Various generating functions, a unifying approach

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Abstract

When investigating various discrete number sequences, it is often fruitful to consider their corresponding generating functions, or exponential generating functions, or their Bell series functions, or... When to use which, often seems to be determined on a whimsical case-by-case basis. Hence, it is natural to ask if there is a unifying algebraic object that reduces to the algebra of various types of generating functions in each special case, and perhaps shows better when which type works better than others. It turns out that, to a large degree, the dual *incidence algebra* $I(P) = C^*(P)$ of a natural coalgebra $(C(P), \Delta, \epsilon)$ provides such a unifying object. Here $C(P)$ is the free $\mathbb{Z}$-module generated by all intervals of a locally finite poset P. We show how $I(P)$ generalizes the notion of a generating function, and how we can often obtain a compact answer to various enumeration questions directly from $I(P)$. – This was first discussed by Goldman and Rota in 1970.

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