East Coast Optimization Meeting 2023

Theme: Optimization and Game Theory

Dates

April 13-14, 2023

Location

Virtually hosted by Center for Mathematics and Artificial Intelligence
George Mason University
Fairfax, VA

Organizing Committee

Harbir Antil (George Mason University)
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Keynote Speakers

1. Michael C. Ferris (University of Wisconsin Madison)

TUTORIAL:

Title. From Complementarity to Risk-averse stochastic equilibria: models and algorithms

Abstract. We present a mechanism for describing and solving collections of optimization problems that are linked by equilibrium conditions. The general framework of MOPEC (multiple optimization problems with equilibrium constraints) captures many example applications that involve independent decisions coupled by shared resources. Included in this class are classical models such as complementarity, general equilibria, and agent based formulations arising from Nash Games. We show how to incorporate stochastic information into these systems and give examples of their use and possible extensions to hierarchical modeling. We outline several algorithms and investigate their computational efficiency. We demonstrate how MOPECs can be implemented in modeling systems.

PUBLIC LECTURE:

Title. Equilibrium, Energy and Environmental Systems

Abstract. Stochastic equilibria can be used to model many data driven applications, including many dynamic models of competition. In this model, players solve risk averse optimization problems that are coupled by a shared scenario tree. Players are linked by equilibrium conditions at nodes of that tree. Applications to energy planning and their interactions with environmental concerns will be outlined.

Energy markets are based on models that capture interactions between producers, transmission and consumers. The recent explosion in the use of renewable supply such as wind, solar and hydro has led to increased volatility in this system at a number of different time scales. We demonstrate how targets for 100

2. Uday V. Shanbhag (Pennsylvania State University)

TUTORIAL:

Title. Analysis and Solution of Equilibrium Problems under Uncertainty

Abstract. In this tutorial, we consider the class of continuous-strategy Nash and Stackellberg equilibrium problems under uncertainty. By leveraging tools from optimization, complementarity, and probability theory, we provide a range of existence and uniqueness results for Nash equilibrium problems under uncertainty. Next, we present a range of gradient response and best-response schemes for the computation of equilibria. We then consider the class of Stackelberg equilibrium problems and discuss a framework for computing such equilibria in uncertain regimes via an inexact smoothing-based framework. TIme permitting, the tutorial will conclude with a discussion of multi-leader multi-follower games together with a summary of some recently developed schemes.

PUBLIC LECTURE:

Title. Hierarchical Games under Uncertainty

Abstract. This will cover algorithms for a range of hierarchical games. The first part of the talk will focus on single-stage and two-stage Stackelberg equilibrium problems where the emphasis will be on presenting rate and complexity guarantees for an inexact variance-reduced smoothed scheme. In the second part of the talk, we will consider a subclass of multi-leader multi-follower games. In the context of such games, we will provide convergent algorithms under suitably defined potentiality or monotonicity requirements.

Guest Speakers

1. Afrooz Jalilzadeh (The University of Arizona)

   Title: Solving Stochastic Nash Equilibrium Problems using Stochastic (Quasi-)Variational Inequalities

   Abstract: Nash equilibrium (NE) is one of the most important concepts in game theory that captures a wide range of phenomena in engineering, economics, and finance. An NE is characterized by observing that in a stable game, no player can lower their cost by changing their action within their designated strategy. An equilibrium in the Nash game can be found by solving a variational inequality (VI) problem. Solving VI and stochastic VI (SVI) problems become more difficult if we consider that the players interact also at the level of the feasible sets. This situation arises naturally if the players share some common resources. This problem can be modeled as Generalized NE (GNE) which can be formulated as Quasi VI (QVI) or Stochastic QVI (SQVI). In this talk, we present efficient iterative schemes with convergence guarantees to solve SVI and SQVI problems. To validate our theoretical findings, we provide some preliminary simulation results on different problems such as stochastic Nash Cournot competition over a network.

2. Rongjie Lai (Rensselaer Polytechnic Institute)

   Title: Computational Methods for Mean-field Games: from conventional numerical methods to deep generative models

   Abstract: Mean field game (MFG) problems study how a large number of similar rational agents make strategic movements to minimize their costs. In this talk, I will discuss our recent work on computational methods for MFGs. I will start from a low-dimensional setting using conventional discretization/optimization methods and discuss some convergence results of the proposed method. After that, I will extend my discussion to high-dimensional problems by bridging the trajectory representation of MFG with a special type of deep generative model, normalizing flows. This connection does not only help solve high-dimensional MFGs, but also provides a way to improve the robustness of normalizing flows. If time permits, I will discuss their extensions to associated inverse problems for learning dynamics.

