

Prime labelings of generalized Petersen graphs

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Abstract

A *prime labeling* of a graph G on n vertices is an assignment of the integers $1, 2, \dots, n$ to the vertices of G such that the labels on adjacent vertices are relatively prime; if G admits a prime labeling then we say G is a *prime graph*. In this talk, we explore the problem of determining if the generalized Petersen graph $P(n, k)$ is prime. A prime labeling of $P(n, 1)$ for even n is provided on the condition that a certain number theoretic conjecture is true; we have demonstrated via computer that this conjecture — which bears resemblance to Bertrand's Postulate and the Goldbach Conjecture — is true for all even $n \leq 2 \times 10^9$. For $k > 1$, an algorithmic approach to labeling $P(n, k)$ is demonstrated; using this approach, we can show $P(n, k)$ is prime for all even $n \leq 50$ and all odd $3 \leq k < n/2$.

Keywords: simple graph, prime labeling, generalized Petersen graph.