Computational Tools for Image and Shape Analysis

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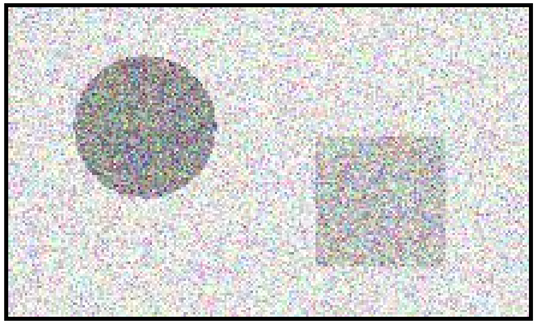
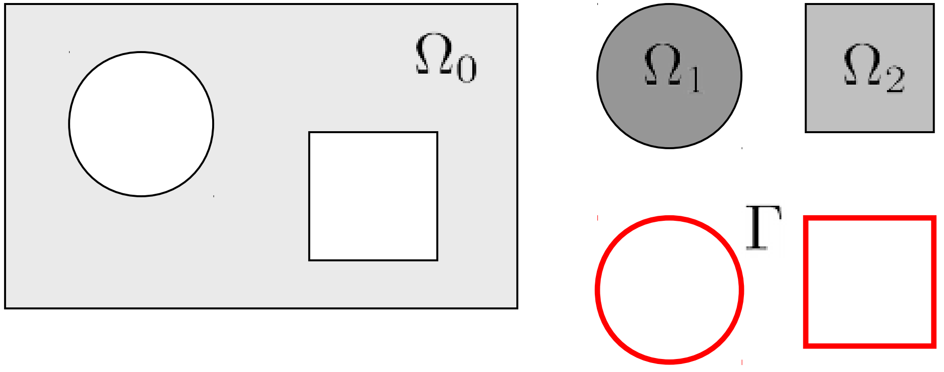
Image segmentation

Task: To identify regions & boundaries in given images.

input image

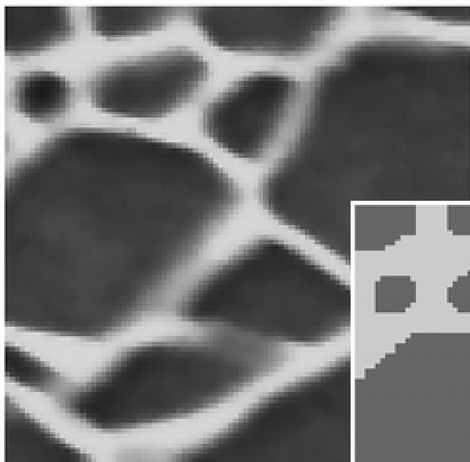


segmentation

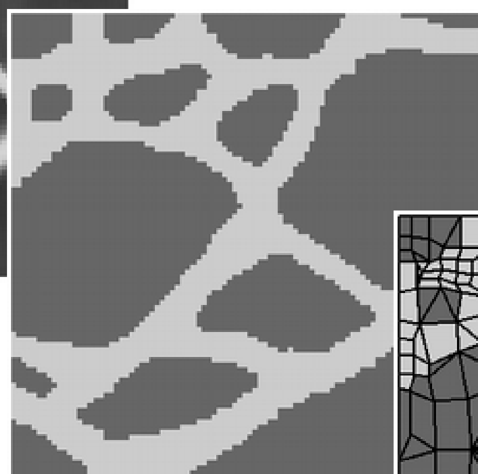


Segmentation of Microstructures

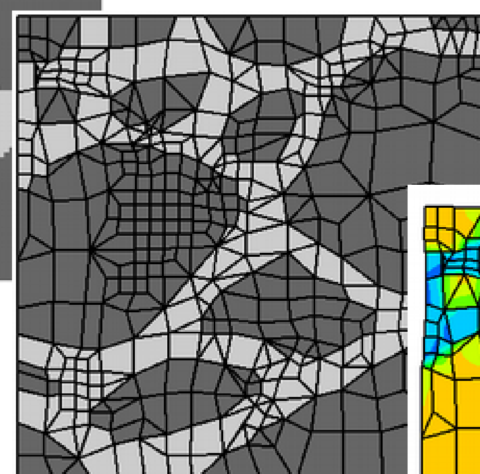
Quantitative measurements of structures, topology etc. and physics simulations



