Equilibrium, Energy and Environmental Systems

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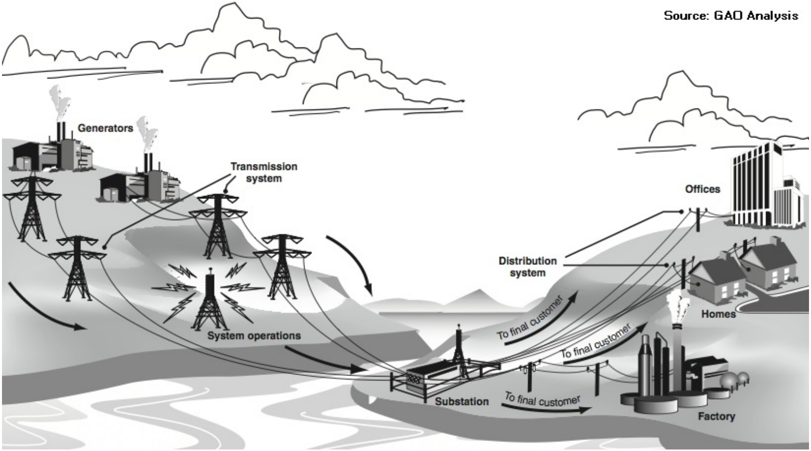
Engineering, Economics and Environment



- Determine generators' output to reliably/economically meet the load
- Power flows cannot exceed lines' transfer capacity
- Tradeoff: Impose environmental regulations/incentives

Perfect competition (MOPEC)

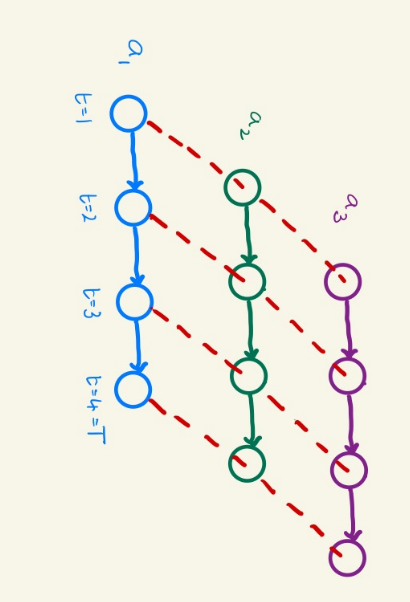
$$\max_{x_i} \pi^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

$$0 \leq \pi \perp \sum_i x_i - d(\pi) \geq 0$$

- When there are many agents, assume none can affect π by themselves
- Each agent is a price taker
- Two agents, $d(\pi) = 24 - \pi$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $\pi = 2$

Simple dynamics (discrete time, finite horizon)

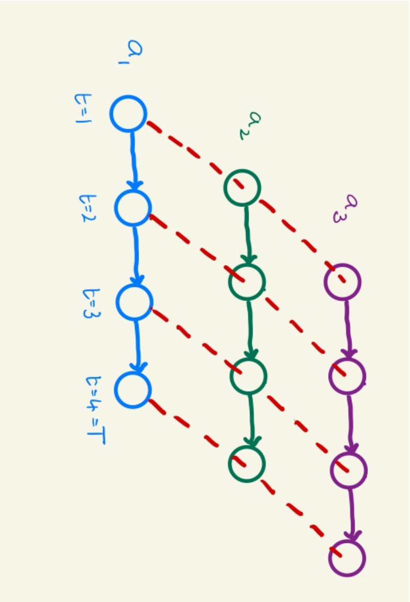


$\forall a \in \mathcal{A}$:

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; \cdot, \cdot) + f_{a2}(x_{a2}; \cdot, \cdot) + \cdots + f_{aT}(x_{aT}; \cdot, \cdot)$$

- Dynamics link over time

Simple dynamics (discrete time, finite horizon)



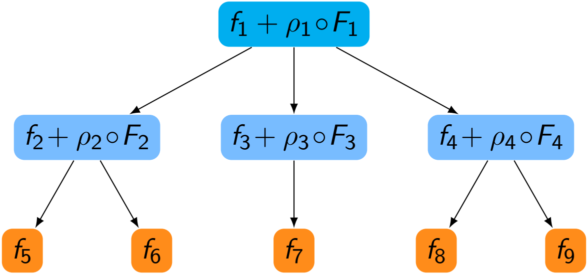
$\forall a \in \mathcal{A}$:

