

Ideals of skew-symmetric matrix Schubert varieties

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Abstract

We consider the set of skew-symmetric matrices whose upper-left $i \times j$ corner has rank at most \mathbf{r}_{ij} , for some integer matrix \mathbf{r} and all i, j . Such a set is defined by the vanishing of a collection of Pfaffians of submatrices, but it is not obvious that these Pfaffians generate a prime ideal, and they do not usually make up a Gröbner basis for the ideal they generate (with respect to reverse lex order). We prove that these ideals are prime (assuming a simple combinatorial condition on \mathbf{r}), and show how to extend the Pfaffian generating set to a Gröbner basis. We also show that their initial ideals are squarefree, describe the associated Stanley-Reisner complexes in terms of combinatorial objects called *skew-symmetric involution pipe dreams*, and prove that these complexes are shellable. Based on joint work with Eric Marberg.

Keywords: Pfaffian, prime ideal, Gröbner basis, Stanley-Reisner complex, shellability.