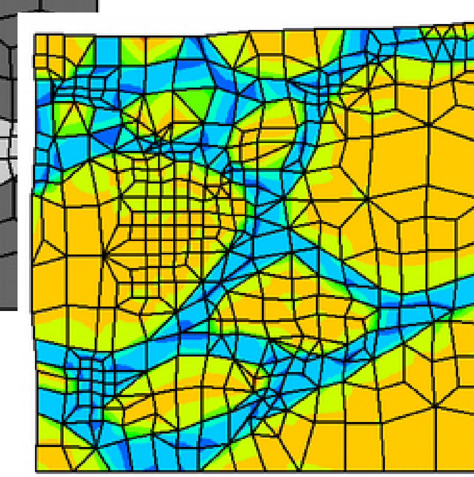
microstructure image



segmentation



meshing

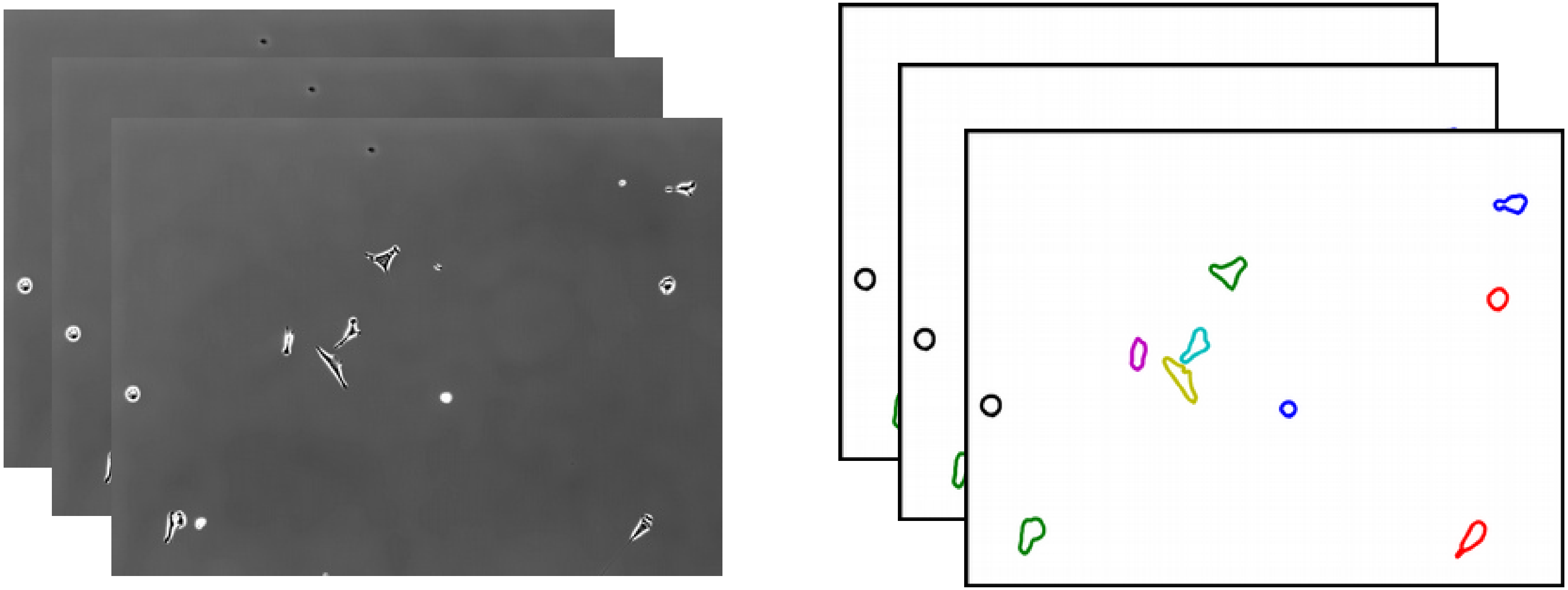


FE soln

Example of an OOF simulation

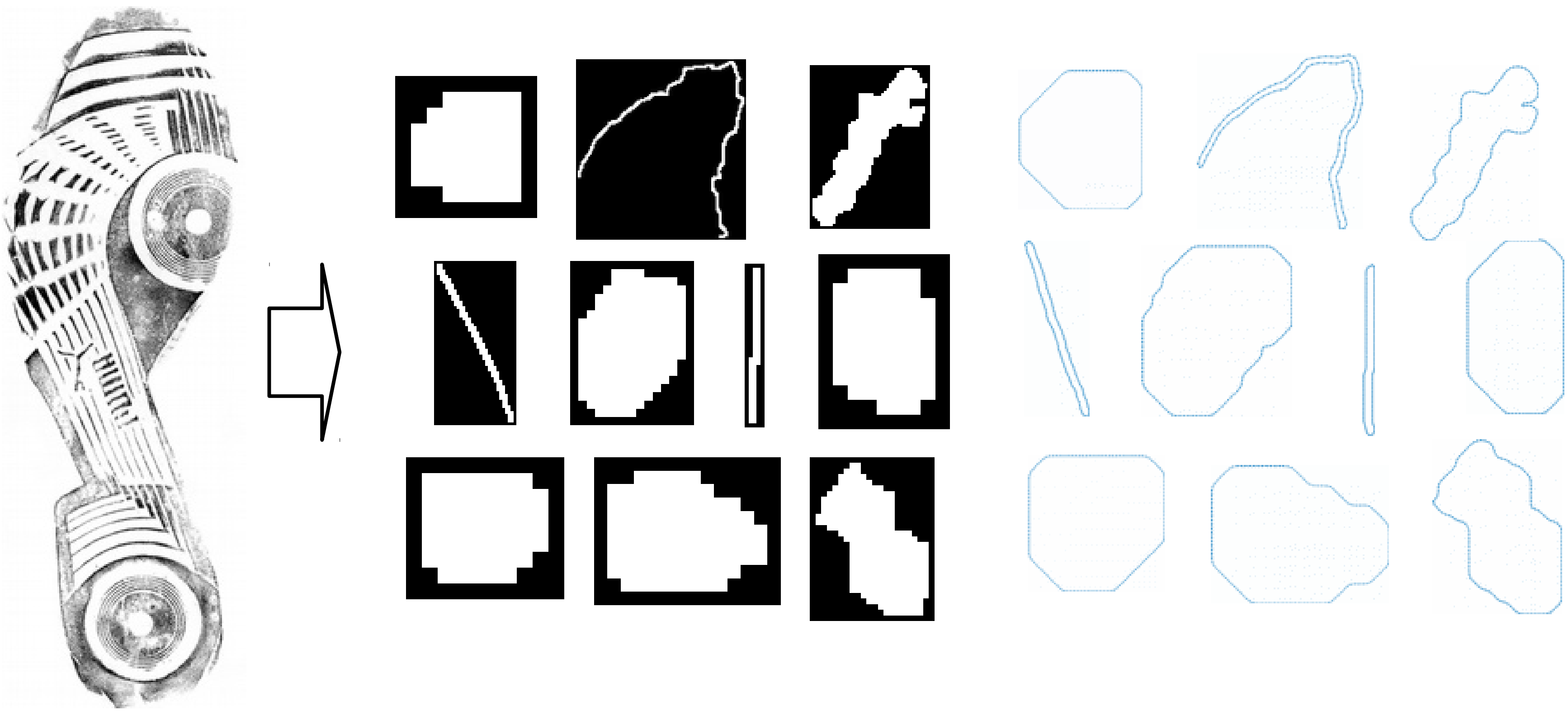
Segmentation of Cell Populations

Detection & evaluation of cells and colonies, e.g. stem cell research



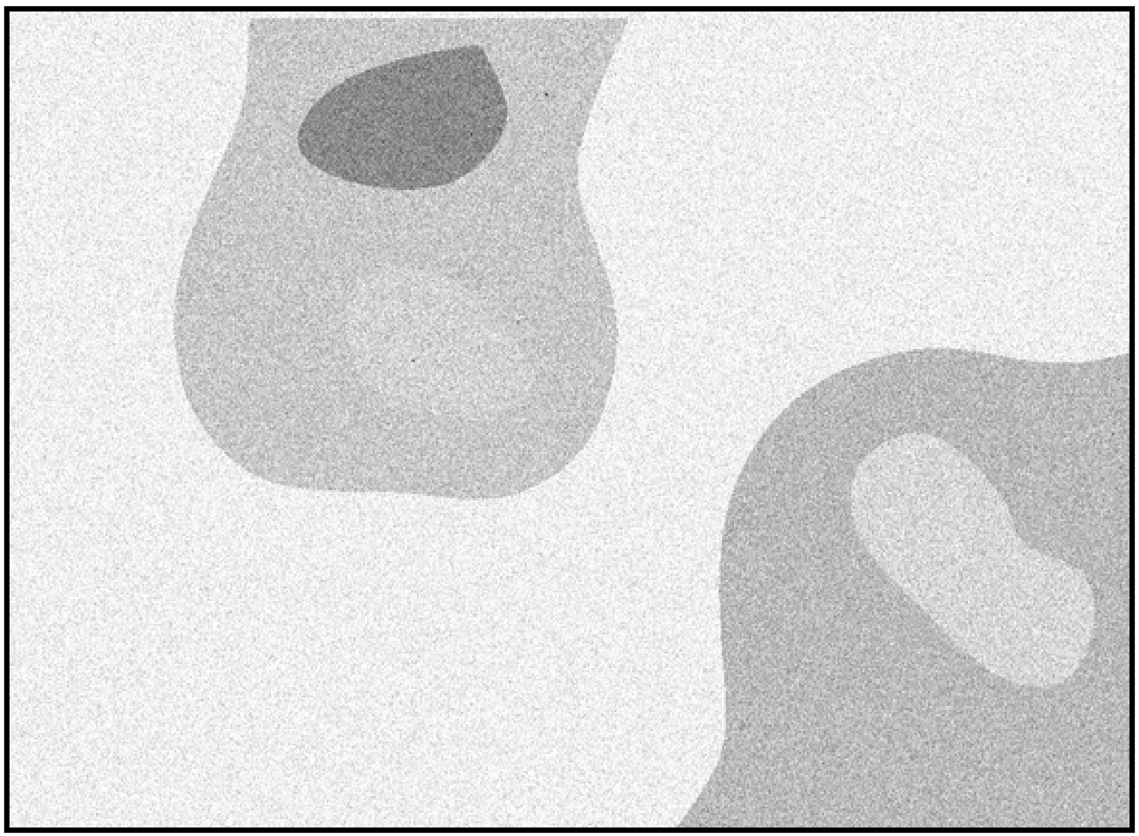
Segmentation for Shoeprint Forensics

Detect patterns and individualizing marks/features in shoeprints



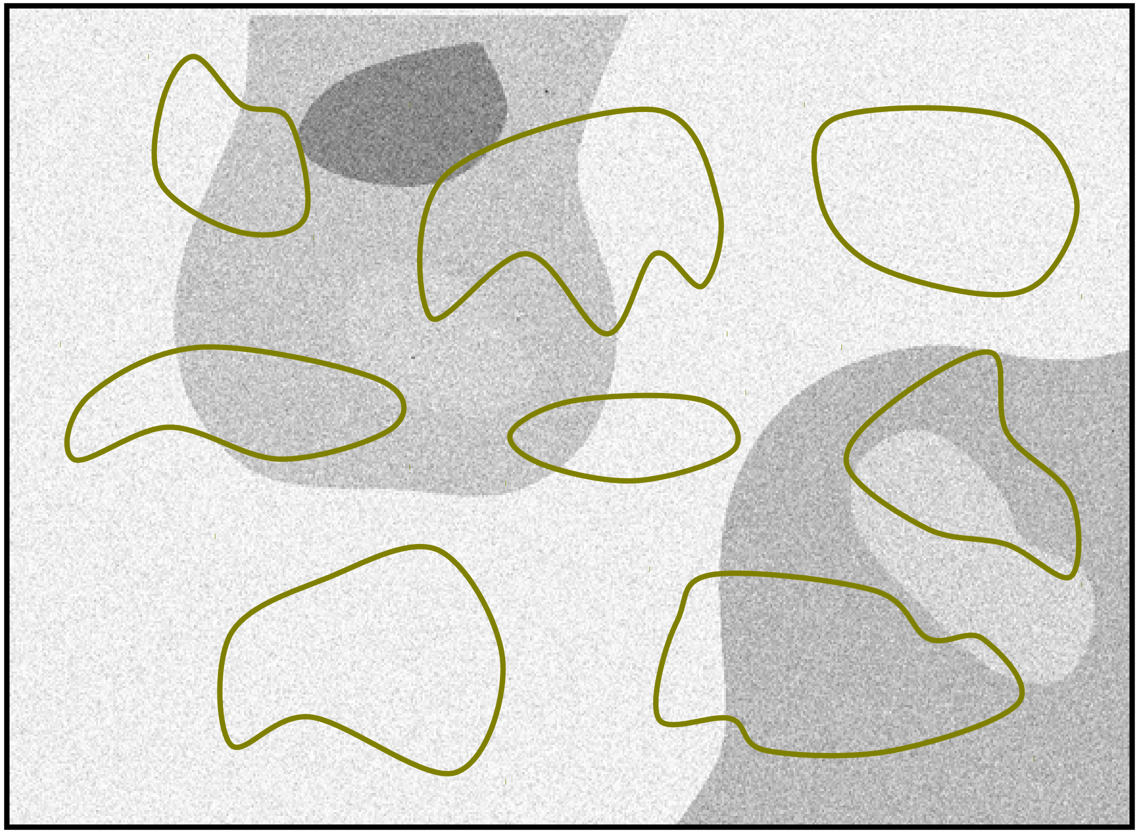
Multiphase Image Segmentation

Goal: Detect regions, boundaries, average values in regions.