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; x_{-a1}, \pi_1) + f_{a2}(x_{a2}; x_{-a2}, \pi_2) + \cdots + f_{aT}(x_{aT}; x_{-aT}, \pi_T)$$

$$0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j),$$

- Dynamics link over time
- Complementarity links nodes across agents

Scenario tree with nodes $\mathcal{N} = \{1, 2, \ldots, 9\}$, and $T = 3$



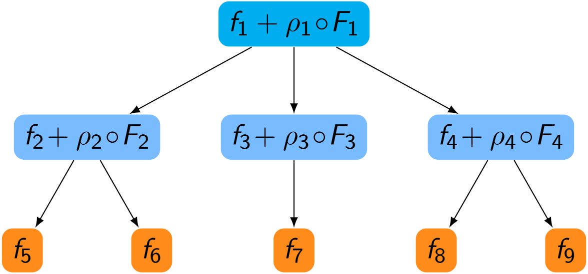
At leaf nodes:

$$\min_{x_{a\ell} \in \mathcal{X}_{a\ell}} \leftarrow f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \quad \forall a \in \mathcal{A},$$

$$0 \in H_\ell(\pi_\ell; x_{\cdot\ell}) + N_{P_\ell}(\pi_\ell)$$

";" separates variables from parameters in function definition

Stochastic equilibrium (MOPEC)



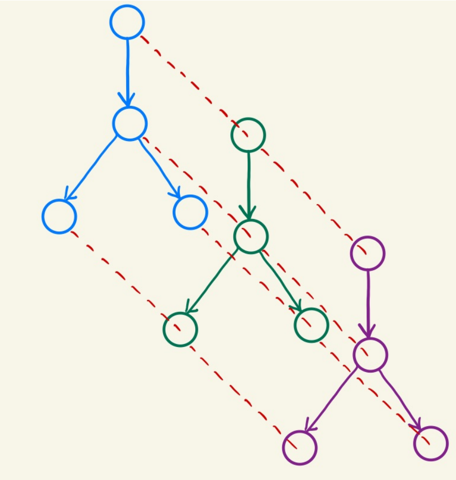
Agents solve problem at root node, linking at all nodes:

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; x_{-a1}, \pi_1)$$

$$+ \rho_{a1}([f_{aj}(x_{aj}; x_{-aj}, \pi_j) + \rho_{aj}([f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell)]_{\ell \in j_+})]_{j \in 1_+}) \quad \forall a \in \mathcal{A},$$

$$0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j), \quad \forall j \in \mathcal{T}.$$

Scenario trees linked across agents



- Dynamics link over time
- Complementarity links nodes of scenario tree across agents

Risk Measures

Problem type

Objective function or Constraint

$$\min_{x \in X} \theta(x) + \rho(F(x))$$

$$\min_{x \in X} \theta(x) \text{ s.t. } \rho(F(x)) \leq \alpha$$

- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{y \in \mathcal{D}} \mathbb{E}_y[Z]$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{y \in [0, p/(1-\alpha)] : \langle \mathbf{1}, y \rangle = 1\}$, then $\rho(Z) = \overline{CVaR}_\alpha(Z)$
- Combinations - increasing risk aversion as λ increases

$$\rho(Z) = (1-\lambda)\mathbb{E}[Z] + \lambda \overline{CVaR}_\alpha(Z)$$

The transformation to complementarity

$$\min_{x \in X} \theta(x) + \rho(F(x))$$

$$\text{where } \rho(u) = \sup_{y \in \mathcal{D}} \left\{ \langle y, u \rangle - \frac{1}{2} \langle y, My \rangle \right\}$$

optimality condition:

$$0 \in \partial\theta(x) + \nabla F(x)^T \partial\rho(F(x)) + N_X(x)$$

calculus:

$$0 \in \partial\theta(x) + \nabla F(x)^T y + N_X(x)$$

$$0 \in -y + \partial\rho(F(x)) \iff 0 \in -F(x) + My + N_{\mathcal{D}}(y)$$

- This is a complementarity problem: opt conds in x coupled with opt conds in y

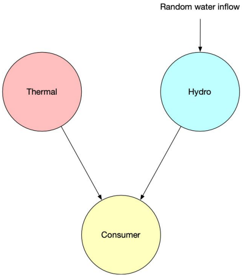
Stochastic Equilibrium as (extended) MOPEC

$$\min_{x_{a\cdot} \in \mathcal{X}_{a0}} f_{a1}(x_{a1}; x_{-a1}, \pi_1) + \sum_{j \in 1_+} y_{aj} \left(f_{aj}(x_{aj}; x_{-aj}, \pi_j) + \sum_{\ell \in j_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \right), \quad \forall a \in \mathcal{A} \tag{1}$$

