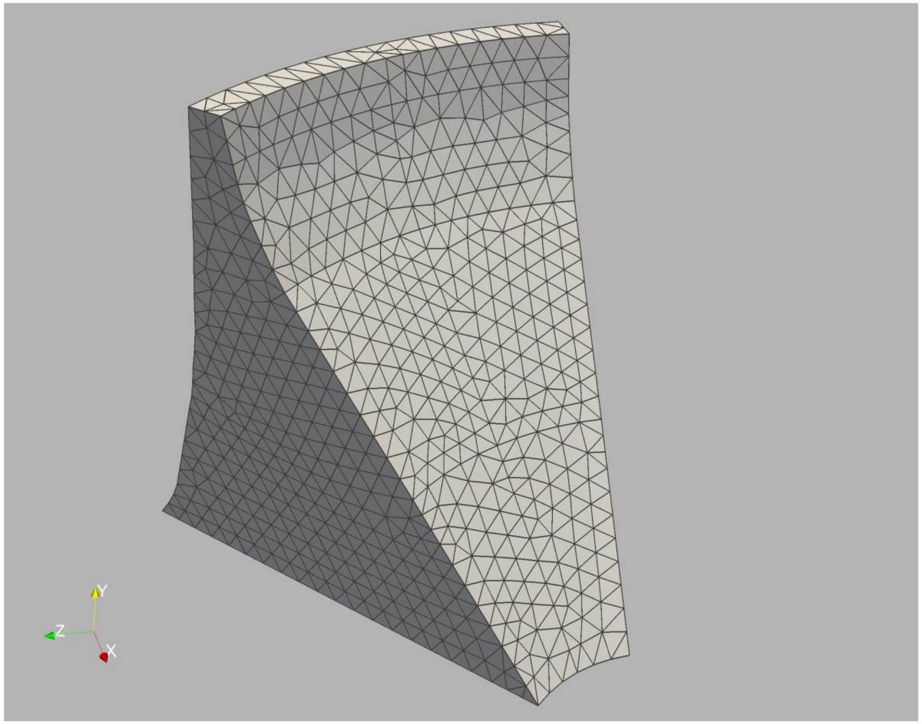
14 Sensors

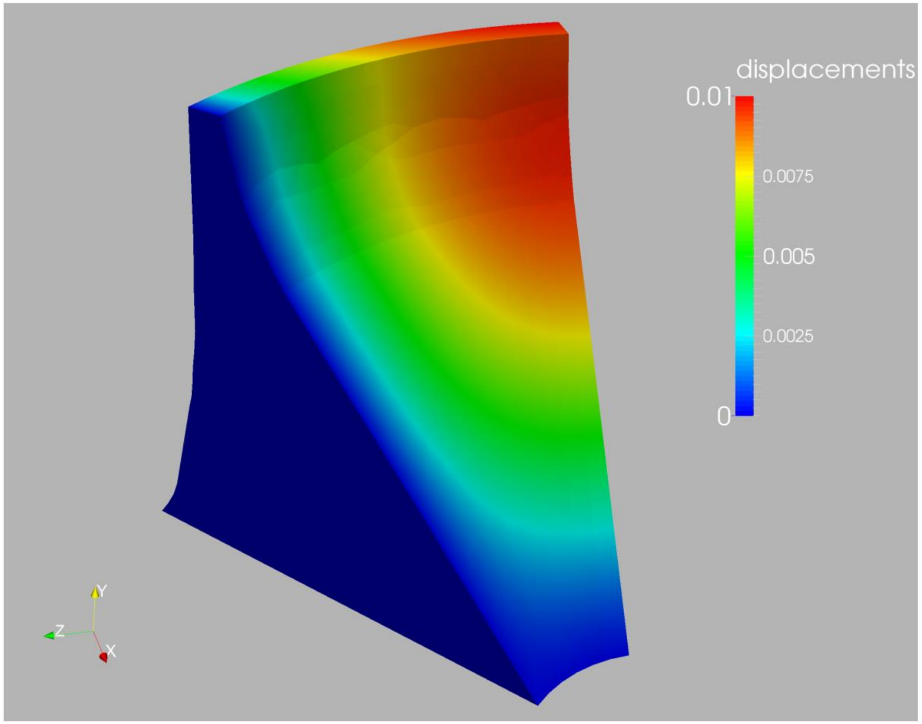