Finding the Right Boundary Curves

Q: How do we decide which are the right curves?



Energy Minimization Formulation

Shape reconstruction usually formulated as energy minimization:

$$J(\Gamma) = Data(\Gamma, I(x)) + Prior(\Gamma) + Regular(\Gamma)$$

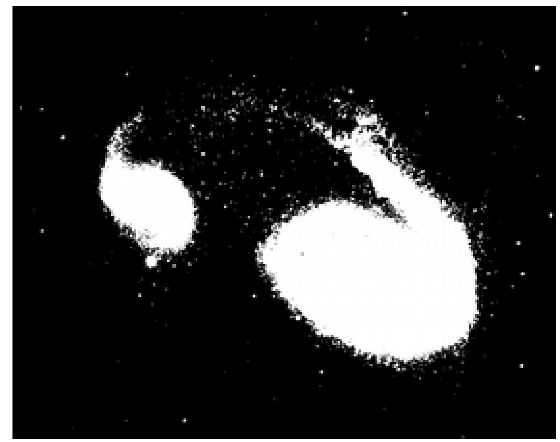
$Data(\Gamma, I(x))$: data fidelity term based on image data

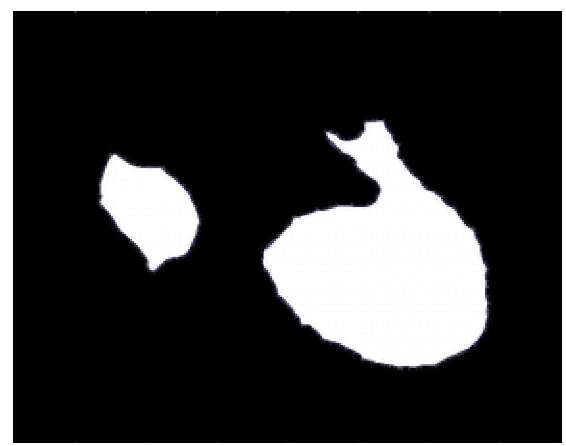
$Prior(\Gamma)$: prior knowledge on the shape

$Regular(\Gamma)$: regularization term for the shape

Motivation for Geometric Regularization

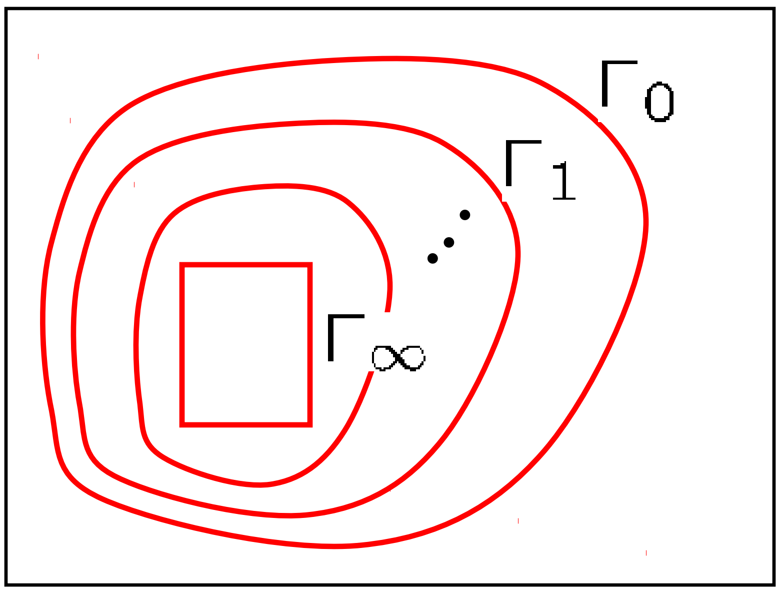
$$Regular(\Gamma) = \gamma \int_{\Gamma} dS \qquad \text{(length penalty)}$$





Shape Optimization

Given shape energy $J(\Gamma)$ for given shape Γ



Compute a sequence

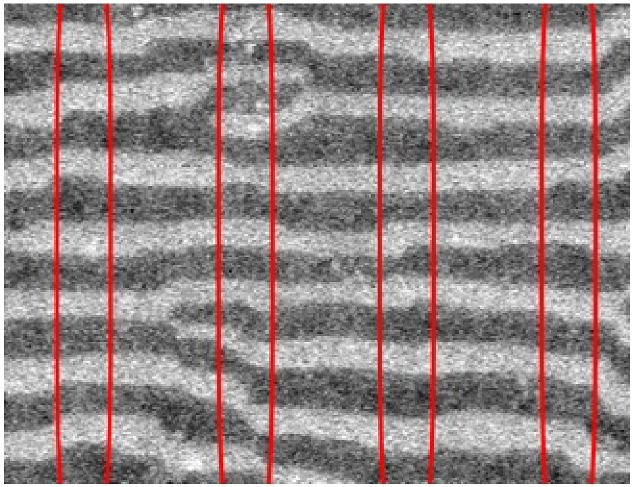
$$\Gamma_0, \Gamma_1, \ldots, \Gamma_\infty$$

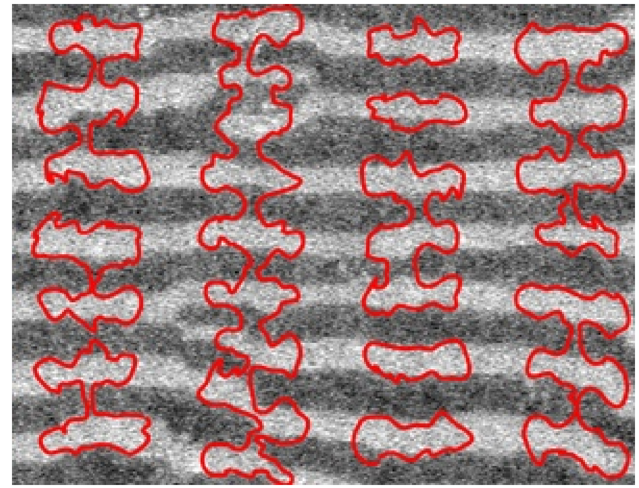
such that

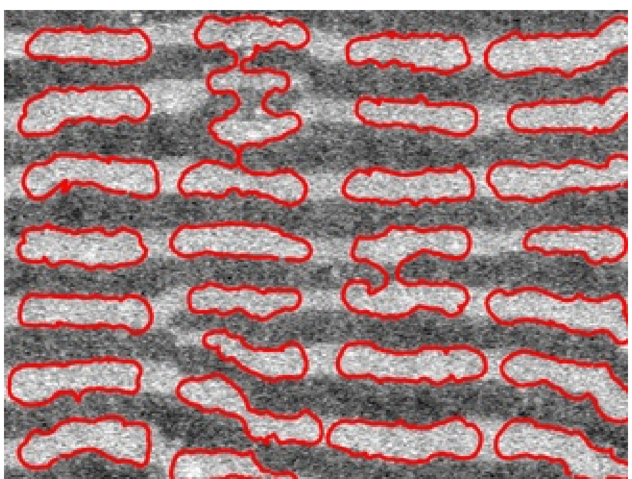
$$J(\Gamma_0) \geqslant \ldots \geqslant J(\Gamma_\infty)$$

