Reciprocal complements: a new construction on integral domains

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Abstract

Let D be an integral domain and F its fraction field. The *reciprocal complement* $R(D)$ of D is the subring of F consisting of all finite sums of elements of the form $1/d$, $d \in D$. Surprisingly, $R(D)$ is always local. D is *Egyptian* if $R(D) = F$, or *Bonaccian* if $R(D)$ is a valuation ring. Any semilocal domain is Egyptian, and any Euclidean domain is Bonaccian, but in general $R(D)$ can have very surprising structure. For example, we showed with Loper that if D is a polynomial ring in $n \geq 2$ variables over a field, then $R(D)$ is non-Noetherian, non-integrally closed, and non-factorial, and primary decomposition is dramatically untrue, but dim $R(D) = n$. A new algebraic structure called a *regular factroid* (within D) can be used to describe which elements of F are in $R(D)$. Variants on the notion of irreducibility of elements of D can be explored in terms of (regular) factroids of D and prime ideals of $R(D)$. We investigate the structure of reciprocal complements, show ways to pass from the reciprocal complement of one domain to another, describe certain quotient domains of $R(k[x, y])$, and show an order-reversing one-to-one correspondence between the regular factroid subrings of D and the prime ideals of $R(D)$. – This work is joint with Lorenzo Guerrieri.

Keywords: integral domain, Egyptian ring, Bonaccian ring, factroid subring, prime ideal.