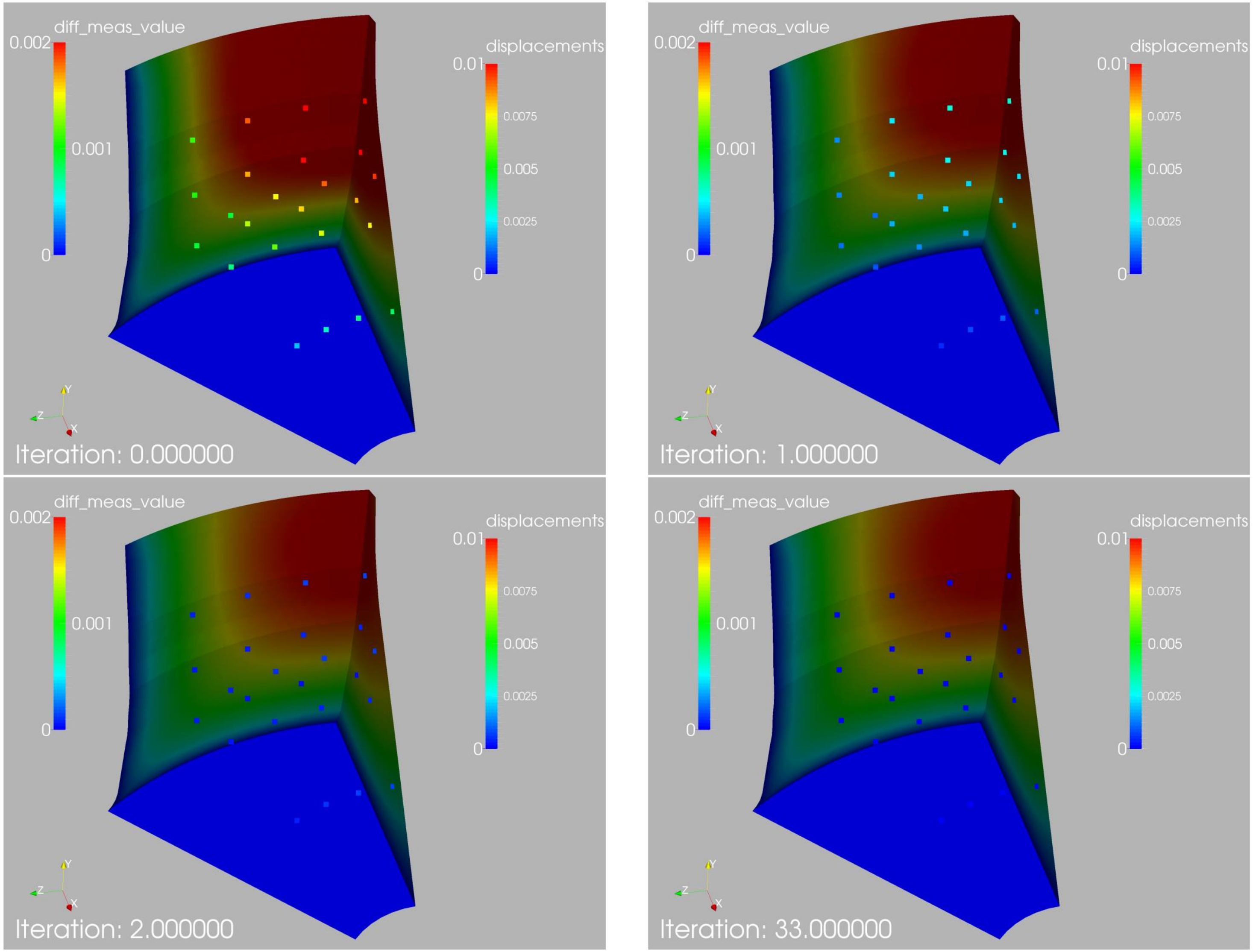
Hoover Dam (MP1)