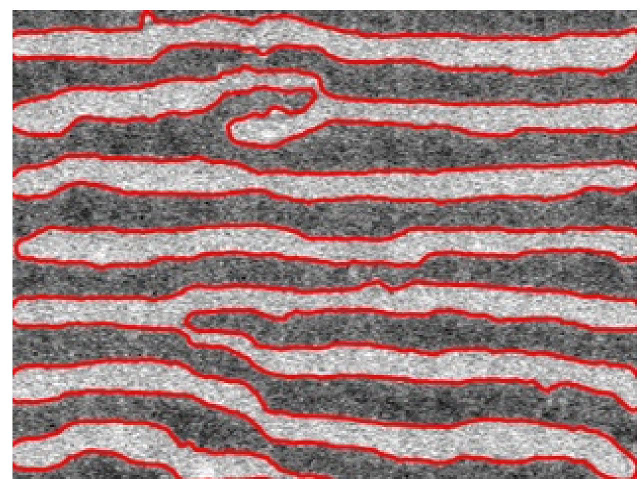
Example of Segmentation Process

Start



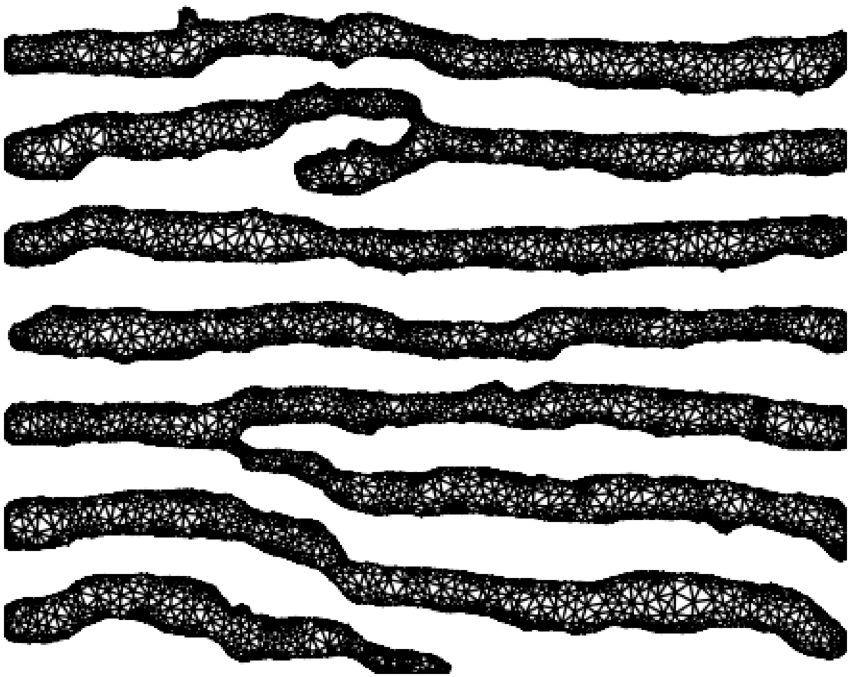


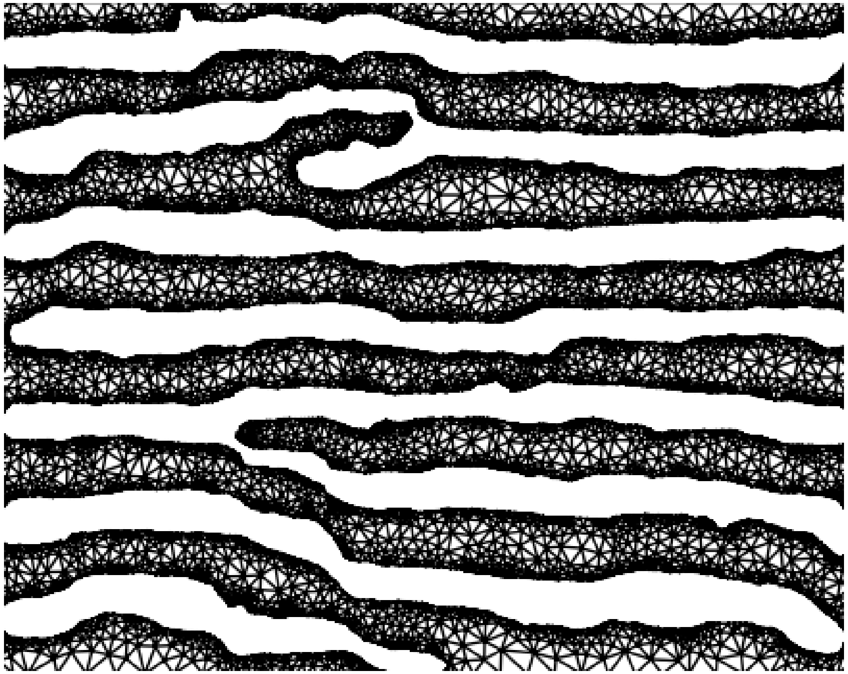




Final

Meshes from Segmentation



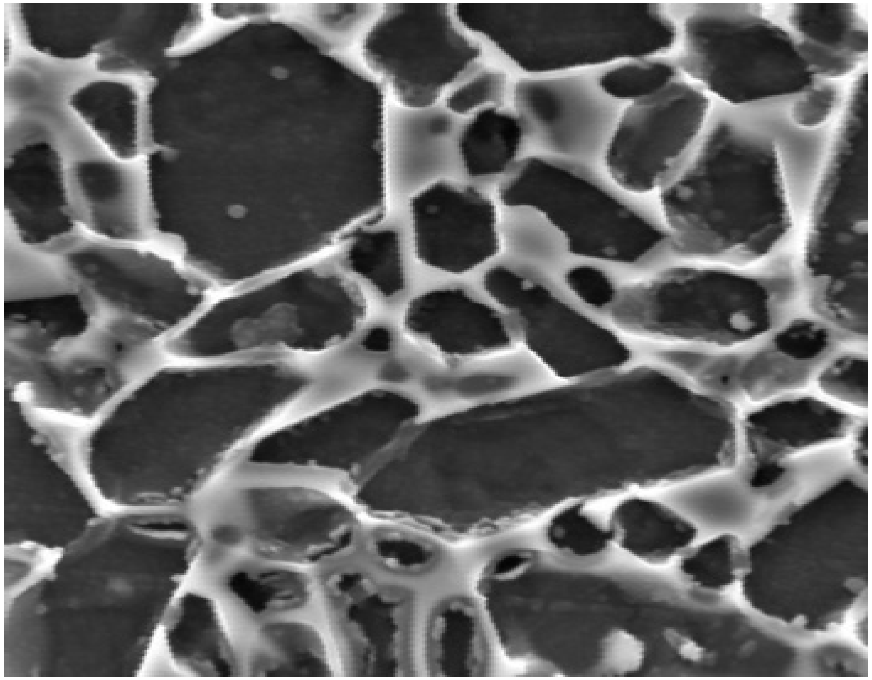


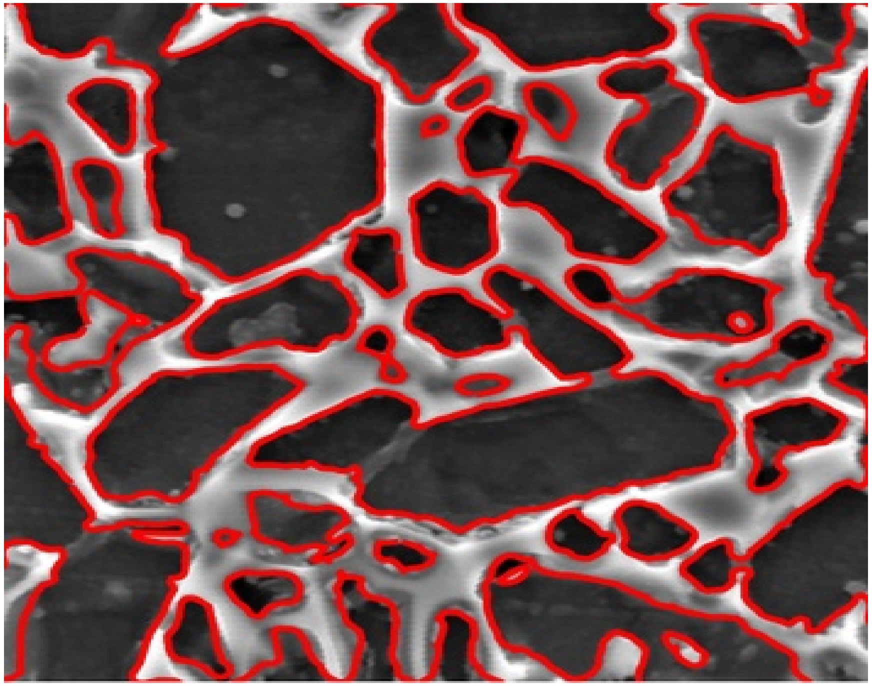
phase 1 : 14178 elements

phase 2 : 15276 elements

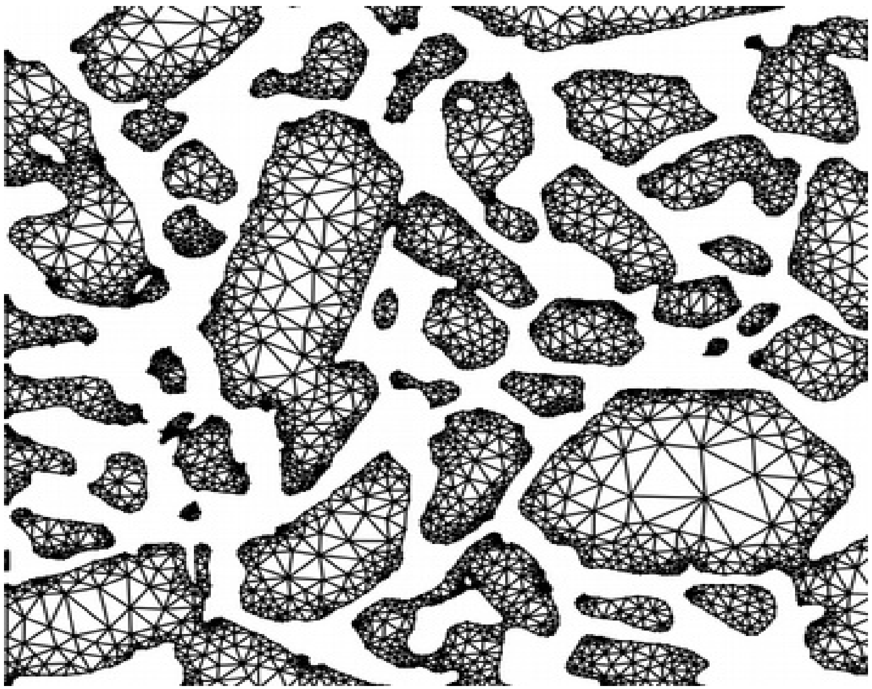
original image has 2850000 pixels

Segmentation of Microstructures

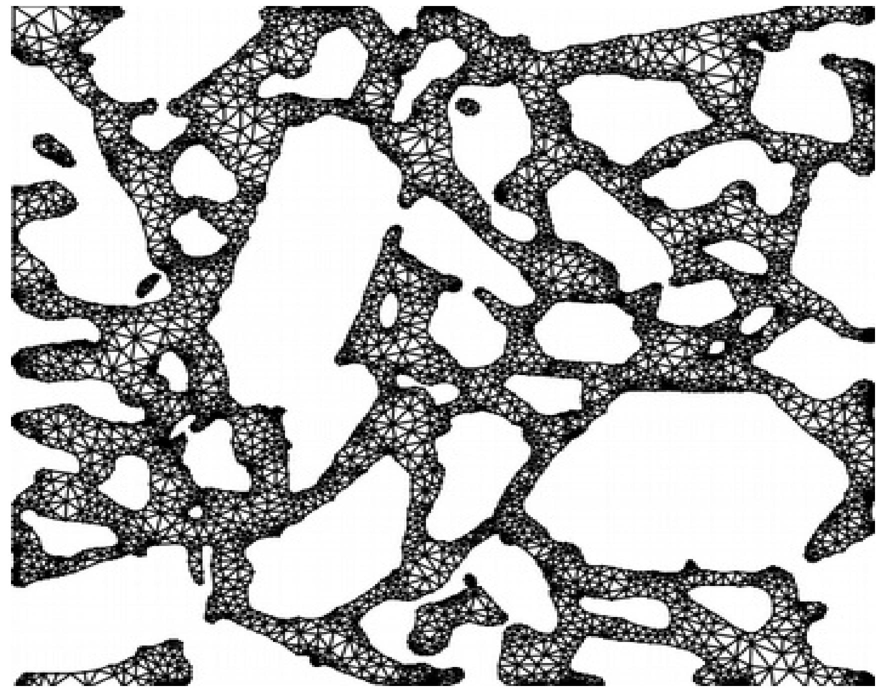




Meshes from Segmentation



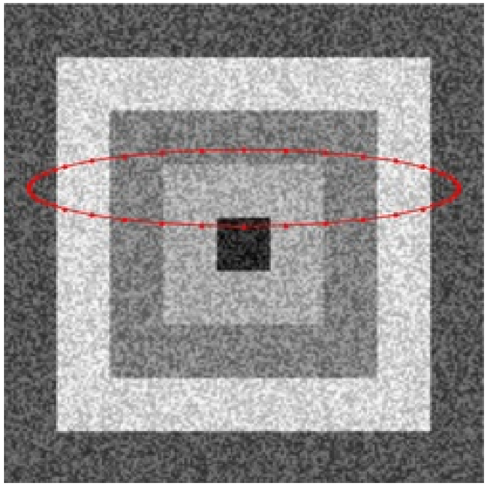
phase 1 : 9118 elements



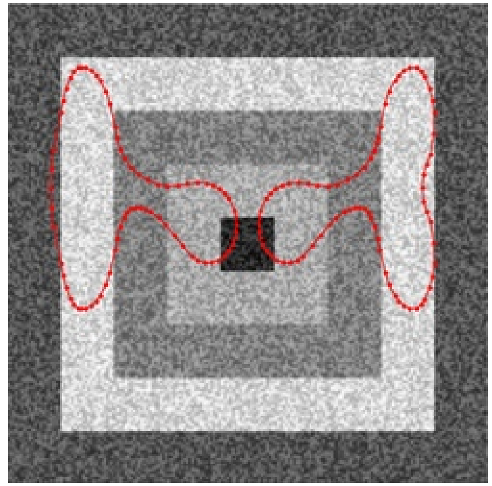
phase 2 : 8946 elements

original image has 1048576 pixels

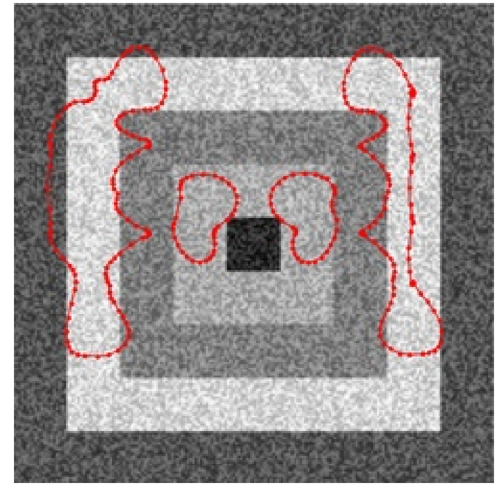
Multiphase Segmentation



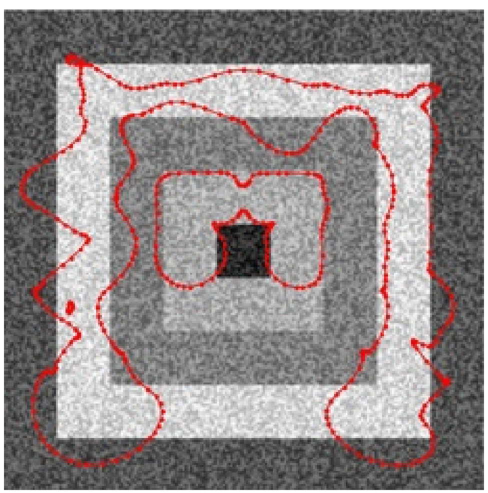
k=0, *2 phases*, J=-0.166



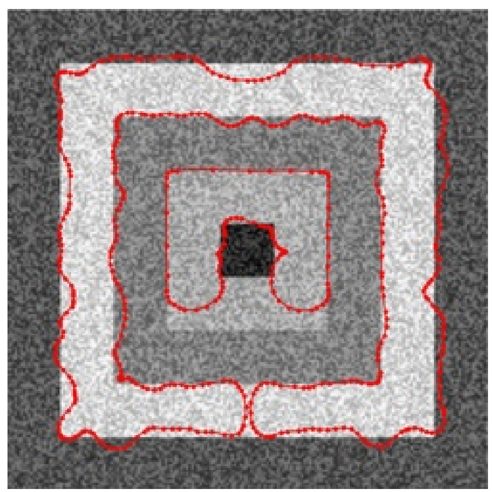
k=1, *3 phases*, J=-0.171



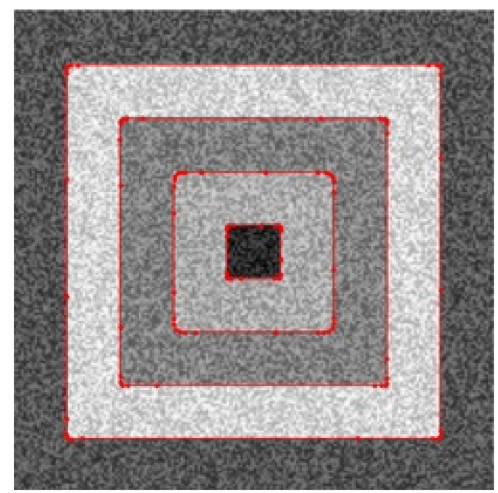
k=3, *5 phases*, J=-0.174



