MATH 125, Discrete Mathematics I

Some additional problems
Fall 2018 - George Mason University
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11th Problem Set

Extra-1: Show that the following are equivalent for a (general) graph G.

1. G has an *Eulerian trail*; a trail in G between two distinct vertices that goes through every edge in G exactly once.
2. G is connected and has exactly two vertices of odd degree.

Extra-2: Let G be an Eulerian graph with edge set $E(G)$ containing no loops. Show that the number of steps in the *Algorithm Eulerian Circuit* presented in class (to find an Eulerian circuit in G) is at most $|E(G)|/2$. If G is a simple graph, show that the number of steps is then at most $|E(G)|/3$.

Extra-3: Suppose that in each step in the *Algorithm Eulerian Circuit* the appended edges (the ones we add to the current circuit) are at least half of the edges in $E(G)$ not yet covered by the current circuit. Show that in this case the number of steps in the algorithm is $\lfloor \lg(|E(G)|) \rfloor + 1$, where lg is the base-2 logarithm (i.e. $\lg x = \log x / \log 2$ if log is the natural logarithm.)

Discussion: Monday, November 12.

November 7, 2018