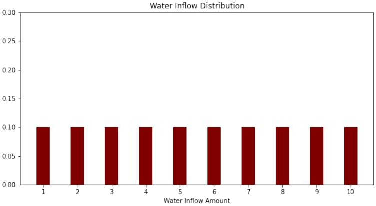
$$0 \in H_j(\pi_j; x_{\cdot j}) + N_{P_j}(\pi_j), \quad \forall j \in \mathcal{T} \tag{2}$$

$$\begin{aligned} r_{a1}(x,\pi) &= \max_{y_{a1_+} \in \mathcal{D}_{a1}} \sum_{j \in 1_+} y_{aj}(f_{aj}(x_{aj}; x_{-aj}, \pi_j) + r_{aj}(x,\pi)) \\ r_{a2}(x,\pi) &= \max_{y_{a2_+} \in \mathcal{D}_{a2}} \sum_{\ell \in 2_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \\ r_{a3}(x,\pi) &= \max_{y_{a3_+} \in \mathcal{D}_{a3}} \sum_{\ell \in 3_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \\ r_{a4}(x,\pi) &= \max_{y_{a4_+} \in \mathcal{D}_{a4}} \sum_{\ell \in 4_+} y_{a\ell} f_{a\ell}(x_{a\ell}; x_{-a\ell}, \pi_\ell) \end{aligned} \tag{3}$$

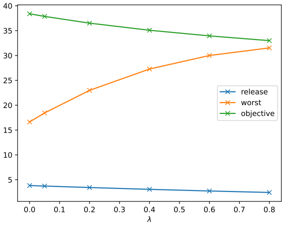
Simple example (3 agents, 2 stages, 10 scenarios)



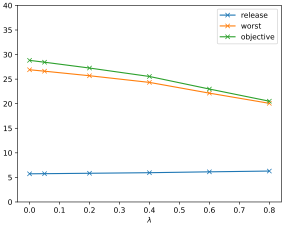
Second stage probabilities:



Low stage 1 inflow:



Higher stage 1 inflow:



Algorithms and problems

- PATH: nonsmooth Newton method (defaults) (blue+red+black)
- GS (Primal-dual): iteratively blue+red then black
- GS-PTH (Primal-dual + PATH)
- GS-CC-PTH (Primal-dual + convex-comb(black) + PATH)
- Homotopy(λ) + Primal-dual + convex-comb(black) + PATH

- Multistage economic dispatch, capacity expansion, hydroelectric system
- 3 types of demand formulation (I,II and III)
- Two scenario trees (4 stages, 40 nodes) and (4 stages, 156 nodes)
- 32 data instances for each formulation
- Several modulus of convexity and risk aversion parameters

$$\rho(Z) = (1-\lambda)\mathbb{E}[Z] + \lambda\overline{CVaR}_\alpha(Z)$$

Hydroelectric example, large tree, type I

quad	λ	PATH(%)	GS(%)	GS-PTH(%)	GS-CC-PTH(%)
0	0.1	12.5	96.9	100.0	100.0
0	0.3	0.0	90.6	100.0	100.0
0	0.5	0.0	96.9	100.0	100.0
0	0.7	0.0	96.9	100.0	100.0
0	0.9	0.0	50.0	78.1	100.0
1e-2	0.1	62.5	100.0	100.0	100.0
1e-2	0.3	9.4	100.0	100.0	100.0
1e-2	0.5	0.0	100.0	100.0	100.0
1e-2	0.7	0.0	100.0	100.0	100.0
1e-2	0.9	0.0	100.0	100.0	100.0
1e-1	0.1	100.0	100.0	100.0	100.0
1e-1	0.3	31.2	96.9	100.0	100.0
1e-1	0.5	9.4	100.0	100.0	100.0
1e-1	0.7	0.0	100.0	100.0	100.0
1e-1	0.9	0.0	100.0	100.0	100.0

Hydroelectric example, large tree, type II

quad	λ	PATH(%)	GS(%)	GS-PTH(%)	GS-CC-PTH(%)
0	0.1	21.9	96.9	100.0	100.0
0	0.3	0.0	93.8	100.0	100.0
0	0.5	0.0	96.9	100.0	100.0
0	0.7	0.0	96.9	100.0	100.0
0	0.9	0.0	75.0	87.5	100.0
1e-2	0.1	65.6	100.0	100.0	100.0
1e-2	0.3	6.2	100.0	100.0	100.0
1e-2	0.5	6.2	100.0	100.0	100.0
1e-2	0.7	0.0	100.0	100.0	100.0
1e-2	0.9	0.0	100.0	100.0	100.0
1e-1	0.1	100.0	100.0	100.0	100.0
1e-1	0.3	65.6	96.9	100.0	100.0
1e-1	0.5	37.5	93.8	100.0	100.0
1e-1	0.7	9.4	93.8	100.0	100.0
1e-1	0.9	0.0	93.8	100.0	100.0

