

Tiling, Terry Tao, Group Characters, and Fuglede's Conjecture

Samuel Ferguson, Metron, Inc., Reston, VA – 20190

Abstract

We first historically motivate the statement of Fuglede's conjecture. We mention that a quantum-mechanical question of Von Neumann was recast as an operator-theoretic problem of Segal which, in turn, was shown to be equivalent to a Fourier-theoretic problem by Fuglede. Fuglede conjectured a relationship between tiling and Fourier series in Euclidean spaces which, if true, would solve it. We suggest how Terry Tao found counterexamples to the analogue of Fuglede's conjecture in finite abelian groups, where group characters replace the Fourier basis functions; we sketch how he used them to disprove Fuglede's conjecture in Euclidean spaces of high dimension. Finally, we obtain sharper results than Tao in all finite abelian groups of the form he considered, by exhibiting counterexamples in spaces of lower dimension than Tao's. Our counterexamples are sets which do not tile, but whose function spaces have orthogonal bases of group characters. If time permits, we will also discuss a simple graph-theoretic algorithm we have developed, which has allowed us to prove the validity of Fuglede's conjecture in vector spaces of dimension at most six over the field with two elements. This talk is based on published joint work with Nat Sothanaphan which may be obtained from: <https://arxiv.org/abs/1901.08734>

Keywords: tiling, Fourier series, finite abelian group, group character, graph theory algorithm.