- Linear Elastic
- Material Parameters: E, ν
- Mesh and Target Displacement



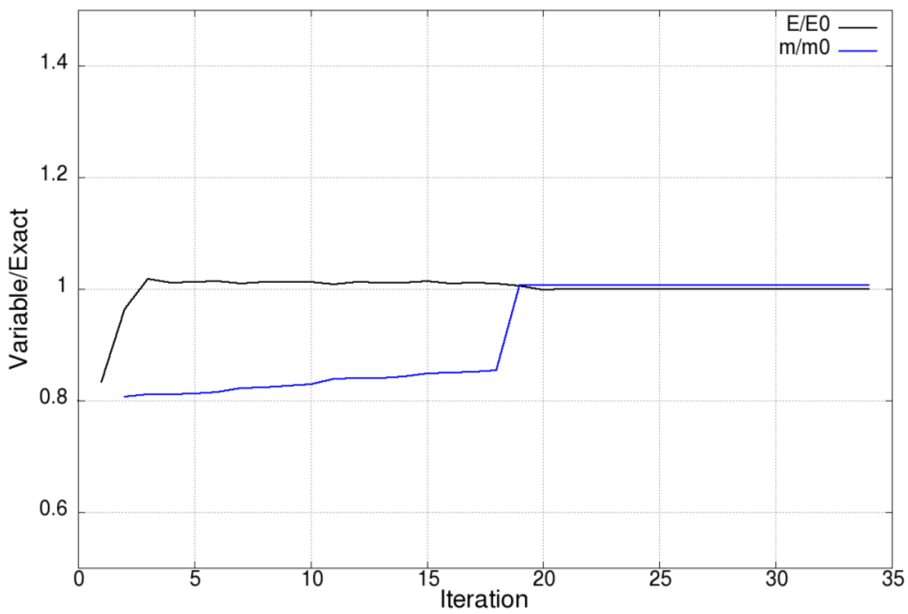


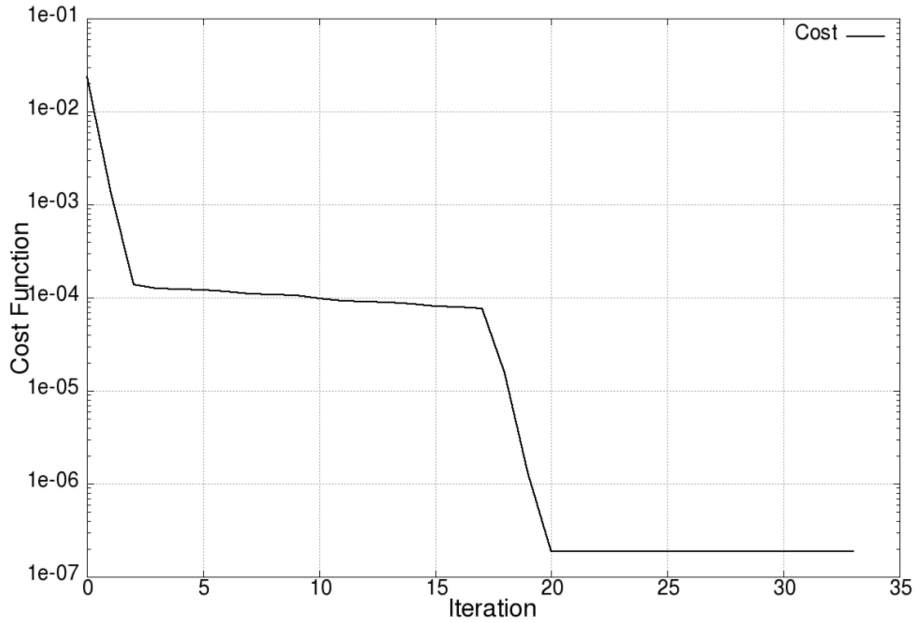
Hoover Dam (MP2)



Hoover Dam (MP3)