Dispatch example, large tree, type I

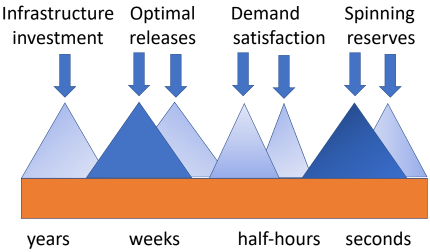
quad	λ	PATH	GS	GS-PTH	GS-CC-PTH	Homotopy
0	0.1	0.0	0.0	59.4	100.0	100.0
0	0.3	0.0	0.0	12.5	96.9	100.0
0	0.5	0.0	0.0	9.4	71.9	87.5
0	0.7	0.0	0.0	3.1	18.8	53.125
0	0.9	0.0	0.0	0.0	9.4	21.875
1e-2	0.1	28.1	15.6	100.0	100.0	100.0
1e-2	0.3	0.0	0.0	90.6	100.0	100.0
1e-2	0.5	0.0	0.0	40.6	100.0	100.0
1e-2	0.7	0.0	0.0	21.9	84.4	93.8
1e-2	0.9	0.0	0.0	6.2	53.1	68.75
1e-1	0.1	0.0	59.4	100.0	100.0	100.0
1e-1	0.3	0.0	43.8	100.0	100.0	100.0
1e-1	0.5	0.0	18.8	96.9	100.0	100.0
1e-1	0.7	0.0	12.5	100.0	100.0	100.0
1e-1	0.9	0.0	15.6	93.8	100.0	100.0

Dispatch example, large tree, type II

quad	λ	PATH	GS	GS-PTH	GS-CC-PTH	Homotopy
0	0.1	62.5	0.0	96.9	100.0	100.0
0	0.3	0.0	0.0	43.8	100.0	100.0
0	0.5	0.0	0.0	9.4	71.9	87.5
0	0.7	0.0	0.0	0.0	31.2	50.0
0	0.9	0.0	0.0	0.0	9.4	12.5
1e-2	0.1	96.9	15.6	100.0	100.0	100.0
1e-2	0.3	9.4	0.0	96.9	100.0	100.0
1e-2	0.5	0.0	0.0	71.9	100.0	100.0
1e-2	0.7	0.0	0.0	40.6	96.9	100.0
1e-2	0.9	0.0	0.0	9.4	65.6	81.25
1e-1	0.1	96.9	53.1	100.0	100.0	100.0
1e-1	0.3	40.6	46.9	100.0	100.0	100.0
1e-1	0.5	3.1	21.9	100.0	100.0	100.0
1e-1	0.7	0.0	18.8	100.0	100.0	100.0
1e-1	0.9	0.0	15.6	93.8	100.0	100.0

Uncertainty is experienced at different time scales

- Demand growth, technology change, capital costs are long-term uncertainties (years)
- Seasonal inflows to hydroelectric reservoirs are medium-term uncertainties (weeks)
- Levels of wind and solar generation are short-term uncertainties (half hours)
- Very short term effects from random variation in renewables and plant failures (seconds)



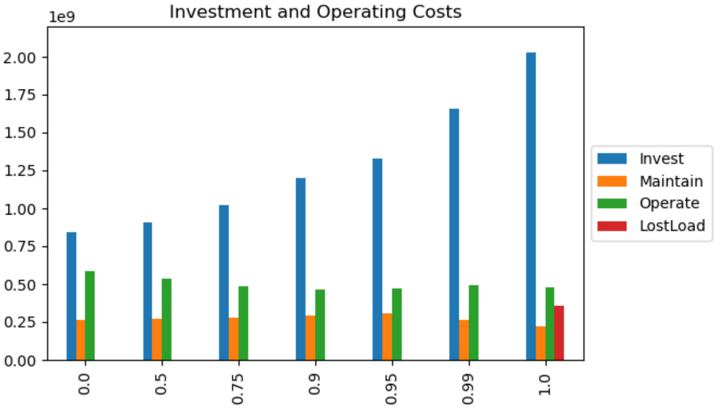
- Tradeoff: Uncertainty, cost and operability, regulations, security/robustness/resilience
- Needs modelling at finer time scales

