

# A directed Steinitz theorem for oriented matroid programming

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## Abstract

Holt and Klee proved that if  $P$  is a  $d$ -dimensional polytope and  $f$  is a linear function on  $P$  that is not constant on any edge of  $P$ , there are  $d$  independent monotone paths from the source to the sink of the digraph defined by the vertices and edges of  $P$  directed according to the directions of increase of  $f$ . Mihalisin and Klee proved that every orientation of the graph of a 3-polytope that is acyclic and admits 3 independent monotone paths from the source to the sink is obtained from some 3-polytope  $P$  and some linear function  $f$  on  $P$ . We prove analogs of Mihalisin and Klee's theorem and the 3 and 4-dimensional versions of Holt and Klee's theorem for oriented matroid programs. Here acyclicity is replaced by the requirement that there be no directed cycle contained in a face of the polytope.

**Keywords:** polytope, directed path, source, sink.