k=6, *4 phases*, J=-0.180



k=11, *3 phases*, J=-0.198

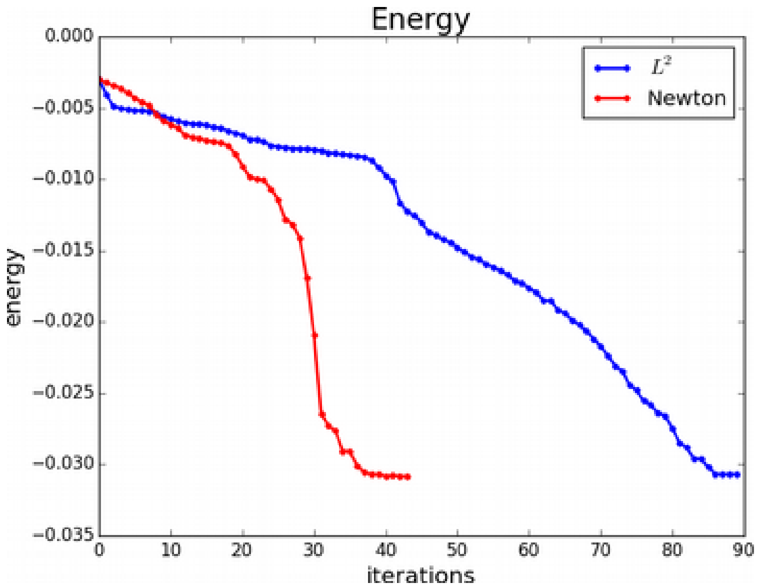


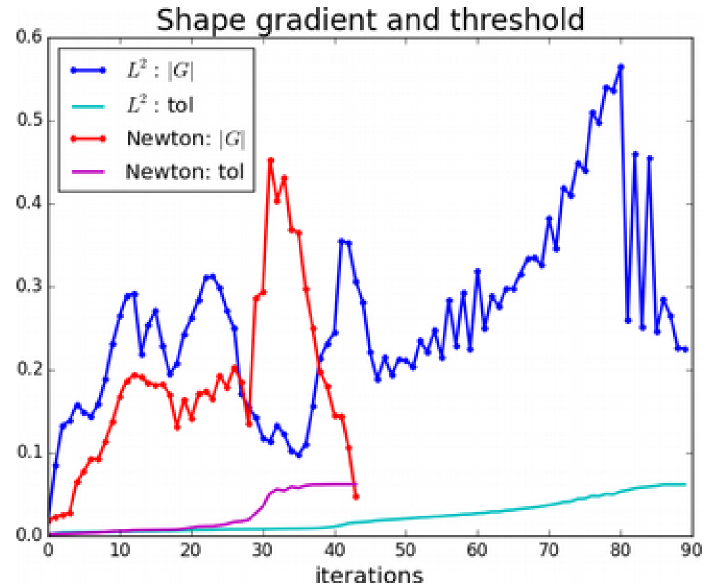
k=56, *5 phases*, J=-0.215

Effective Iterative Solution?

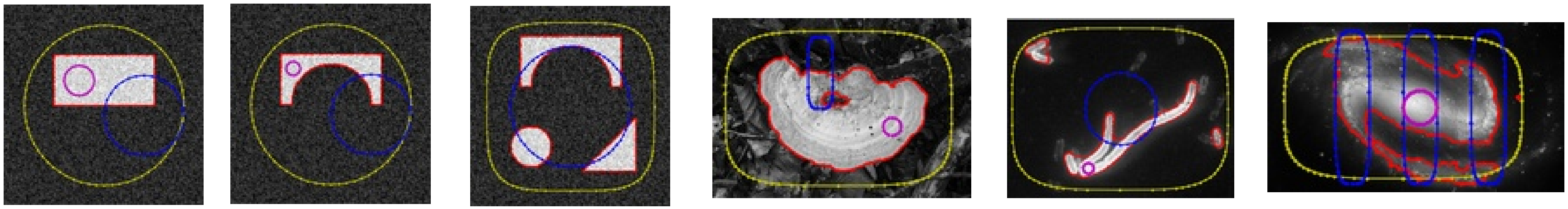
- Efficient representation for the problem and geometry
- (Choose a good starting configuration)
- Reduce number of iterations:
  - Stable gradient descent for larger steps
  - Aggressive energy decrease at each iteration
  - Practical stopping criterion
- Reduce cost of each iteration:
  - Economical adaptive representation for regions and boundaries
  - Velocity computation in linear time

Faster Convergence with Shape Newton





Faster Convergence with Shape Newton



	rectangle			concave			multiple	
	Γ_{in}	Γ_{out}	Γ_{part}	Γ_{in}	Γ_{out}	Γ_{part}	Γ_{out}	Γ_{part}
L^2 (m)	35 (16)	47 (23!)	33 (19!)	35 (18)	99 (58)	40 (19)	54 (35)	54 (26!)
L^2 (n)	46 (20!)	46 (24)	40 (19)	51 (31)	83 (51!)	45 (20!)	59 (33!)	56 (27!)