Simplified two-stage stochastic optimization model

- Capacity decisions are z at cost $K(z)$
- Operating decisions are: generation y at cost $C(y)$, loadshedding q at cost Vq.
- Random demand is $d(\omega)$.
- Minimize capital cost plus expected operating cost:

$$
\begin{array}{llrcl}
\text{P:} & \min\limits_{z,y,q \in X} & K(z) & + & \mathbb{E}_\omega[C(y(\omega)) + Vq(\omega)] \\
& \text{s.t.} & y(\omega) & \leq & z, \\
& & y(\omega) & \geq & d(\omega) - q(\omega), \\
& & z_{\mathcal{N}} & \leq & (1-\theta) z_{\mathcal{N}}(2017)
\end{array}
$$

Costs as we impose tighter emission restrictions



- Markets based on marginal (operating) prices
- Tradeoff: Building more (renewable) capacity costs more, but makes operations cheaper - how to recover the fixed cost investment
- Operational costs dominated (at 100% renewable) by load shedding

More realistic model

Plant k has current capacity U_k, expansion x_k at capital cost K_k per MW, maintenance cost L_k per MW, and operating cost C_k. Minimize fixed and expected variable costs. Here $t = 0, 1, 2, 3$, is a season and $w(t)$ is reservoir storage at end of season t.

$$
\begin{aligned}
\text{P:} \quad \min \psi &= \textstyle\sum_k (K_k x_k + L_k z_k) + \sum_t \mathbb{E}_\omega [Z(t,\omega)] \\
\text{s.t.} \quad Z(t,\omega) &= \textstyle\sum_b T(b) \left(\sum_k C_k y_k(t,\omega,b) + Vq(t,\omega,b)\right), \\
x_k &\le u_k, \\
z_k &\le x_k + U_k, \\
y_k(t,\omega,b) &\le \mu_k(t,\omega,b) z_k, \\
\textstyle\sum_b T(b) y_k(t,\omega,b) &\le \nu_k(t,\omega) \textstyle\sum_b T(b) z_k \; + w(t-1) - w(t), \\
q(t,\omega,b) &\le d(t,\omega,b), \\
d(t,\omega,b) &\le \textstyle\sum_k y_k(t,\omega,b) + q(t,\omega,b), \\
w(t) &\le W, \\
y, q, w &\ge 0.
\end{aligned}
$$

Environmental constraints

Some capacity z_k, $k \in \mathcal{N}$, is "non renewable". Some generation $y_k(\omega)$, $k \in \mathcal{E}$ emits $\beta_k y_k(\omega)$ tonnes of CO2. For a choice of $\theta \in [0,1]$ constraint is either:

$$\mathbb{E}_\omega[\sum_{k\in\mathcal{E}} \beta_k y_k(\omega)] \leq (1-\theta)\mathbb{E}_\omega[\sum_{k\in\mathcal{E}} \beta_k y_k(\omega, 2017)],$$

(reduce CO2 emissions compared with 2017)

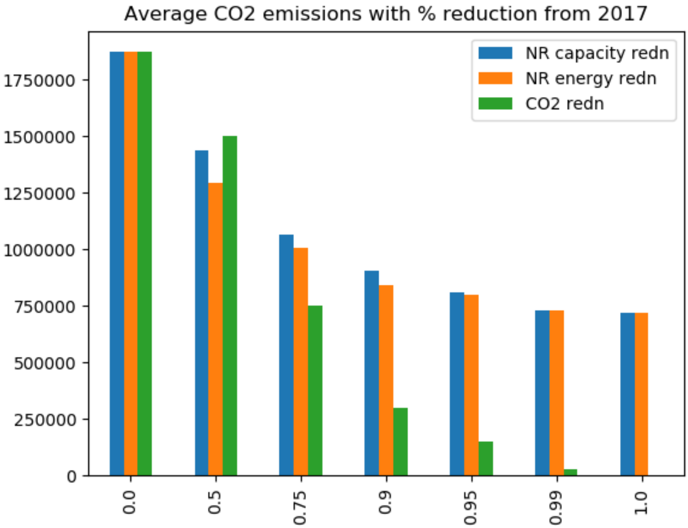
$$\sum_{k\in\mathcal{N}} z_k \leq (1-\theta) \sum_{k\in\mathcal{N}} z_k(2017),$$

(reduce non-renewable capacity compared with 2017)

$$\mathbb{E}_\omega[\sum_{k\in\mathcal{N}} y_k(\omega)] \leq (1-\theta)\mathbb{E}_\omega[\sum_{k\in\mathcal{N}} y_k(\omega, 2017)],$$

