What makes a diagram commute?

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Abstract

We are concerned with commutative diagrams in the usual sense: A **diagram** is a digraph whose vertices are objects in a category and whose arcs are morphisms between objects. A diagram **commutes** if any two route pairs from one vertex x to another vertex y compose to equal morphisms $x \to y$. We pose a simple question that has an unexpected answer: How can one most efficiently determine if a diagram commutes? For example, consider a diagram whose underlying digraph is the transitive tournament on n vertices, which has exponentially many route pairs. Does one have to check commutativity of all these pairs before deciding that the diagram commutes? If not, how many? Which ones? – We introduce the idea of a so-called **minimal CS-generating set** for a diagram, a smallest set $\mathcal{B}$ of route pairs for which commutativity on the elements of $\mathcal{B}$ propagates to commutativity of the entire diagram. For a given diagram, all such sets have the same size, which is no greater than $\binom{n-1}{2}$, for a diagram on n vertices (with equality holding precisely when the diagram is a transitive tournament). – This is joint work with Paul Kainen (Georgetown University).

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