- Convergence History





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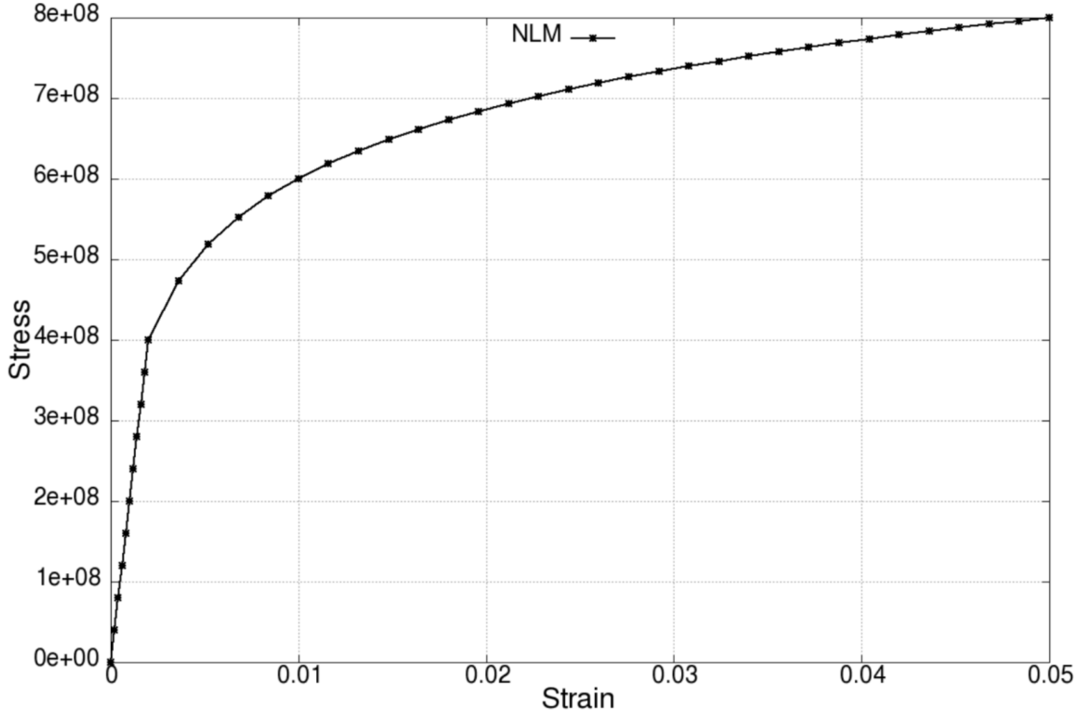
Truss (MP1)

- Nonlinear Material

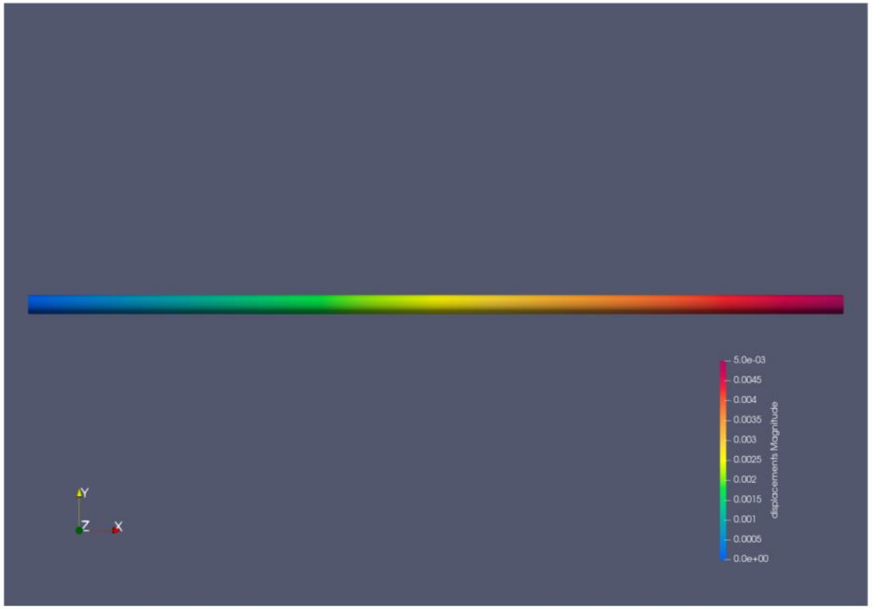
$$\epsilon \leq \epsilon_0 : E = E_0$$

$$\epsilon \geq \epsilon_0 : E_0\epsilon_0 + (E_1\epsilon_1 - E_0\epsilon_0)(\log(\epsilon/\epsilon_0))/(\log(\epsilon_1/\epsilon_0))$$