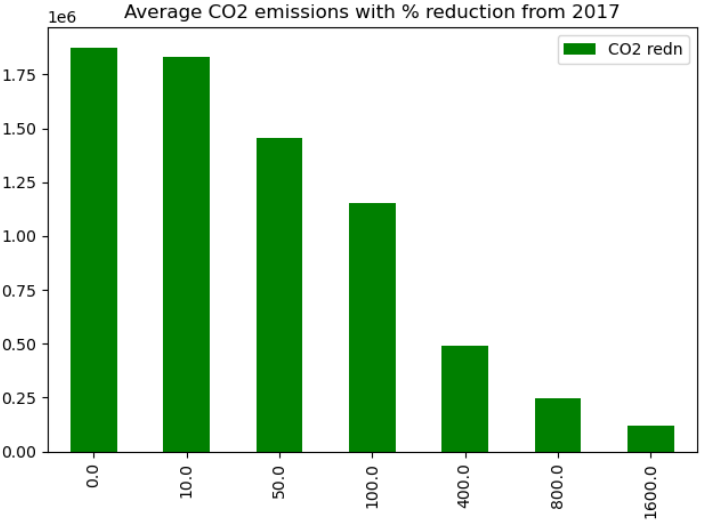
(reduce non-renewable generation compared with 2017)

Could impose constraints almost surely instead of in expectation or with risk measure (small impact) or use chance constraints

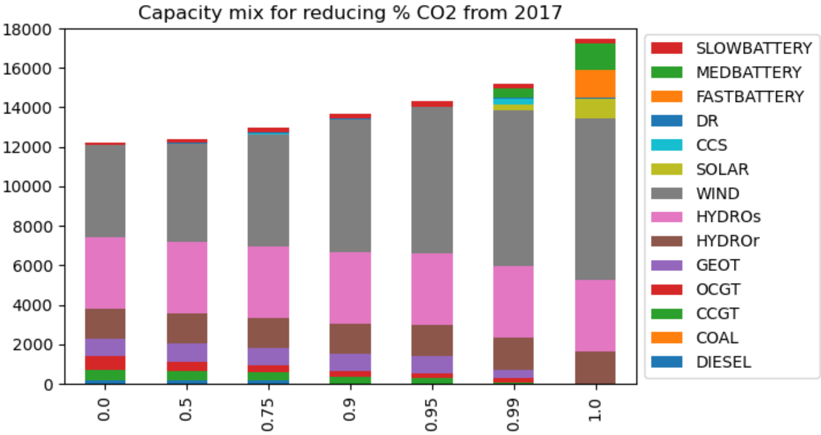


Since (renewable) geothermal and CCS emit some CO_2 100% renewable yields modest reductions in CO_2 emissions.

Average emissions for increasing carbon price ($ / tonne)



Technology choices as θ increases (% CO2 redn)



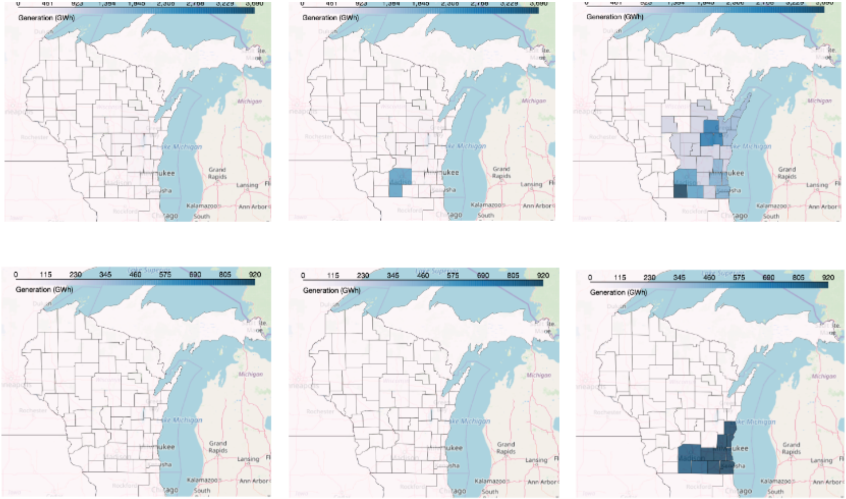
- Rich portfolio of renewable technologies used
- More capacity needed as more uncertain generation

Large pumped storage investment: Lake Onslow

Technology	Without			With		
	SI	HAY	NI	SI	HAY	NI
ONSLOW	0.0	0.0	0.0	1000.0	0.0	0.0
SLOWBATT	500.0	500.0	500.0	0.0	500.0	500.0
WIND	0.0	2049.9	5000.0	0.0	1407.4	5000.0