3. Mathias Staudigl (Maastricht University, Netherlands)

   Title: Recent Advances in Computational Game Theory

   Abstract: Game theory is a powerful methodological tool to mathematically formalize and study strategic optimization problems between self-interested agents. Historically, game theoretic models played a fundamental role in economics and operations research as a qualitative model for economic decision making. However, given its intimate connection with the theory of variational inequalities, a more quantitative line of research quickly emerged in order to numerically compute in challenging game problems in engineering. More recently, game theory plays a significant role in machine learning and AI in order to generate robust predictions (in the sense of theory of Min-Max) and deep learning architectures (GANs). From the point of view of distributed control and multi-agent systems, the game paradigm has also received increased attention with the motivation to solve complex games involving dynamics in abstract spaces (such as PDE-constrained optimization). These recent developments led to significant advancements in a field I would call computational game theory (CGT). In this talk, we review the available theory and computational methods from the point of view of variational analysis and operator splitting methods for (generalized) Nash equilibrium seeking and discuss recent research directions.

Contributed Talks

(1) Shima Mohebbi (George Mason University)
Cooperative Game Theory Models for Resilient Infrastructure Systems

(2) Suhan Zhong (Texas A&M University)
Distributionally Robust Optimization with Moment Ambiguity Sets

(3) Matthias Chung (Emory University)
The Art to Repeatedly Project your Problems

(4) Tina Mai (Duy Tan University at Da Nang, Vietnam (DTU) and Texas A&M University at College Station, USA (TAMU))
Prediction of numerical homogenization for Richards equation using deep learning

(5) Lucas Bouck (University of Maryland)
Numerical Approximation of a Membrane Model for Liquid Crystal Elastomers Counterexamples for (stochastic) gradient descent

(6) Ratna Khatri (U.S. Naval Research Laboratory)
Optimal Control based deep neural networks

(7) Konstantin Riedl (Technical University of Munich)
A Global Convergence Analysis of Consensus-Based Optimization

(8) Guillaume Wang (École polytechnique fédérale de Lausanne)
An Exponentially Converging Particle Method for the Mixed Nash Equilibrium of Continuous Games

(9) Oliver Hinder (University of Pittsburgh)
Making SGD parameter-free

(10) Agnieszka Wiszniewska-Matyszkiel (University of Warsaw)
Infinite horizon optimal control with unbounded payoffs with appealing counterexamples to overuse of heuristics

Abstracts

Cooperative Game Theory Models for Resilient Infrastructure Systems

Shima Mohebbi[1]

[1]George Mason University

Critical infrastructure systems are interdependent at different levels and are governed by several sectors working together to maintain social, economic, and environmental well-being. Many models focus on a centralized view for the optimal design and operation of these systems, but rarely is there only one decision maker for the infrastructure systems. In the decentralized decision-making paradigm, individual decision makers need to decide how to allocate resources and eventually improve the aggregated infrastructure systems resilience. Existing literature demonstrates the importance of understanding collective behavior for decision models and Operational Research (OR) interventions. There is still a dearth of studies examining how decision makers collaborate to shape OR-supported processes in infrastructure systems. This talk focuses on cooperative game theory, coupled with network optimization, enabling players/decision makers to explore coalitional structures and incentive schemes to coordinate their decisions towards system optimality. This game framework is first formulated and solved for a healthcare system, representing one infrastructure sector. It is then extended for interdependent water and transportation infrastructure systems. The proposed approach is applied to water distribution and road networks in the City of Tampa, Florida. The obtained solutions are compared to centralized solutions in different scenarios to demonstrate the feasibility of the approach for the city-scale infrastructure networks.

Distributionally Robust Optimization with Moment Ambiguity Sets

Suhan Zhong[1]

[1]Texas A&M University

The Art to Repeatedly Project your Problems

Matthias Chung [1]

[1]Emory University

Inference by means of mathematical modeling from a collection of observations remains a crucial tool for scientific discovery and is ubiquitous in application areas such as signal compression, imaging restoration, and supervised machine learning. With ever-increasing model complexities and growing data size, new specially designed methods are urgently needed to recover meaningful quantities of interest. We consider the broad spectrum of linear inverse problems where the aim is to reconstruct quantities with a sparse representation on some vector space; often solved using the (generalized) least absolute shrinkage and selection operator (lasso). The associated optimization problems have received significant attention, in particular in the early 2000s, because of their connection to compressed sensing and the reconstruction of solutions with favorable sparsity properties using augmented Lagrangians, alternating directions and splitting methods. We provide a new perspective on the underlying l1 regularized inverse problem by exploring the generalized lasso problem through variable projection methods. We arrive at our proposed variable projected augmented Lagrangian (vpal) method. We provide numerical examples demonstrating the computational efficiency for various imaging problems.

