Egyptian integral domains

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Abstract

In ancient Egypt, the preferred way to represent a fraction was as the sum of reciprocals of distinct positive integers – e.g. $\frac{5}{9} = \frac{1}{2} + \frac{1}{18}$. Fibonacci showed that any fraction can be so represented. Accordingly, Guerrieri, Loper, and Oman call an integral domain D *Egyptian* if any element of its fraction field can be written as a sum of reciprocals of distinct elements of D. In a recent preprint [GLO], they expanded on Fibonacci's result by showing that various classes of domains are Egyptian or non-Egyptian.

In the current talk, I will speak of several generalizations of their results. For instance, I show that $D[X]_W$ is Egyptian whenever D is Egyptian and W contains a polynomial of positive degree. I show that any domain with a nontrivial positive grading fails to be Egyptian. I show that a domain that is an affine semigroup ring cannot be Egyptian unless the semigroup is a group. I also introduce the notions of *generically Egyptian* (i.e. some open localization is Egyptian) and *locally Egyptian* (i.e. Egyptian on an affine-local open cover of the prime spectrum) domains. With this, I can show that any finitely generated algebra over a locally (resp. generically) Egyptian domain is locally (resp. generically) Egyptian. I also show that ring-theoretic pullbacks of Egyptian (resp. generically Egyptian) domains are Egyptian (resp. generically Egyptian).

If there is time, we will explore the case of rings with zero-divisors.

Keywords: reciprocal, fraction, integral domain, polynomial, generically Egyptian, locally Egyptian.