Nwt (m)	26 (19!)	**23 (17)**	**23 (17)**	40 (19!)	38 (32)	**26 (17)**	43 (36)	32 (22)
Nwt (n)	**22 (20)**	25 (19)	28 (21)	**27 (19)**	**37 (34)**	27 (18)	**40 (33)**	**28 (22)**

	fungus			bacteria			galaxy		
	Γ_{in}	Γ_{out}	Γ_{part}	Γ_{in}	Γ_{out}	Γ_{part}	Γ_{in}	Γ_{out}	Γ_{part}
L^2 (m)	44 (21)	63 (34!X)	49 (24!)	74 (40)	117 (60!X)	86 (47)	46 (26)	33 (20!X)	60 (38)
L^2 (n)	52 (29!)	54 (31!X)	35 (21!)	64 (35)	MAX!	64 (35)	54 (29!)	83 (46!)	56 (36)
Nwt (m)	**17 (14)**	**22 (21)**	**36 (15!)**	**27 (25)**	**33 (28)**	**12 (11)**	**26 (18)**	32 (25!)	**26 (22)**
Nwt (n)	**17 (14)**	**22 (21)**	60 (58)	**27 (25)**	**33 (28)**	**12 (11)**	34 (26)	**29 (28)**	37 (34)

Tables of energy evaluations (and iterations)

Topology Optim. for Statistical Energy

Statistical region energy for labeling pixels

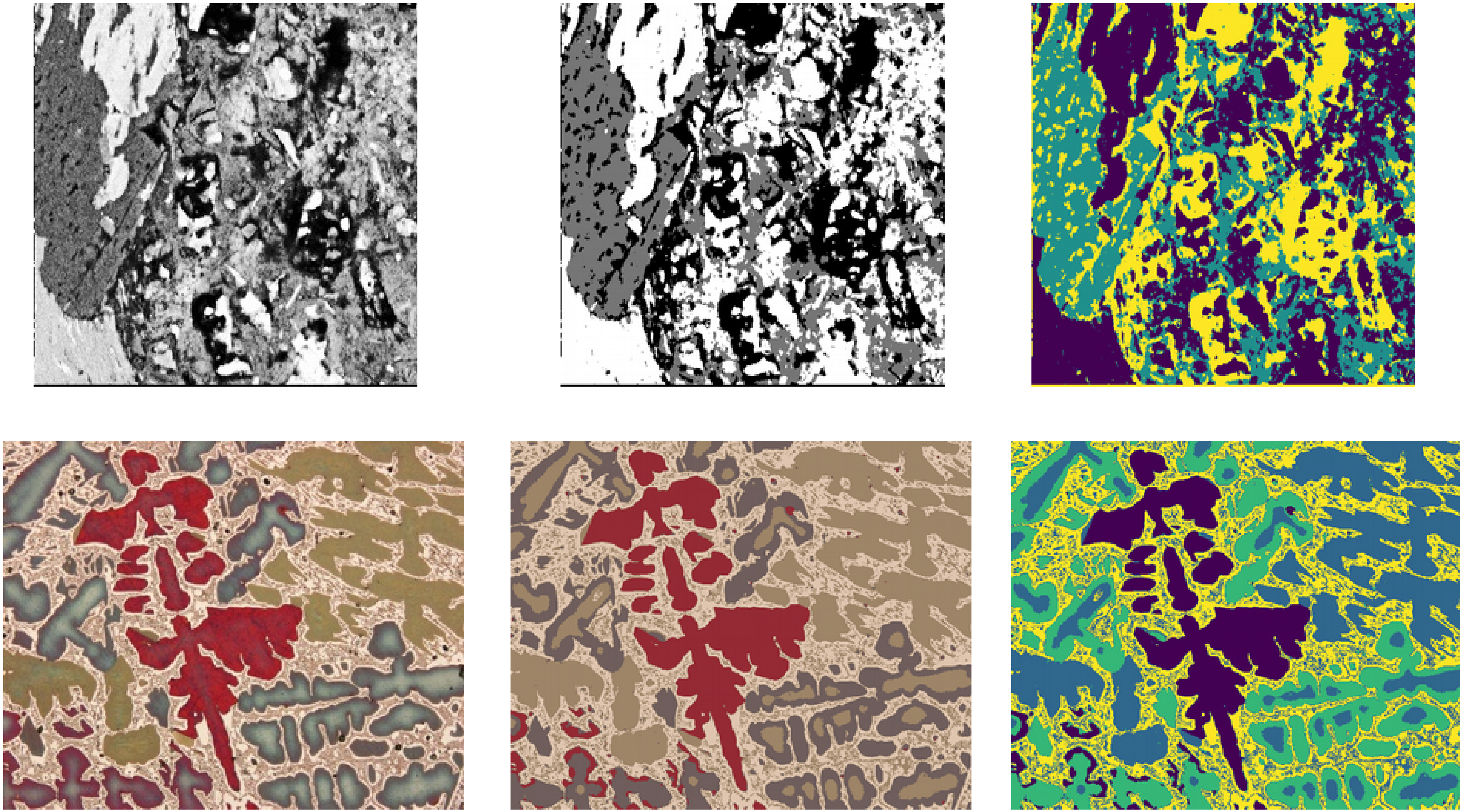
$$J(\{\Omega_i\}_{i=1}^{k}) = -\sum_{i=1}^{k} \int_{\Omega_i} \log p(I(x)|\theta_i) + \nu \sum_{i \neq j}^{k} \int_{\Omega_i} \int_{\Omega_j} w(x,y),$$