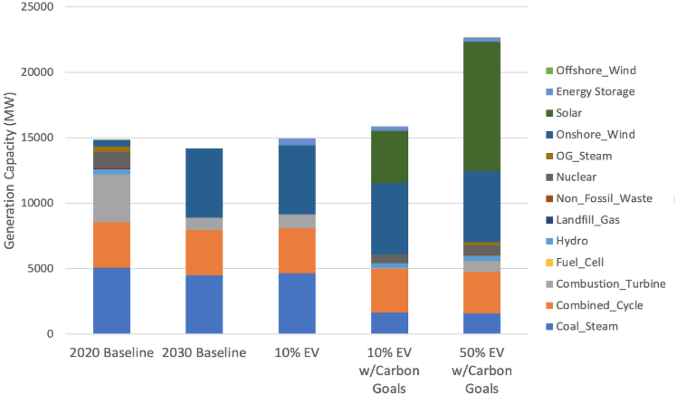
- Worried about the effects of dry winters and excess wind capacity
- Pumped storage costs amortized over long period
- Economical if emissions constraint is strict enough (i.e. no more than 5% of 2017 levels)
- Remove large battery in SI, reduce wind capacity at HAY

Wisconsin: wind and solar penetration

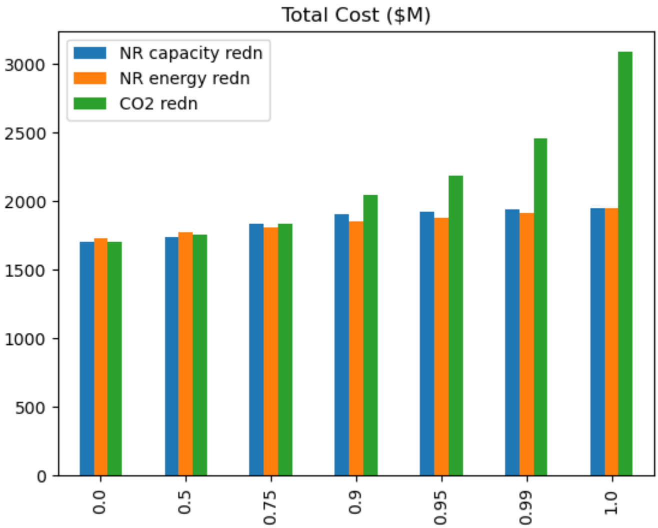


WEREWOLF model outputs: Renewable increases (wind and solar) for 0%, 40%, 80% carbon reduction policy scenarios in Wisconsin

Impact of Electric Vehicles on Generator Investments

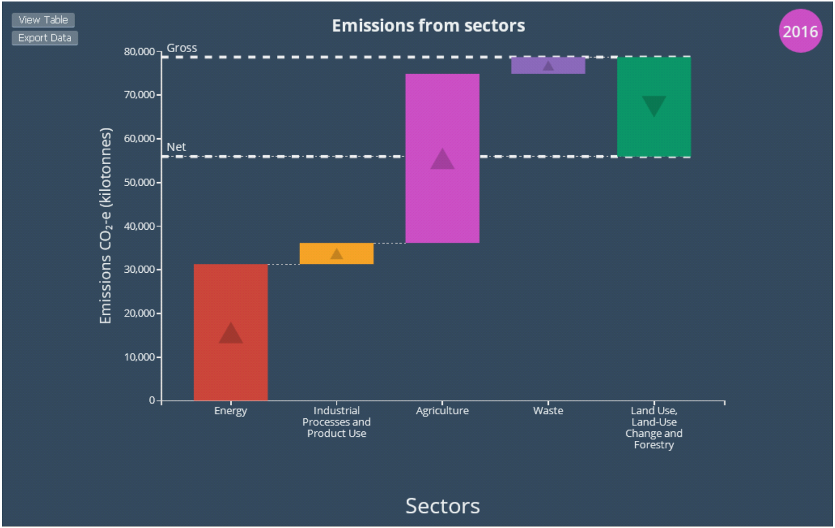


- Carbon Goals: 60% reduction on in-state carbon emissions
- Nuclear (low-carbon) used
- Coal steam generators shut down, supplanted by renewables
- Additional 180,000 MWh demand for EVs
- Storage investment needed
- Additional demand or carbon goals give more dramatic effects