Truss (MP2)

- 4 Load Cases: $F_x = (10^5, 4\cdot 10^7, 5\cdot 10^7, 6\cdot 10^7)$
- Target:
- $E_0 = 2\cdot 10^{11}$, $E_1 = 1.6\cdot 10^{10}$
- $\varepsilon_0 = 2\cdot 10^{-3}$, $\varepsilon_1 = 5\cdot 10^{-2}$
- $\nu = 0.3$
- 4 Sensors Along the Truss



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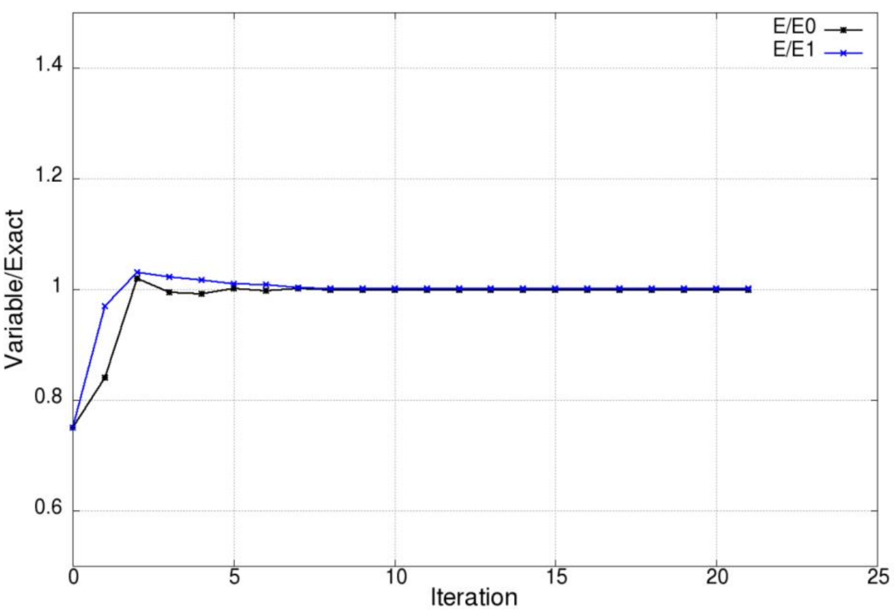
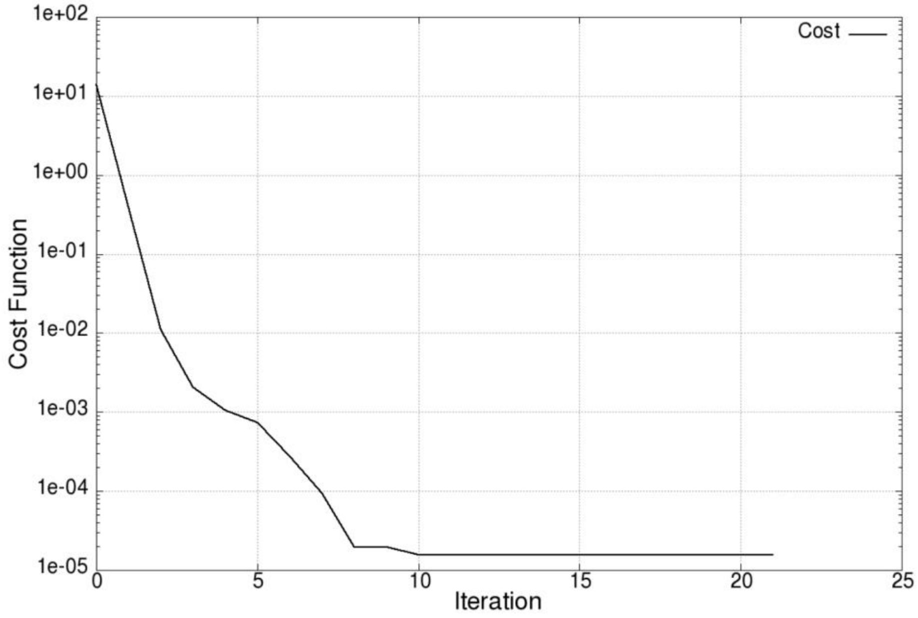
Truss (MP3)