This has a statistical term & nonlocal region regularization.

Example of parametric probability distributions:

$$p(I(x)|\theta_i) = \frac{1}{2\sigma_i} e^{-\frac{|I(x)-\mu_i|}{\sigma_i}}, \quad \theta_i = (\mu_i, \sigma_i)$$

TP: Pwconst Segmentation



original image assigned colors segmentation labels

TP: Statistical Region Models

original image

pw. const image model

Gaussian distribution of pixels

Laplacian distribution of pixels

TP: Statistical Region Models

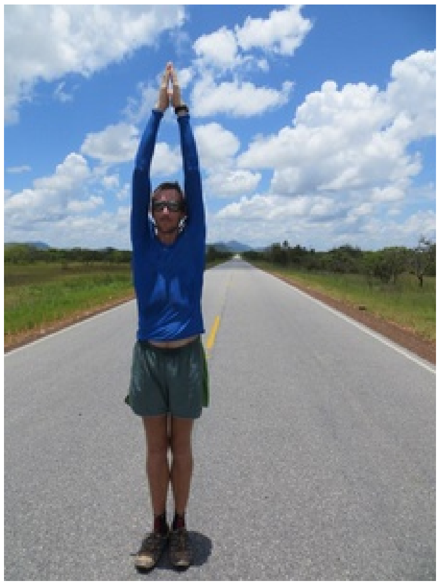

original image


pw.const. model


graph cuts


Laplacian distr.

TP: Statistical Region Models


original image


pw.const. model


graph cuts


Laplacian distr.

Part II - The Need for Shape Analysis

Real objects have shapes.

Analysis and inference on objects in data often relies on shapes of the objects (rather than plain numbers, vectors or matrices).

With shapes, we can ask questions such as:

- Does this object look like this other one?
- What is a typical object like in this data?

We need to quantify and compare shapes from data for many applications.

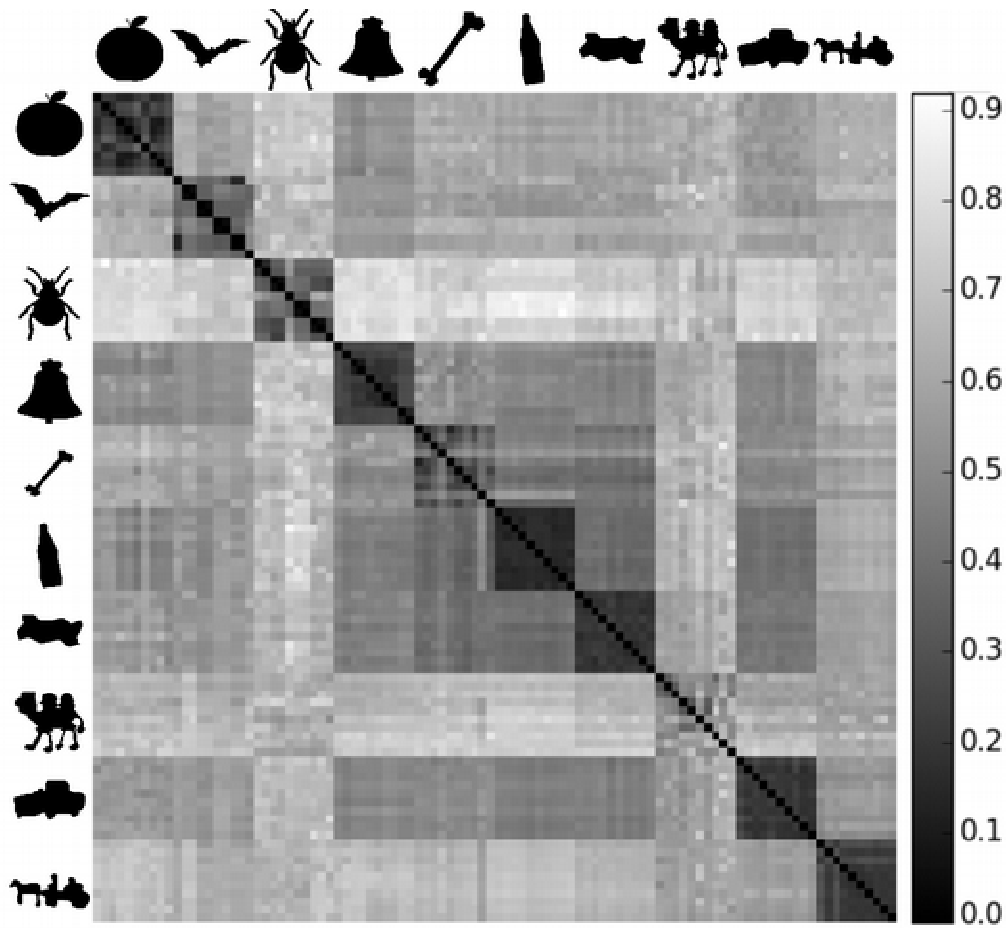
Many Applications

Many other areas and applications, in which objects in data are analyzed and evaluated based on shape:

- Security, surveillance,
- Biometry,
- Machine vision,
- Analysis of spectra, functional data
- Medical diagnostics,
- Cell biology, molecular biology

....

Example: Clustering with Shapes

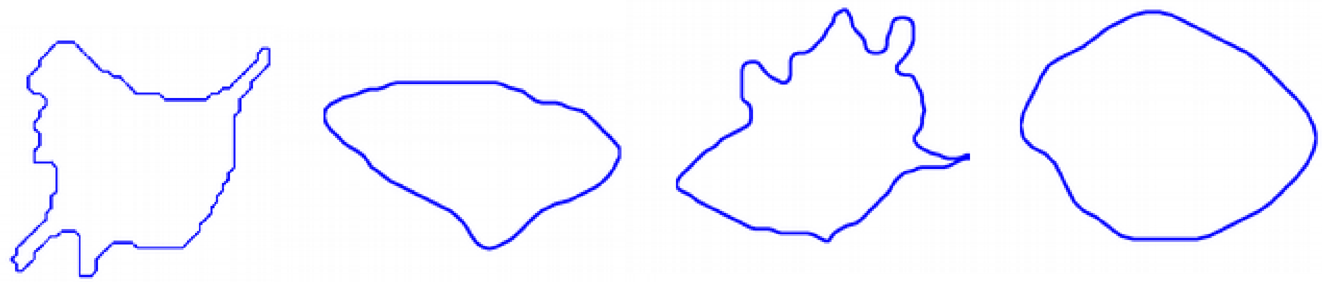


Pairwise distance matrix for MPEG7 shape data set

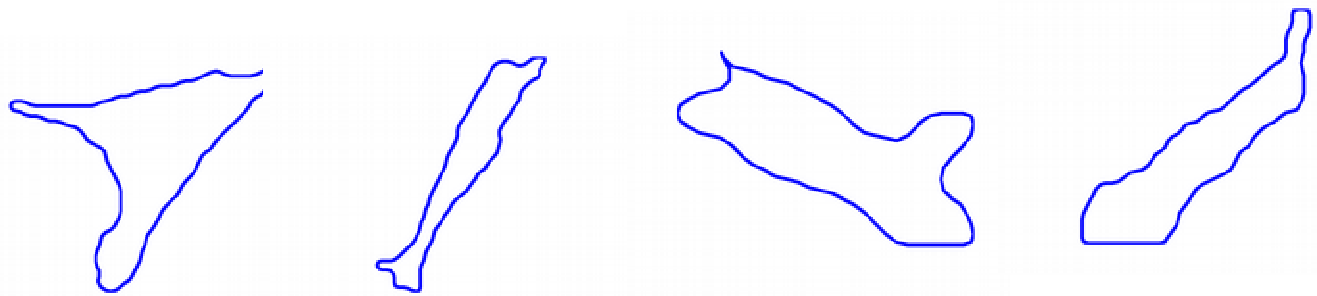
Example: Cell Populations

Classifying cell populations:

Sample DLEX

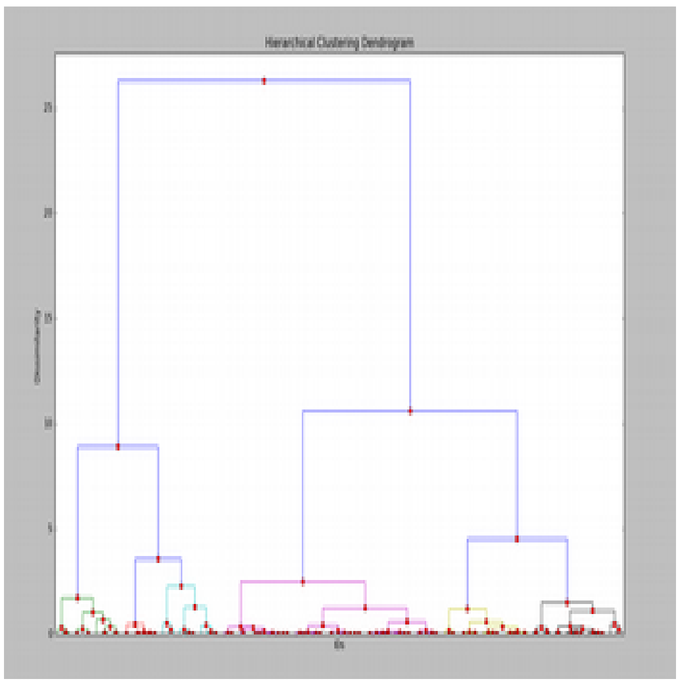


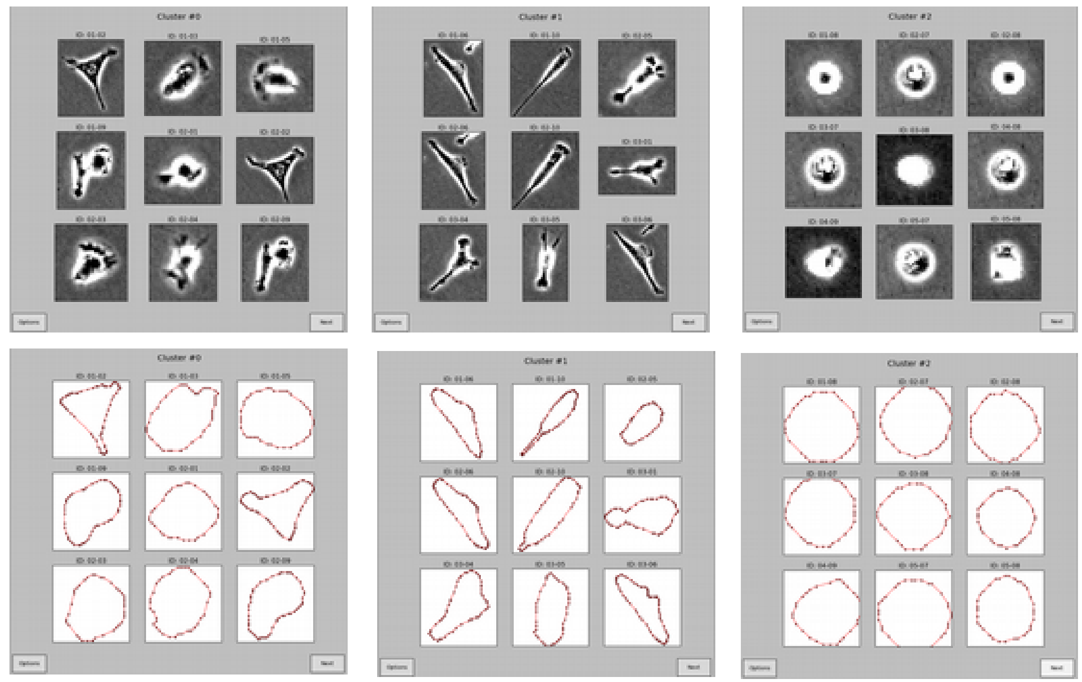
Sample NIH1



Example: Cell Populations

Grouping cell populations by shape to investigate cell function

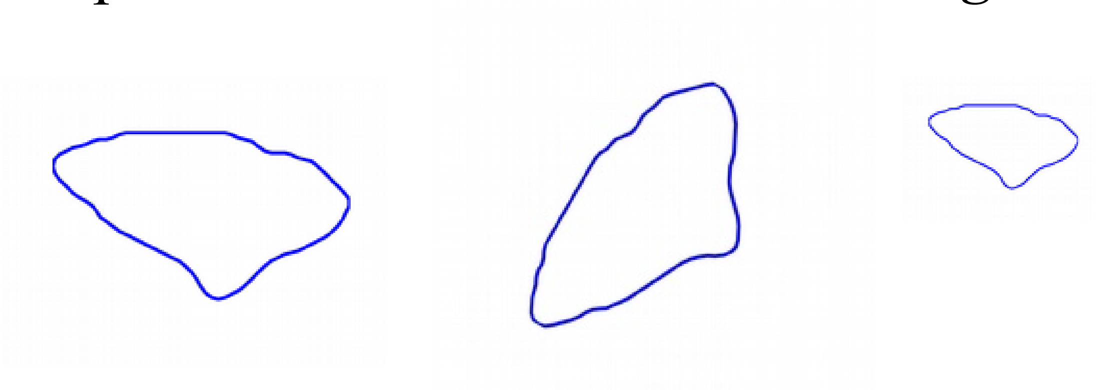




Dissimilarity of Closed Curves in 2d

How to define shape dissimilarity or distance for 2d curves?

Need representation invariant w.r.t. *scaling, translation, rotation.*

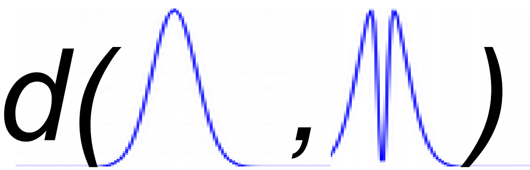
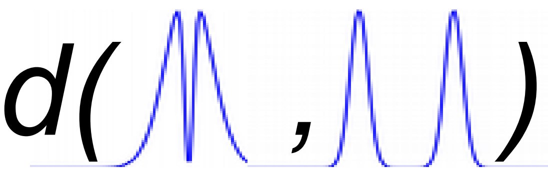


Moreover, we want a fast algorithm, because

- Complex curves can have a large number of nodes,
- Large-scale statistical shape analysis computes distance many many times.

Elastic Shape Distances

We want ***elastic*** shape comparisons.

	d()	*d*()
simple difference	*small*	*large*
elastic distance	*large*	*small*

Key trade-off:

- Simple formulations do not capture natural variability of shapes.
- Effective formulations are computationally expensive.

Minimization for Shape Distance

We write a more practical version of the minimization problem.
We need to find global minimizer $(t_0, \theta, \gamma(t))$ of

$$E(t_0, \theta, \gamma) = \int_0^1 \|q_1(t) - \sqrt{\dot{\gamma}(t)} R(\theta) q_2(t_0 + \gamma(t))\|^2 dt$$

so that we can compute shape distance of β_1, β_2 by

$$d([q_1], [q_2]) = d(\ q_1, \sqrt{\dot{\gamma}} R(\theta) q_2(t_0 + \gamma)\ )$$

(*we need to ensure* $\gamma(0) = 0, \gamma(1) = 1, \dot{\gamma} > 0$)

Tools for Image and Shape Analysis

- Efficient algorithms for image segmentation and geometry detection.
- Efficient algorithms for elastic shape analysis.
- Rigorous algorithms to advance measurement science.
- Impact on a various project in material science, biology, forensics.