Prediction of numerical homogenization for Richards equation using deep learning

Tina Mai [1]

[1] Duy Tan University at Da Nang, Vietnam (DTU) and Texas A&M University at College Station, USA (TAMU)

In [Sergei Stepanov, Denis Spiridonov, and Tina Mai. Prediction of numerical homogenization using deep learning for the Richards equation. Journal of Computational and Applied Mathematics, 424:114980, 2023], we construct a new coarse-scale approximation algorithm based on numerical homogenization for the nonlinear Richards equation as an unsaturated flow through heterogeneous media. This method offers frequent and quick calculations of macroscopic parameters utilizing deep neural networks (DNNs). To be more specific, during training a neural network, we use a training set of stochastic permeability realizations and accordingly computed macroscopic targets (effective permeability tensor, homogenized stiffness matrix, and right-hand side vector). Our proposed deep learning method, which generates nonlinear maps between such permeability fields and macroscopic characteristics,

is innovative in that it incorporates the treatment for the nonlinearity of Richards equation in the predicted coarse-scale homogenized stiffness matrix. Several numerical tests on problems involving two-dimensional models demonstrate how well this method performs in terms of predicting the macroscopic properties and, consequently, solutions.

Numerical Approximation of a Membrane Model for Liquid Crystal Elastomers

Lucas Bouck [1]

[1] University of Maryland

Liquid crystal elastomers are materials where a nematic liquid crystal is coupled with a rubbery material. When actuated with heat, the interaction of the liquid crystal with the rubber creates complex shapes. Starting from the classical 3D energy of Bladon, Warner and Terentjev (1994), we derive a 2D membrane energy as the formal asymptotic limit of the 3D energy. We characterize the zero energy deformations and prove that the energy lacks certain convexity properties. We discretize with a finite element method and a bending energy regularization, and we prove that minimizers of the discrete energy converge to minimizers of the continuous energy. We employ a nonlinear gradient flow scheme for the discrete problem. Finally, we present computations of liquid crystal defects and nonisometric origami.

Optimal Control based deep neural networks

Ratna Khatri[1]

[1] U.S. Naval Research Laboratory

A Global Convergence Analysis of Consensus-Based Optimization

Konstantin Riedl [1]

[1] Technical University of Munich

Consensus-based optimization (CBO) is a multi-particle derivative-free optimization method capable of globally minimizing high-dimensional nonconvex and nonsmooth functions, a task which is ubiquitous in machine learning. In this talk we shed a light on the internal mechanisms of CBO, which are responsible for its success. Based on an experimentally supported intuition that, in the mean-field limit (as the number of particles goes to infinity), CBO always performs a gradient descent of the squared Euclidean distance to the global minimizer, we develop a novel technique for proving global convergence in mean-field law for a rich class of objective functions. In particular, we prove that CBO performs a convexification of a very large class of optimization problems as the number of optimizing particles tends to infinity. From this result it becomes apparent that the hardness of any global optimization problem is necessarily encoded in the mean-field approximation, or, more precisely, in the way how the empirical measure of the finite particle dynamics is used to approximate the mean-field limit. In consideration of the central significance of such approximation with regards to the overall computational complexity of the implemented numerical scheme, we establish a novel probabilistic quantitative result about the convergence of the interacting particle system towards the corresponding mean-field dynamics. By combining both results we provide the first holistic convergence proof of CBO methods on the plane. To demonstrate its practicability we provide experiments for well-understood machine learning problems.

An Exponentially Converging Particle Method for the Mixed Nash Equilibrium of Continuous Games

Guillaume Wang[1]

[1] École polytechnique fédérale de Lausanne

We consider the problem of computing mixed Nash equilibria of two-player zero-sum games with continuous sets of pure strategies and with first-order access to the payoff function. This problem arises for example in game-theory-inspired machine learning applications, such as distributionally-robust learning. In those applications, the strategy sets are high-dimensional and thus methods based on discretisation cannot tractably return high-accuracy solutions. In this paper, we introduce and analyze a particle-based method that enjoys guaranteed local convergence for this problem. This method consists in parametrizing the mixed strategies as atomic measures and applying proximal point updates to both the

atoms' weights and positions. It can be interpreted as a time-implicit discretization of the ""interacting"" Wasserstein-Fisher-Rao gradient flow. We prove that, under non-degeneracy assumptions, this method converges at an exponential rate to the exact mixed Nash equilibrium from any initialization satisfying a natural notion of closeness to optimality. We illustrate our results with numerical experiments and discuss applications to max-margin and distributionally-robust classification using two-layer neural networks, where our method has a natural interpretation as a simultaneous training of the network's weights and of the adversarial distribution.

Making SGD parameter-free

Oliver Hinder [1]

[1] University of Pittsburgh

We develop an algorithm for parameter-free stochastic convex optimization (SCO) whose rate of convergence is only a double-logarithmic factor larger than the optimal rate for the corresponding known-parameter setting. In contrast, the best previously known rates for parameter-free SCO are based on online parameter-free regret bounds, which contain unavoidable excess logarithmic terms compared to their known-parameter counterparts. Our algorithm is conceptually simple, has high probability guarantees, and is also partially adaptive to unknown gradient norms, smoothness, and strong convexity. At the heart of our results is a novel parameter-free certificate for SGD step size choice, and a time-uniform concentration result that assumes no a-priori bounds on SGD iterates.

Infinite horizon optimal control with unbounded payoffs with appealing counterexamples to overuse of heuristics

Agnieszka Wiszniewska-Matyszkiel[1]

[1] University of Warsaw