- Convergence History





Extension to Transient Problems

Transient Problems (1)

- Many Cases Transient
- ➔ History ➔ Large Amount of Data (Some Redundant ?)
- ➔ Should be More Accurate
- Optimize:

$$I(\mathbf{u}_{1,..,n},\alpha)=\frac{1}{2}\int_0^T\sum_{i=1}^{n}\sum_{j=1}^{\mathrm{m}} w_{ij}^{\mathrm{md}}(\mathbf{u}_{ij}^{\mathrm{md}}-\mathbf{I}_{ij}^{\mathrm{d}}\mathbf{u}_i)^2dt$$

- Subject to: $\mathbf{M}\ddot{\mathbf{u}}_i+\mathbf{C}\dot{\mathbf{u}}_i+\mathbf{K}\mathbf{u}_i=\mathbf{f}_i,\quad i=1,n$

$$\mathbf{u}_i(0)=\bar{\mathbf{u}}_i,\quad i=1,n$$

$$\dot{\mathbf{u}}_i(0)=\bar{\mathbf{v}}_i,\quad i=1,n$$

Transient Problems (2)

- Augmented Lagrangian

$$L(\mathbf{u}_{1,..,n},\alpha,\tilde{\mathbf{u}}_{1,..,n}) = I(\mathbf{u}_{1,..,n},\alpha)$$

$$+\int_0^T \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^T(\mathbf{M}\ddot{\mathbf{u}}_i + \mathbf{C}\dot{\mathbf{u}}_i + \mathbf{K}\mathbf{u}_i - \mathbf{f}_i)dt$$

- Same Derivatives as Before ➔

$$\mathbf{M}\ddot{\tilde{\mathbf{u}}}_i - \mathbf{C}\dot{\tilde{\mathbf{u}}}_i + \mathbf{K}\tilde{\mathbf{u}}_i = -\sum_{j=1}^{\mathrm{m}} w_{ij}^{\mathrm{md}}\mathbf{I}_{ij}^{\mathrm{d}}(\mathbf{u}_{ij}^{\mathrm{md}} - \mathbf{I}_{ij}^{\mathrm{d}}\mathbf{u}_i), \quad i = 1,n$$

$$\tilde{\mathbf{u}}_i(T) = \mathbf{0}, \quad i = 1,n$$

$$\dot{\tilde{\mathbf{u}}}_i(0) = \mathbf{0}, \quad i = 1,n$$

Transient Problems (3)

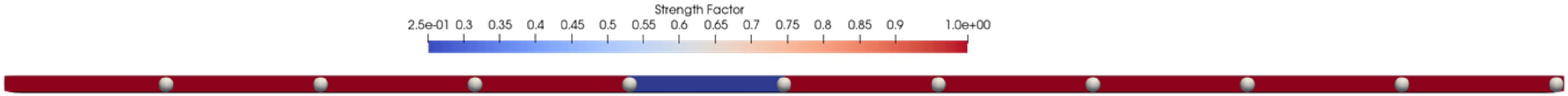
- Gradient:

$$\frac{dL}{d\alpha_e}\langle v\rangle = \int_0^T \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^T \frac{d\mathbf{K}}{d\alpha_e}\langle v\rangle \mathbf{u}_i dt = \int_0^T \sum_{i=1}^{n} \tilde{\mathbf{u}}_i^T \mathbf{K}_e \mathbf{u}_i v dt$$