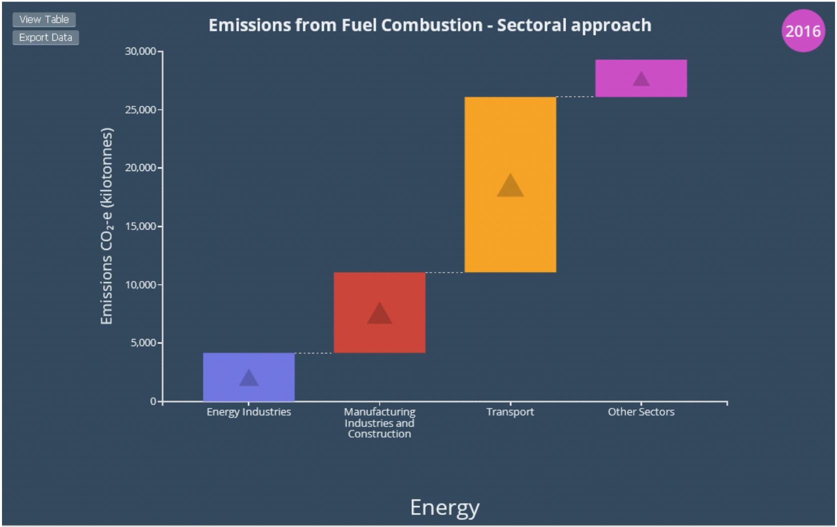
Cost of actually reaching zero CO_2 emissions (without geothermal or CCS) increases as we approach the limit.

New Zealand greenhouse gas emissions



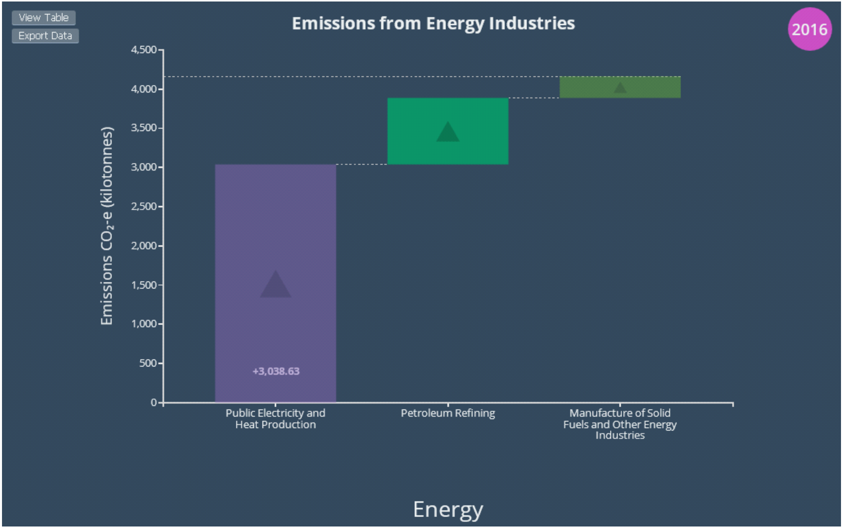
Total GHG emissions in 2016 were 80 M t CO2 equivalent.

New Zealand greenhouse gas emissions



Total CO2 emissions in 2016 were 30 M t.

New Zealand greenhouse gas emissions



Total CO2 emissions from electricity in 2016 were 3 M t.

General equilibrium (with contracts/incentives)

Consumption d_k, energy y_j, flows f, prices π, σ

$$\begin{array}{rl} \text{Consumers} & \max_{d_k \in \mathcal{C}} \text{utility}(d_k) - T_C(\sigma, d, f, y) - \pi^T d_k \\ \text{Generators} & \max_{(y_j) \in \mathcal{G}} \text{profit}(y_j, \pi) - T_G(\sigma, d, f, y) \\ \text{Transport} & \min_{f \in \mathcal{F}} \text{cost}(f, \pi, \sigma) \end{array}$$

Market clearing

$$0 \leq \pi \perp \sum_j y_j - \sum_k d_k - \mathcal{A}f \geq 0$$

$$0 \leq \sigma \perp E - \sum_j \mathcal{E}_j(y_j) \geq 0$$

Conclusions

- 100% renewable electricity system has several interpretations with different implications.
- Policy should choose the objective function not the action: e.g. reducing thermal capacity ceteris paribus can increase average emissions.
- Uncertainty in the model makes a difference.
- Electricity system has uncertainties at many time scales. Can include these in a single model with some approximations.
- 100% emission reduction in (NZ) electricity is needlessly expensive given proportion of electricity emissions.
- Next steps: A multistage model, and its competitive equilibrium counterpart.

A mathematical modelling approach to planning

- Build and solve a social plannning model that optimizes electricity capacity investment with constraints on CO2 emissions.
- Social planning solution should be stochastic: i.e. account for future uncertainty
- Social planning solution should be risk-averse: because the industry is.
- Approximate the outcomes of the social plan by a competitive equilibrium with risk-averse investors.
- Compensate for market failures from imperfect competition or incomplete markets.