On the number of discrete chains

Eyvindur Ari Pálsson, Virginia Tech, Blacksburg, VA 24061

Abstract

One of Erdős' greatest contributions to geometry was his problem on distinct distances where he asked: what is the least number of distinct distances among N points? This seemingly innocent question inspired many other related questions, such as the Erdős unit distance problem, which asks: how often can a particular distance at most arise among N points? In this talk we will recount the history of these questions and then focus on a new result that studies a generalization of the Erdős unit distance problem to chains of k distances. In particular, given $\mathcal{P}$, a set of N points, and a sequence of distances $(\delta_1, \ldots, \delta_k)$, we study the maximum possible number of tuples of distinct points $(p_1, \ldots, p_{k+1}) \in \mathcal{P}^{k+1}$ satisfying $|p_j p_{j+1}| = \delta_j$ for every $1 \leq j \leq k$. We derive upper and lower bounds for this family of problems.

Keywords: discrete geometry, Paul Erdős' distance problem.