- Implementation:
  - Solve Forward Problem: ($T=0 \rightarrow T=T_{end}$): Store **u**(t)
  - Solve Backward Problem: ($T=T_{end} \rightarrow T=0$)
  - Obtain Gradient
  - Smooth Gradient
  - Update Weakness Factor

Example: Cantilever Beam (1)

- Beam
- Fixed at x=0, Free/Loaded at x=L
- $\nu = 0.3$, $E = 2.0\times10^{11}$ Pa, $\rho = 7800$ kg/m^3
- Load: $f = 4\times10^5$ N, Applied for 0.25 sec; Then Released
- Start: α=1.0
- 10 Elements
- Test 1: Sensor Locations At Every Gridpoint
- Target: Element 5 Weakened



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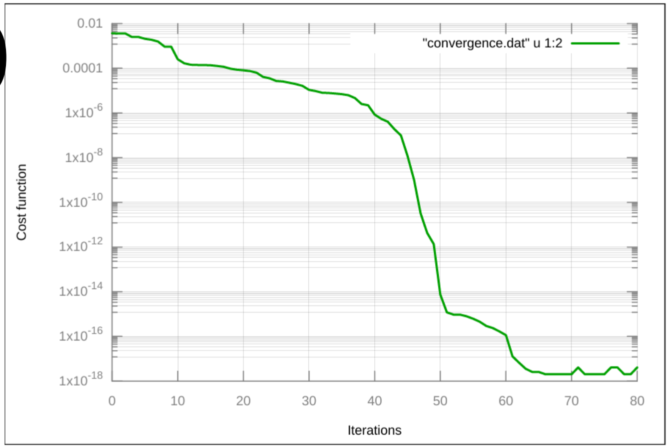
Example: Cantilever Beam (2)



Conclusions and Outlook

Conclusions and Outlook (1)

- Adjoint-Based System Identification for Structures
- Gradient `Lives in H^{-1}' ➔ Need Smoothing
- Explored Several Types of Smoothing
  - Simple Element/Point/Element Averaging
  - Laplacian Smoothing
  - Similar Results for All
  - Some Unsuitable for Quasi-Newton/Newton
- Overall: Seems to Work (!)
  - Trusses, Beams, Plates, Solids
  - FEELAST, CALCULIX, KRATOS
- Benchmark Suite for Regression Tests

Conclusions and Outlook (2)

- Developed Optimal Force Selection Algorithm(s)
- Developed Optimal Sensor Placement Algorithm(s)
- Many Possible Extensions/Variants [Siemens Fisher Senior Fellow]
  - Multiple Load Cases
  - Transient Load Cases
  - `Local’ Zones for Optimal Sensor Placement
  - Sensor Placement for Large Structures
    - Element Grouping/Zoning/… ?
  - Treatment of Singularities
  - Mixing of Sensors
  - Faulty Sensors
- Sensor Argument: `Best Sensor’
  - But Did Not Take Multiple Weakening Into Account

Conclusions and Outlook (3)

- DTs: Here to Stay
  - Compelling Advantages for Safety, Maintenance, Longevity

- DTs: Many Scientific Challenges/Opportunities I
  - CAD/CompMech Software:
    - DT-Ready Modeling (Across Multiple Disciplines)
    - Automatic Update [Grid Gen, Re-Runs, Checking, …]
  - Forward Problem:
    - Gaps in Knowledge, Uncertainty in Physical Parameters
    - Multi-Physics, Multi-Lenthscale, Multi-Timescale, …

Conclusions and Outlook (4)

- DTs: Many Scientific Challenges/Opportunities II
  - Inverse/Optimization:
    - Difficult/Impossible Adjoints
    - Non-Smooth, Non-Convex, Multi-Valued, High Dimensional
  - Large Data:
    - Storage, Retrieval, Comparison, Reduction, Abstraction, …

- Development of DT Workforce
  - Change of `Silo Mentality'

In Short:

- Much remains to be done: let us get on with it !
- Es gibt viel zu tun: packen wir’s an !
- Queda mucho por hacer: manos a la obra !
- It reste beaucoup a faire: allez-y !
- لا يزال هناك الكثير للقيام به: هيا
- …